The weird world of photons

- Why is a E&M nature of light not sufficient to explain experiments ?

- Do photons exist ?

- Some quantum properties of photons
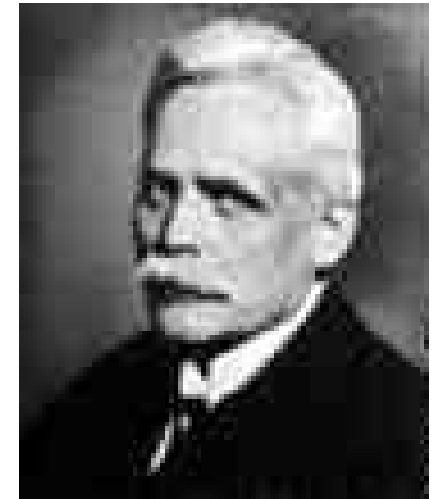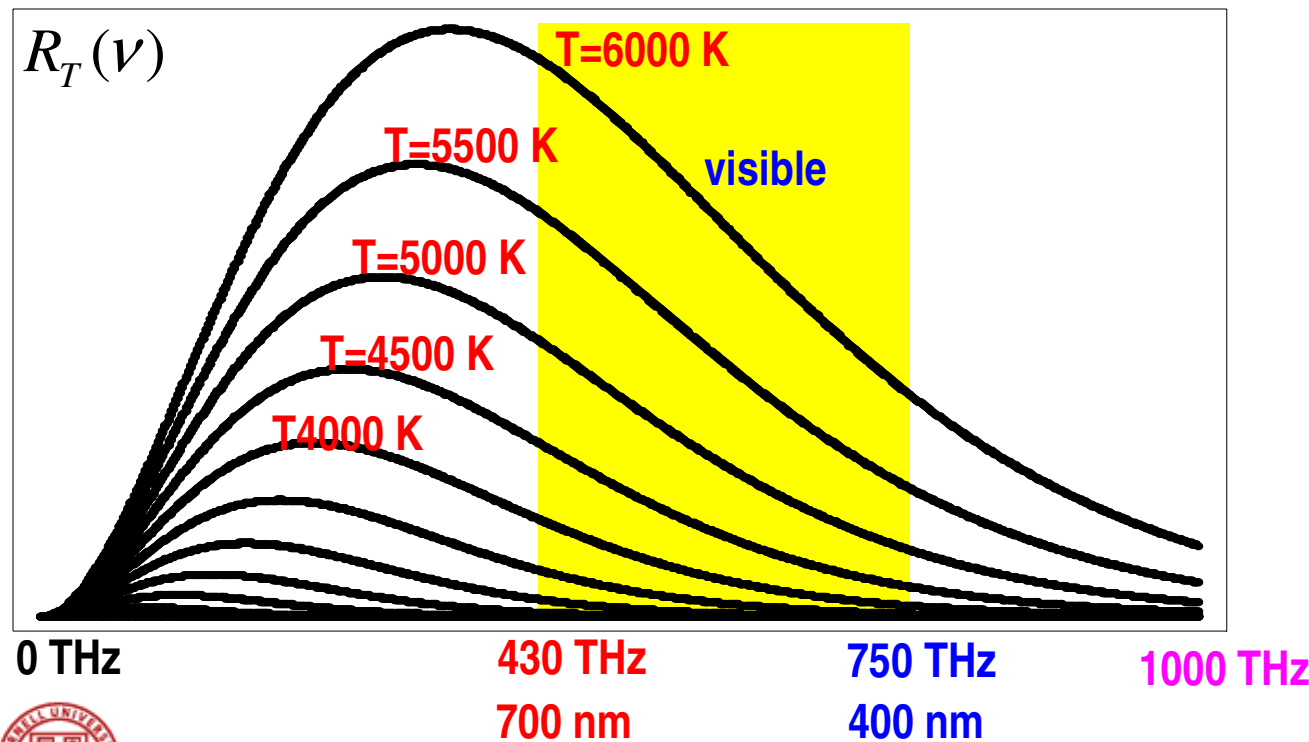
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Black body radiation

Stefan's law: Energy/ area/ time = $\int\limits_0^\infty R_T(\nu)d\nu = T^4 \cdot \sigma$, $\quad \sigma = 5.670 \cdot 10^{-8} \frac{1}{K^4} \frac{J}{m^2 \cdot s}$

Wien's displacement law: $\quad R_T(\nu)d\nu = T^4 \cdot f(\frac{\nu}{T})d(\frac{\nu}{T})$, $\quad \int\limits_0^\infty f(\frac{\nu}{T})d(\frac{\nu}{T}) = \sigma$

➡ $R_T(\nu)/T^3$ only depends on ν/T

$R_T(\nu)$

T=6000 K

T=5500 K

T=5000 K

T=4500 K

T4000 K

visible

0 THz

430 THz
700 nm

750 THz
400 nm

1000 THz

Wilhelm C.W.O.F.F. Wien
1864-1928
Nobel Prize in Physics, 1911

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Derivation of black body radiation

The first quantum property discussed was that of light by Max Plank (1900).

By proposing light quanta he derived the function $f\left(\frac{v}{T}\right)$

Derivation:

1. Represent the black body as a black body box.

2. In order to find what radiation escapes from the hole, compute the energy in the box in the frequency interval from **v** to **v+dv**.

3. Derive the number of radiation modes for this interval in the black body box: $dZ = g(v)dv$

4. Use the fact from statistical mechanics that for a temperature T, the probability for system to have energy E is given by **exp(-E/kT)** .

5a. Initially: Each radiation mode can have arbitrary intensity

5b. Plank: **Assume the each mode can only have an energy that is a multiple of some v–dependent quantum ε(v)** .

6. Compute the average energy

   per mode and sum over all modes.

Result: Wien's displacement law and Stefan's law

$$R_T(v) = \tfrac{c}{4}u = T^3 \frac{2\pi h}{c^2} \frac{(v/T)^3}{\exp\left(\frac{h}{k}\frac{v}{T}\right)-1}$$
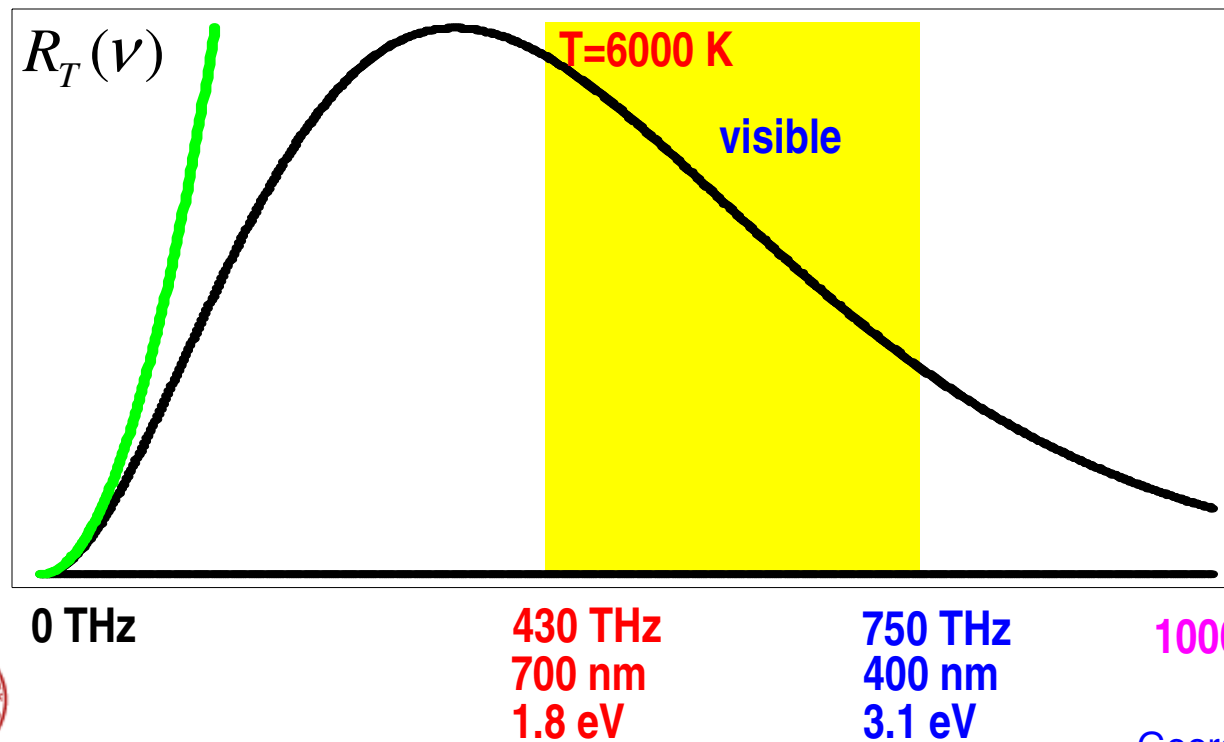
Max   , 1858-1947
Nobel Prize in Physics, 1918

Interpretation

A electromagnetic wave with frequency ν contains light quanta (photons) with the energy $h\nu$.
The energy of the wave determines the number of such photons that make up the wave.

➔ For small ν, the wave can have nearly all energies $nh\nu$ and one obtains the classical limit
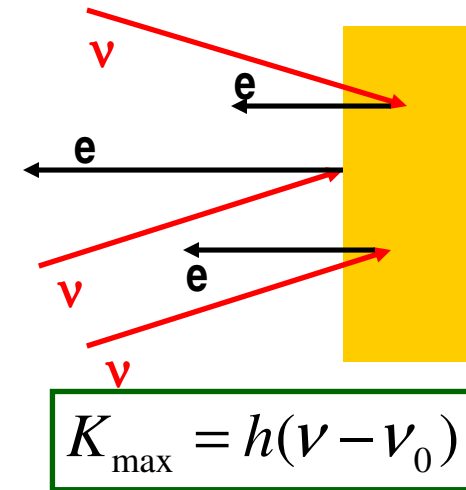
$$\nu_{max} = 0.06 \frac{\text{THz}}{\text{K}} T$$

$R_T(\nu)$

T=6000 K

visible

0 THz	430 THz	750 THz	1000 THz
	700 nm	400 nm	
	1.8 eV	3.1 eV	

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Further evidence for photons

The photoelectric effect:

1) The maximum electron energy depends only on ν and increases linearly with ν: $E_{max} = h\nu - W$ (Millikan 1916)

$$K_{max} = h(\nu - \nu_0)$$

2) There is a minimum ν independent of intensity. It is the Work function **W** divided by **h**.

3) The first emission happens faster than a wave would deposit energy (Lawrence and Beams 1928)

4) Different metals lead to the same value for **h**

Robert A. Millikan
(1868-1953, USA)
1923 Nobel prize

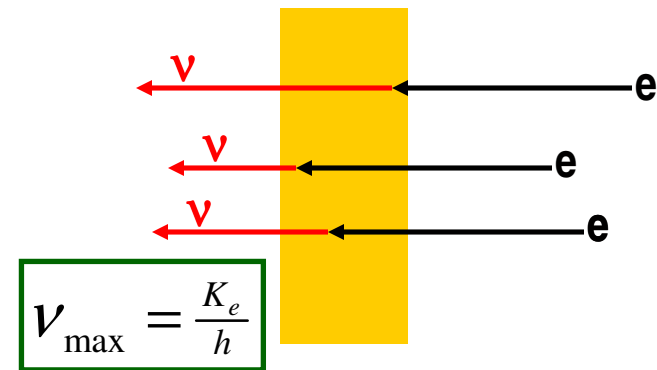Georg.Hoffstaetter@cornell.edu

Further evidence for photons
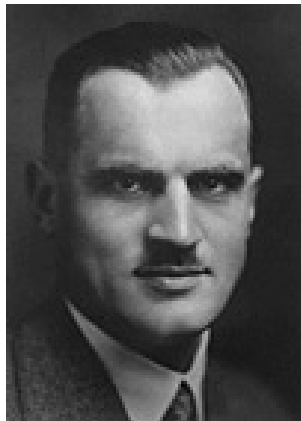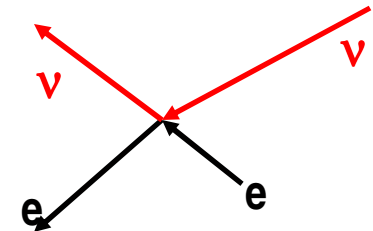
Bremsstrahlung

The maximum emitted frequency depends only on the electron energy, not on the e-beam intensity.

Highest frequency is linear in electron energy.

Typical electron energies are keV, and the work function of a few eV can therefore be neglected.

(Duane and Hunt 1915)

$$\nu_{\max} = \frac{K_e}{h}$$

The Compton effect:

Scattering of light with free electrons follows the formulas of classical scattering of free particles when one assumes an energy **E=hν** and momentum **p=E/c=h/λ** for the photons.
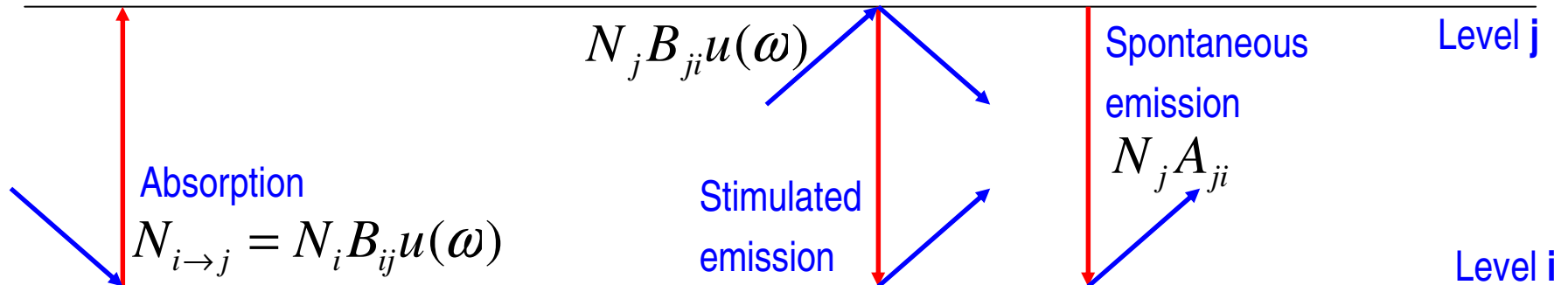
(Compton 1919-1923)

Artur H. Compton, 1892-1962
Nobel Prize in Physics, 1927

Stimulated emission for balck-body radiation

Einstein's explanation from 1917 for the energy density $u(\omega)$ in a black body box.

The light in a black body is emitted by electrons that change their energy level:

$$N_j B_{ji} u(\omega)$$

Spontaneous emission

$$N_j A_{ji}$$

Level **j**

Absorption

$$N_{i \to j} = N_i B_{ij} u(\omega)$$

Stimulated emission

Level **i**

Equilibrium:

$$N_{i \to j} = N_{j \to i} \to N_i B_{ij} u(\omega) = N_j [A_{ji} + B_{ji} u(\omega)] \to u(\omega) = \frac{A_{ji}/B_{ji}}{\frac{B_{ij}}{B_{ji}} \frac{N_i}{N_j} - 1}$$

Thermodynamic population of electron energy states:

$$N_i = C e^{-\frac{E_i}{kT}}, \quad N_j = C e^{-\frac{E_j}{kT}} \to \frac{N_i}{N_j} = e^{\frac{E_j - E_i}{kT}} = e^{\frac{\hbar\omega}{kT}}$$

To obtain **Plank's black-body radiation formula**:

$$u(\omega) \propto \frac{\omega^3}{e^{\frac{\hbar\omega}{kT}} - 1}$$

● Probability for stimulated emission = probability of absorbtion, $\quad B_{ij} = B_{ji}$

● Probability of spontaneous emission increases with ω³ : $\quad A_{ji}/B_{ji} \propto \omega^3$

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Formation of optical images

At low exposure, the few photons that lead to a reaction in a photographic plate are statistically distributed. Hits of individual photons can be observed.

With increasing exposure, the interferences of electromagnetic waves that refract in the lens of the camera and form the image become apparent. The image formed by many photons forms corresponds to the image formed by interfering electromagnetic waves.
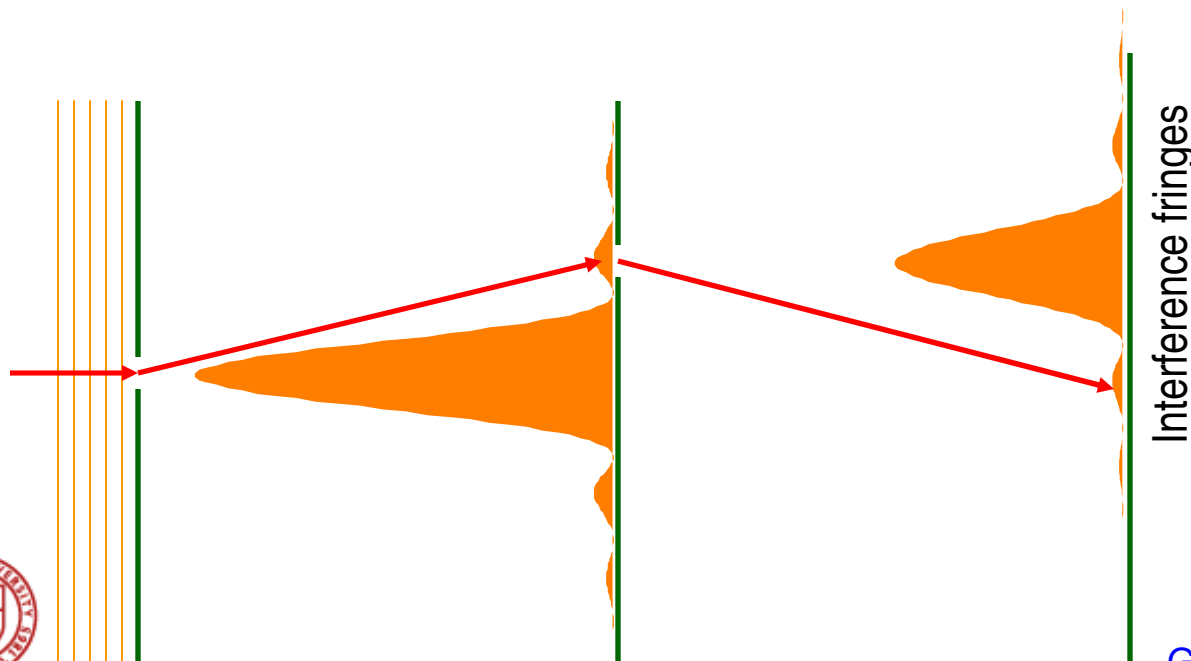
Non-particle like properties of photons

- Photons move into shadowed regions by diffraction

- One cannot associate a field vector to an individual photon.

- Wave properties of light can only be found when very large numbers of photons are investigated.
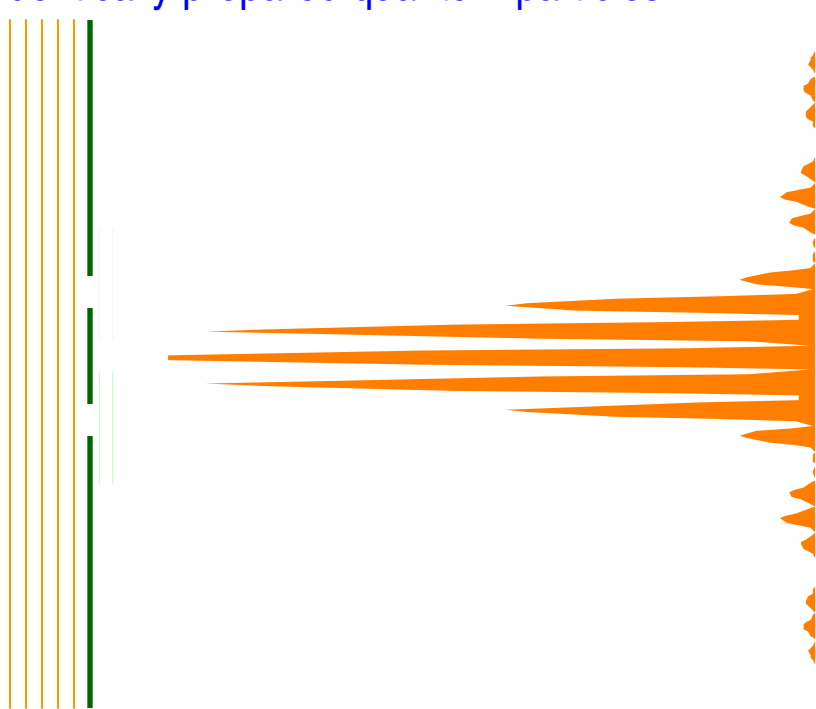
Particle wave duality:

Wave properties are an expression of the probabilistic or statistical

behavior of large numbers of identically prepared quantum particles.

Interference fringes

Wave function and probability amplitude

Particle wave duality:

Wave properties are an expression of the probabilistic or statistical

behavior of large numbers of identically prepared quantum particles.

The paths of a photon:

Information about the photon must have traveled through both slots. Otherwise

the intensity distribution due to the two openings would be added.

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Photons and quantum states

A electromagnetic plain wave can be uniquely defined by specifying four things:

1. Frequency
2. Direction of propagation
3. Polarization
4. Amplitude of the electric field (and thereby the energy in the field)

Similarly the state of a photon is uniquely defined by specifying three things:

1. Frequency (and therefore the energy of the photon **hν**)
2. Direction of motion
3. Polarization state

The energy in the wave is then determined by the number of photons.

The **state of a photon** completely determines a photon in the following sense:

Everything that can be known about a photon is specified.

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Probability or hidden variables

Hidden variables: To avoid the introduction of probability into the propagation of completely determined particles, some people have tried to introduce hidden variables, that distinguish the photons that behave differently at the screen.

➔ This would not be satisfying, since the wave is completely determined before the slids and the photons should therefore all be equally determined.

➔ It has been shown that this approach **cannot work**.
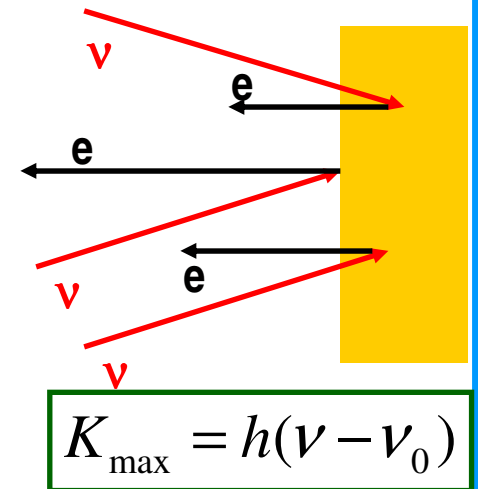
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Note on the existence of photons

Is the particle structure of light a good picture, or is it only an artifact of the tools that have been used to investigate light ?

After all, the quantized number of leaves falling from trees does not force one to adopt a quantum picture of wind.

While Einstein received his Nobel Prize for his interpretation of the photo-electric effect, ironically, his conclusions about properties of light were not totally justified, other interpretations of the photoelectric effect are possible.

$$K_{max} = h(\nu - \nu_0)$$

Observations about the photoelectric effect that have to be explained are:

1) When the incident light has a frequency below a threshold, no electrons are emitted.
2) The energy of the electrons increases linearly with the light frequency.
3) The number of electrons increases linearly with the light intensity.
4) Electrons can be emitted at times very shortly after the onset of illumination.

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Alternate explanation of the photoelectric effect

Observations about the photoelectric effect that have to be explained are:
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4) Electrons can be emitted at times very shortly after the onset of illumination.

Investigating an electron immersed in a classical sinusoidally varying electric field, where the electron is bound in an energy level E_0 and can make a transition to a free electron with energy E_k above E_0, then the time dependent Schroedinger equation leads to:

1) An appreciable transition probability only if $h\nu = E_k$. A emission thus only occurs when there is a free state available at energy $h\nu$ above the ground state.
2) The energy Ek is thus linearly related to w.
3) The probability of the transition is quadratic with the electric field.
4) The probability to have a transition to any energy turns out to increase linearly with time, leading to some probability even at very small times.

The photoelectric effect does not consitute a prove of existence of photons.
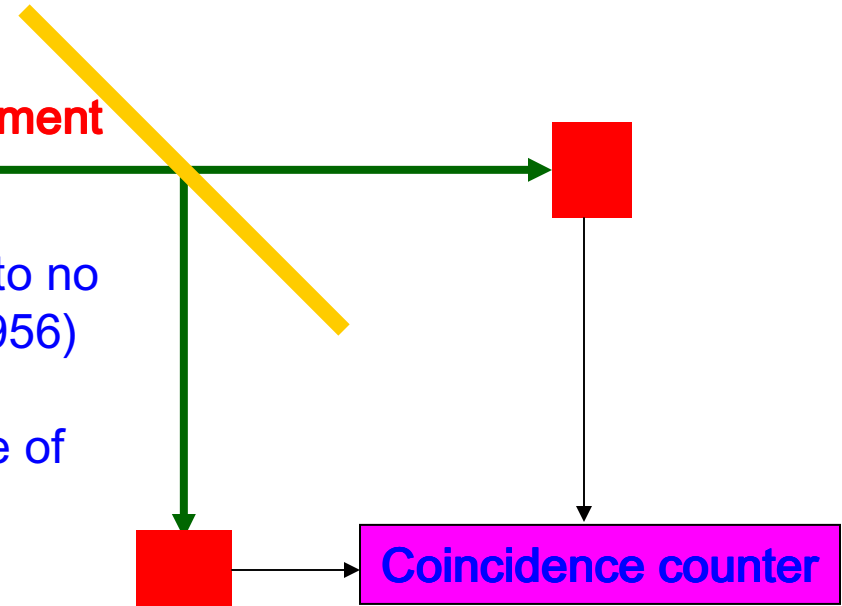
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Experimental verification of photons

For a photon to have particle characteristics it can only be at one place at any time.

Test: The Hanbury-Brown and Twiss Experiment

While the existence of photons should lead to no coincidence of detection, the experiment (1956) with a mercury emission line showed more coincidence than even a random occurrence of coincident electron emissions should allow. This lead to the discipline of studying the quantum nature of light.

Coincidence counter

Reason: Conventional light sources are not in a state with a well defined number of photons so that coincidence is likely to occur.

$$|\Psi\rangle = A_1|1\rangle + A_2|2\rangle + A_3|3\rangle + \ldots$$

Finally (Clauser,1974), a single photon state was produced, and in deed no coincidences were counted: **Photons at last !**

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Photons and waves

Now coincidence is counted: Photon is detected only at one point at each time.

Interference when the photon can take two paths: Some information about the photon must have taken both paths.

How can these two concepts be combined?

One try: **The conspiracy theory**
The photon changes its nature from particle to wave depending on which experiment is chosen, i.e. sometimes it acts as a particle, sometimes as a wave, depending whether the experiment is chosen to test for particle properties or for wave properties.
This has been refuted by delayed choice experiments (John Wheeler, 1978)

Another try: **Non-local theory**
The photon is influenced by the existence of the second path, even though it does not travel through it (David Bohm, 1952). Problems: This influence is instantaneous and does not diminish with the distance to the second path.
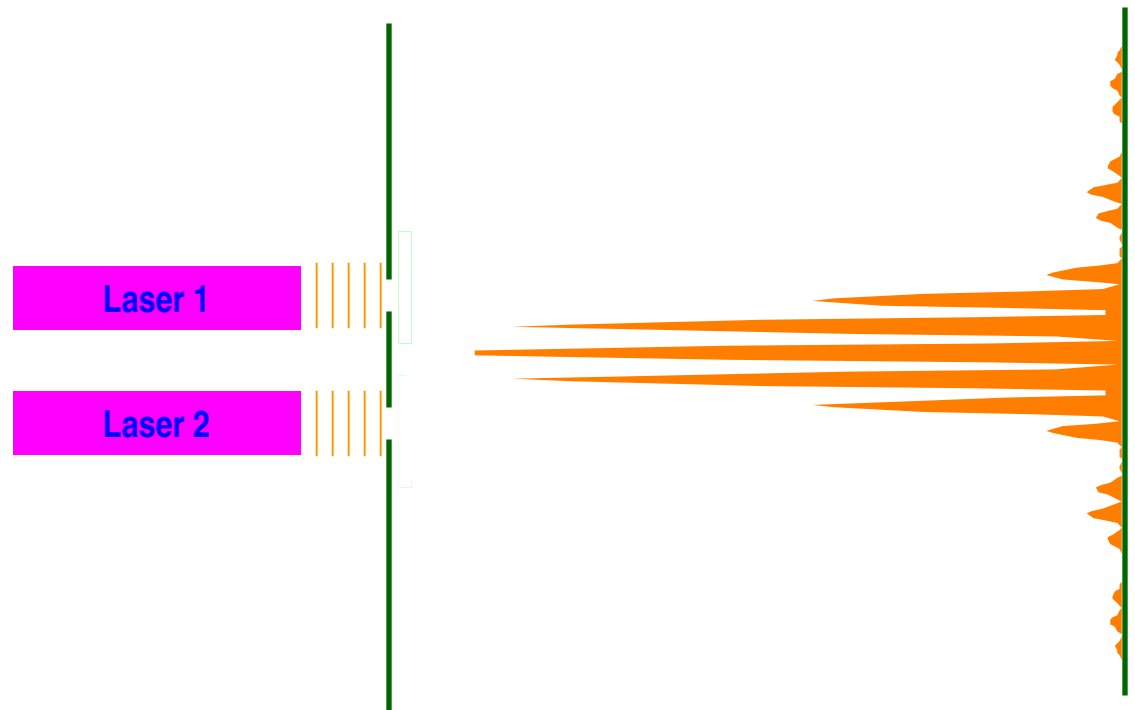It is therefore not favored.

What is left: The probability interpretation of quantum mechanics.

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The uncertainty of birth

Interference from two lasers (Pfleegor-Mandel, 1967)

Light from two lasers can interfere, even when light is emerging with very low intensity:

The photon wave function interferes as if the photon could have been born in either laser.

There is quantum uncertainty of the birth place of the photon.

$$\left|\Psi\right\rangle \propto \left|\text{born at 1}\right\rangle + \left|\text{born at 2}\right\rangle$$

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