Exact CSR Wakes for the 1-D Model

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Abstract

The forces from Coherent Synchrotron Radiation (CSR) on the source particle bunch can be computed exactly for a line charge. Modeling a finite bunch by a line charge often produces a very good model of the CSR forces, and the full bunch can then be propagated under these forces. This 1-D model of CSR has often been used with a small angle approximation, an ultra relativistic approximation, and the approximation that radiation originating in one dipole can be neglected in the next dipole. Here we use Jefimenko's forms of Maxwell's equations, without such approximations, to calculate the wake-fields due to the longitudinal CSR force in multiple bends and drifts. Several interesting observations are presented, including multiple bend effects, shielding by conducting parallel plates, and bunch compression.

INTRODUCTION

Coherent Synchrotron Radiation (CSR) is an important detrimental effect in modern particle accelerators with high bunch charges and short bunch lengths. It is a collective phenomenon where the energy radiated at wavelengths on the order of the bunch length is enhanced by the number of charges in the bunch. The energy radiated is energy lost by individual particles, which subsequently affects their motion.

This paper is a collection of results from Ref. [1], to which the reader is referred for a full exposition. It uses a 1-dimensional model that projects the transverse particle density onto the longitudinal dimension, and calculates the CSR force from the electric field on this line.

Unfortunately, the electromagnetic fields on the world-sheet of a charged line are singular. Pioneering efforts described in Refs. [2, 3] circumvent this problem by examining the non-singular terms only. In more detail, Ref. [4] regularizes the longitudinal force between two charges traveling on the arc of a circle by subtracting off the Coulomb force, calculated as if the charges traveled on a straight line, from the force calculated using Liénard-Wiechert fields. Such an approach is what is currently used in the particle tracking codes Elegant [6] and Bmad [7].

EXACT 1-D MODEL

The exact 1-D approach in Ref. [1] uses the less widely known Jefimenko forms of Maxwell's equations [5], which allow one to calculate electromagnetic fields by directly using the evolving charge and current densities, and which internally incorporate all retardation effects. According to these equations, for given charge and current density $\rho(\mathbf{x},t)$ and $\mathbf{J}(\mathbf{x},t)$ at position $\mathbf{x}$ and time t, the electric field $\mathbf{E}(\mathbf{x},t)$ is

$$\mathbf{E}(\mathbf{x},t) = \frac{1}{4\pi\epsilon_0}\int \mathrm{d}^3x' \left[\frac{\mathbf{r}}{r^3}\,\rho(\mathbf{x}',t') + \frac{\mathbf{r}}{c\,r^2}\,\partial_{t'}\rho(\mathbf{x}',t') - \frac{1}{c^2\,r}\,\partial_{t'}\mathbf{J}(\mathbf{x}',t')\right]_{t'=t-r/c}, \quad (1)$$

in which $\mathbf{r} \equiv \mathbf{x}-\mathbf{x}'$, $r \equiv \|\mathbf{r}\|$, ϵ_0 is the vacuum permittivity and t' is the retarded time. In this formulation, the retarded points $\mathbf{x}'$ and times t' are independent variables, so there are no functions that need to be inverted. Therefore, if one knows ρ, $\dot\rho$, and $\dot{\mathbf{J}}$ at all points in space $\mathbf{x}'$ and times $t' \le t$, with a dot denoting the time derivative, then this formula gives the electric field by direct integration.

Now consider a line charge distribution, which follows a path $\mathbf{X}(s)$ parameterized by distance s, has a unit tangent $\mathbf{u}(s) = d\mathbf{X}(s)/ds$, and moves with constant speed βc along this path. A bunch with total charge Q and normalized line density λ therefore has one-dimensional charge density and current

$$\begin{aligned}\rho(s,t) &= Q\,\lambda(s-s_b-\beta\,ct),\\ \mathbf{J}(s,t) &= Q\,\beta c\,\mathbf{u}(s)\,\lambda(s-s_b-\beta\,ct),\end{aligned} \quad (2)$$

where s_b is the location of the bunch center at time $t=0$.

The rate of energy change per unit length of an elementary charge q at position s is $d\mathcal{E}/ds = q\,\mathbf{u}(s)\cdot\mathbf{E}(s,t)$. Functions of this type are called wake-fields, and for CSR

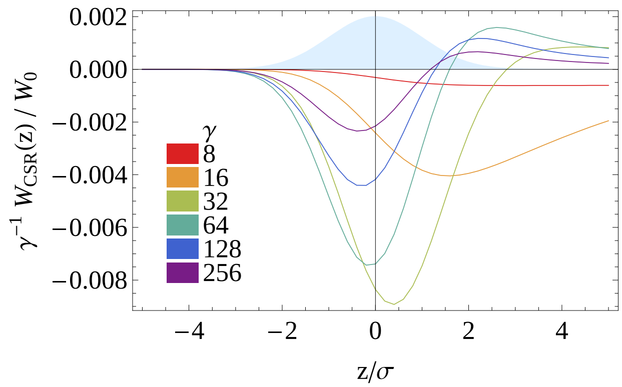

Figure 1: The steady-state CSR-wake for various relativistic γ using the exact 1-D model. Here $\kappa\,\sigma = 3\times10^{-5}$ for a Gaussian bunch, represented in light blue. The wake is scaled by W_0 defined in Eq. (6), as well as γ.

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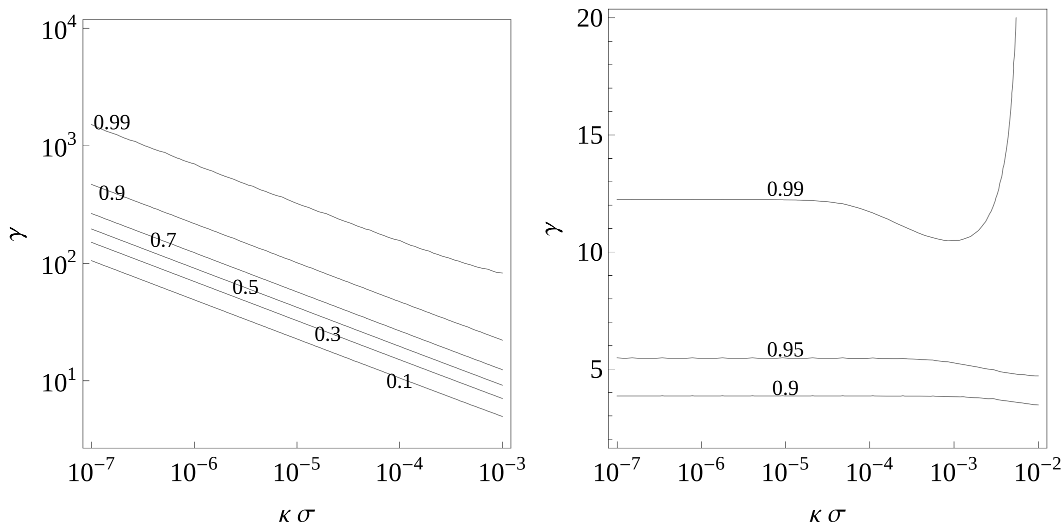

Figure 2: The ratio of the average energy lost by a Gaussian bunch using the exact CSR-wake to that using the ultra-relativistic, small angle approximation (left plot) described in Ref. [6] and to that using a better approximation (right plot) described in Ref. [7]. Lower numbers therefore imply an overestimation of the CSR effect.

we call them CSR-wakes. Using Eq. (1) with the one-dimensional bunch in Eq. (2) gives

$$\frac{d\mathcal{E}}{ds}(s,t) = N r_c m c^2 \int_{-\infty}^{\infty} \mathrm{d}s' \left[\frac{\mathbf{u}(s)\cdot\mathbf{r}(s,s')}{r(s,s')^3}\lambda(s_r) - \beta\frac{\mathbf{u}(s)\cdot\mathbf{r}(s,s')}{r(s,s')^2}\lambda'(s_r) + \beta^2\frac{\mathbf{u}(s)\cdot\mathbf{u}(s')}{r(s,s')}\lambda'(s_r)\right], \tag{3}$$

with the definitions $s_r \equiv s' - s_0 + \beta\, r(s,s')$, $s_0 \equiv s_b + \beta\, ct$ $\mathbf{r}(s,s') \equiv \mathbf{X}(s) - \mathbf{X}(s')$, and $r(s,s') \equiv \|\mathbf{r}(s,s')\|$. Additionally, $N = Q/q$ is the number of elementary particles with mass m and classical radius $r_c = q^2/\left(4\pi\epsilon_0 m c^2\right)$, and the prime on λ indicates a derivative of this function with respect to its argument, *i.e.* $\lambda'(x) = d\lambda/dx$. Here s_0 is the center of the bunch at time t, and this is the only place where the time dependence appears.

Unfortunately, the integral in Eq. (3) diverges as $s - s' \to 0$, which is a consequence of the one-dimensional line charge model. This problem can be alleviated by using the regularization procedure originating in Saldin *et al.* [4], where the electric field $\mathbf{E}$ is split into two parts as $\mathbf{E} = \mathbf{E}_{\text{CSR}} + \mathbf{E}_{\text{SC}}$. The space charge (SC) part $\mathbf{E}_{\text{SC}}$ is the electric field of a line charge moving on a straight path, and subtracting this from $\mathbf{E}$ results in the always finite Csr-wake

$$\left(\frac{d\mathcal{E}_{\text{CSR}}}{ds}\right) = q\,\mathbf{u}(s)\cdot\left[\mathbf{E}(s,t) - \mathbf{E}_{\text{SC}}(s,t)\right]. \tag{4}$$

We are usually concerned with the field near the bunch center at s_0, so the notation

$$W_{\text{CSR}}(z) \equiv \frac{d\mathcal{E}_{\text{CSR}}}{ds}(s_0 + z) \tag{5}$$

is used to refer to the CSR-wake immediately surrounding the bunch center at s_0.

RESULTS

Steady-State

Due to the rotational symmetry, there will be an angle into a bending magnet beyond which the CSR-wake does not change. This is often referred to as the steady-state regime. The exact CSR-wakes in this case are shown in Fig. 1 for various energies, and one sees that CSR-wake changes form for low energies. All CSR-wakes are normalized by

$$W_0 \equiv N r_c m c^2 \frac{(\kappa\,\sigma)^{2/3}}{\sigma^2}, \tag{6}$$

in which κ is the curvature (inverse radius) of the bending magnet, and σ is the bunch length. For ultra-relativistic ($\gamma \to \infty$) particles, the steady-state CSR-wake is always proportional to this factor and, for example, the average energy loss per unit length (per particle) of ultra-relativistic Gaussian bunch is approximately $-0.35\,W_0$.

The exact model can be used to obtain the limits of validity of the approximations used in particle tracking codes. This is done in Fig. 2, which shows that the method used in Elegant [6] is invalid at low energies and very short bunches, while the method used in Bmad [7] is more accurate.

Multiple Bends with Shielding

The exact 1-D model can be used for any orbit, including orbits through multiple bends separated by drifts. To quantify how the CSR-wake changes through a bend, we plot the average $\langle W_{\text{CSR}}\rangle$ and standard deviation σ_W of the CSR-wake over a Gaussian bunch distribution in Fig. 3 as the bunch center s_0 progresses through a bend. The non-zero energy losses when the bunch is at the entrance of the bend ($s_0 = 0$) show the influence of a previous bend.

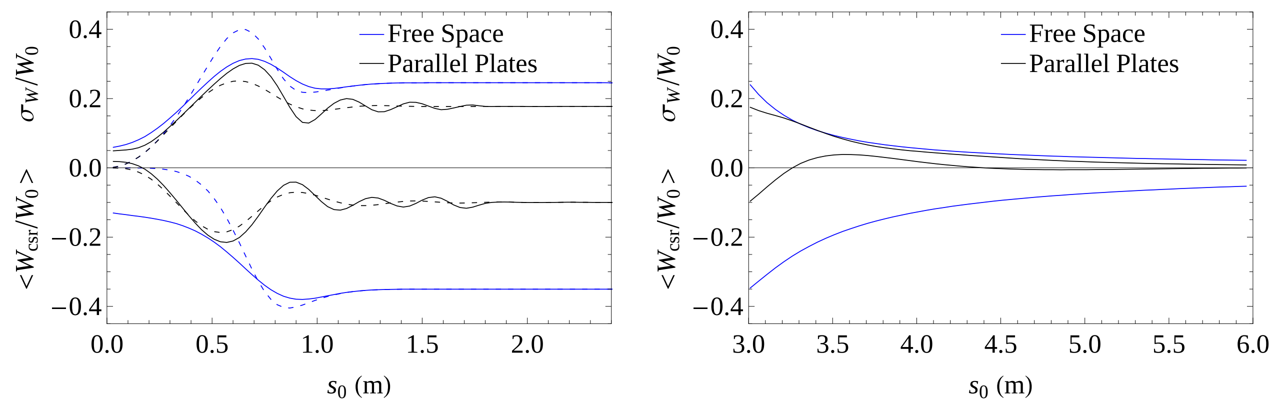

Figure 3: The average energy loss and energy spread induced, per unit length, of the CSR-wake for a Gaussian bunch through the length of a bend (left plot) and in the drift region (right plot) following that bend. Solid lines take into account a previous bend separated by 1 m from the current bend, while dashed lines have no previous bend. The bending radii are 10 m, and the bunch has a length $\sigma = 0.3$ mm with an energy of 1 GeV. In this example, shielding by parallel plates (separated by 2 cm) drastically reduces the energy loss, but only moderately reduces the energy spread, when compared to the free space calculations.

The presence of a conducting beam chamber can strongly change CSR wake-fields. If particle trajectories are planar within a chamber of finite height and infinite width, then such a chamber can be modeled exactly using the image charge method for infinite parallel plates. The electric field due to these image charges can be calculated directly using Eq. (3), and does not need to be regularized. Calculations of shielding with multiple bends are shown in Fig. 3. There we see that shielding can strongly suppress the total energy loss, but less strongly suppress the energy spread induced (integrated σ_W), from the coherent radiation.

Also shown in Fig. 3 are these calculations in the exit drift following the bend. There we see that shielding greatly reduces the total energy loss, but the energy spread induced is similar to the free space calculation.

Bunch Compression

Bunch compression can be achieved in a bending magnet if there is a correlation between energy and longitudinal position of particles in the bunch. To exactly calculate CSR for this, however, requires at least a 2-dimensional model, because particles of different energies travel on different orbits. In the framework of the 1-D model described by Eq. (3), this effect can be approximately modeled by allowing the bunch length to be time dependent, and neglecting variations in the velocity βc.

Calculations for a compressing bunch between parallel plates in Fig. 4. Method 1 uses the exact 1-D model. Method 2 calculates the CSR-wake at any s_0 *as if* it always had the same length in the past. This is the approach used in Elegant [6] and Bmad [7]. Finally, for reference, Method 3 calculates the CSR-wake for a bunch that always has the compressed length. In this example, Method 2 overestimates the CSR effect compared to the more realistic Method 1.

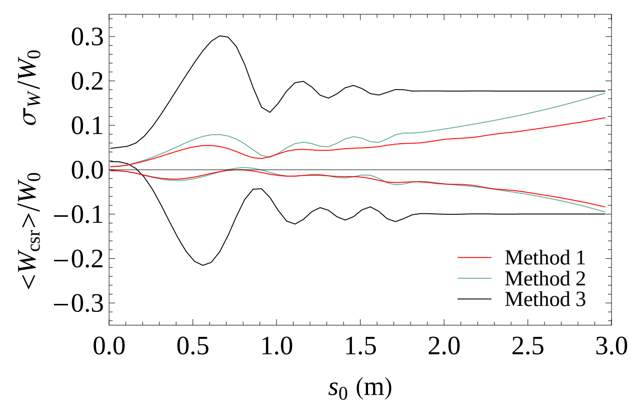

Figure 4: Similar to Fig. 3, but with a bunch that compresses linearly from $\sigma = 0.9$ mm to $\sigma = 0.6$ mm through a previous bend, travels through a 1 m drift, and then compresses linearly from $\sigma = 0.6$ mm to $\sigma = 0.3$ mm in the plotted bend. The methods are described in the text.

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