Advanced Accelerator Physics and Accelerator Simulation Homework 3

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Exercise 1 : This was a part of Homework 2.

Exercise 2:
When the coordinates $w = x + iy$ and $\bar{w} = x - iy$ are used, the Laplace operator has been derived to be $\vec{\nabla}^2 = 4\partial_w \partial_{\bar{w}} + \partial_z^2$.
(a) Check that this is correct.
(b) The static magnetic field in a charge free space is given by $\vec{B} = -\vec{\nabla}\psi$. Writing the magnetic field in x and y direction in complex notation as $B = B_x + iB_y$, derive a formula that expresses B and B_z in terms of $\Psi(w, \bar{w}, z)$ and only ∂_w, $\partial_{\bar{w}}$, and ∂_z.
(c) Given the vector potential in complex notation as $A = A_x + iA_y$ and A_z, derive a formula that expresses B and B_z given by $\vec{B} = \nabla \times \vec{A}$, again only using ∂_w, $\partial_{\bar{w}}$, and ∂_z and A, A_z.

Exercise 3:
(a) The field in a bending magnet has usually two symmetries: Midplane symmetry since the upper and lower part of the magnet are built identically, and a mirror symmetry with respect to the vertical plane, since each pole is build with right/left symmetry when viewed along the beam pipe. Which multipoles, in addition to the main dipole component, satisfy this symmetry and can therefore be associated with such a bending magnet.
(b) Similarly, a focusing magnet has C_2 and midplane symmetry. Which multipoles, in addition to the main quadrupole term, satisfy this symmetry and can therefore appear when such a magnet is built.
(c) Generalize your observation to a magnet which is built with exact C_n symmetry and midplane symmetry. Which multipole terms can the field have?