

# Advanced Accelerator Physics and Accelerator Simulation Homework 4

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## Exercise 1:

Consider a box shape solenoid field. On the central axis, the solenoid field is given by

$$\vec{B}(z) = \begin{cases} B_0 \vec{e}_z & \text{for } z \in [0, L] \\ 0 & \text{else} \end{cases} \quad (1)$$

(a) A particle flies into the solenoid parallel to the central axis with a horizontal distance  $x_0$ . Describe its trajectory after the solenoid.

(b) If it touches the central axis somewhere after the solenoid, where would that be? How does the focal length depend on the field  $B_0$  and the length  $L$ ?

(c) Show that the magnetic field  $\vec{B} = \{\frac{x}{2}\Psi_0'', \frac{y}{2}\Psi_0'', -\Psi_0'\}$  can be derived from the vector potential  $\vec{A} = \{\frac{y}{2}\Psi_0', -\frac{x}{2}\Psi_0', 0\}$ , where  $\Psi_0$  is a function of  $z$ .

(d) Show that during this motion, the particle's angular momentum around the z-axis is not conserved. Also show that the z component of its canonical angular momentum  $L_z = \left\{ \vec{r} \times (\vec{p} + e\vec{A}) \right\}_z$  is conserved. To do this, you can show that  $\frac{dL_z}{dt} = 0$ .

(e) Given a proton beam of  $E_k = 5\text{keV}$ , how many turns of a 100A current is approximately needed for a 10cm coil to have a 1 meter focal length.

(f) Show that within the solenoid the particles perform helical motion of radius  $\frac{x_0}{2}$  around the axis  $x = \frac{x_0}{2}$ .

## Exercise 2:

(a) Describe the magnetic field and the magnetic scalar potential in a duodecapole ?

(b) How strong is a duodecapole for which the distance from the central axis to the iron pole is given by  $a$  and around each pole is a winding of  $n$  wires each having a current  $I$  ?

(c) Show what fields are created when a  $n$  pole is shifted by a distance  $\Delta$  in the transverse direction. For example, show that a shifted sextupole has a quadrupole field.

## Exercise 3:

Dipole magnets are used to guiding charged particles in a beam line or circular accelerator. Neutrons cannot be guided by homogeneous magnetic fields since they have no charge. However, due to their magnetic dipole moment the Stern-Gerlach Force could be used to guide

them.

(a) Show that the force, produced by a quadrupole on a particle with horizontal spin, corresponds to the force on a charged particle in a dipole magnet.

(b) Show that a skewed sextupole magnet can be used for focusing neutral particles with horizontal spin.

(c) What would the multipole coefficient need to be to produce an instantaneous bending radius of 10m for a neutron with an energy of 1MeV?

**Exercise 4:**

(a) Suppose an air-coil magnet has four wires parallel to a beam pipe with the  $(x, y)$  coordinates  $(a, 0)$ ,  $(0, a)$ ,  $(-a, 0)$ , and  $(0, -a)$ . The first and the third wire have the current is  $I$ , the second and fourth have  $-I$ . What multipole components  $\Psi_\nu$  will be created in the center of the beam pipe?

(b) Given an electron beam with 2GeV energy and  $a = 5\text{cm}$ . How much current would one need to create a quadrupole component of  $k_1 = 0.01\text{m}^{-2}$ ?

**Exercise 5:**

A magnetic field has midplane symmetry. The particle motion through this field is described by a transport map  $\vec{M}$  with  $\vec{z}(s) = \vec{M}(s, \vec{z}_0)$  for the phase space vector  $\vec{z} = (x, p_x, y, p_y)$ .

(a) Explain why midplane symmetry requires the condition

$$\underline{S} = \begin{pmatrix} \underline{0} & \underline{0} \\ \underline{0} & -\underline{1} \end{pmatrix}, \quad \vec{M}(s, \vec{z}_0) = \underline{S}\vec{M}(s, \underline{S}\vec{z}_0). \tag{2}$$

The matrices  $\underline{0}$  and  $\underline{1}$  are the  $2 \times 2$  dimensional zero and unit matrix.

(b) What conditions do the Taylor coefficients  $M_i^{\vec{k}}$  of the transport map satisfy when it is expanded as

$$M_i(\vec{z}_0) = \sum_{j=1}^4 \sum_{k_j=1}^{\infty} M_i^{\vec{k}} x_0^{k_1} p_{x0}^{k_2} y_0^{k_3} p_{y0}^{k_4}. \tag{3}$$