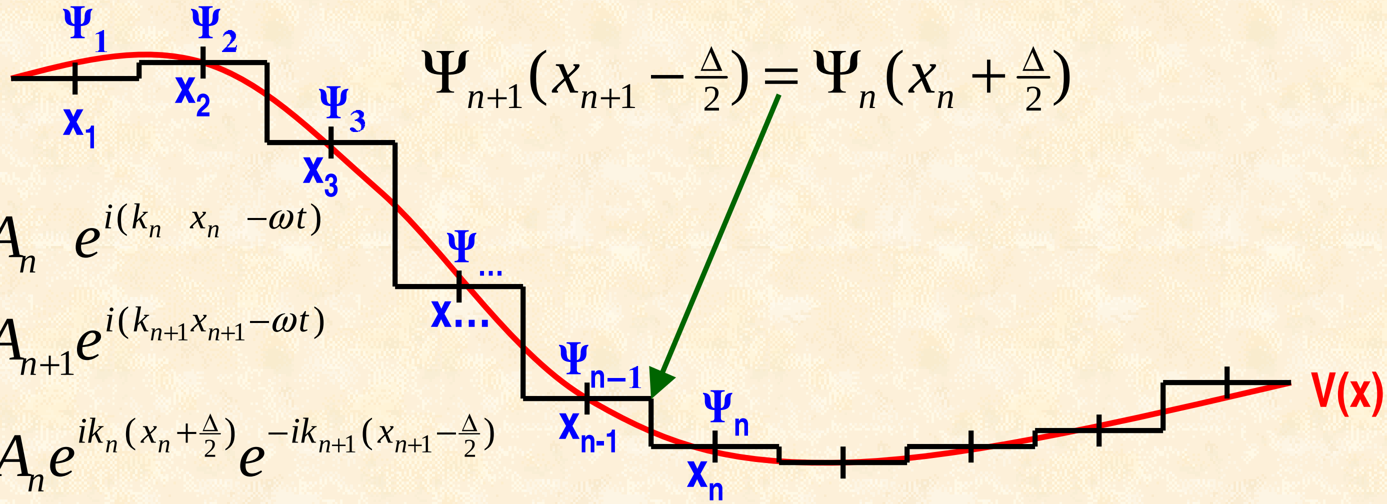
Expectation values for p_x

Review of one dimension:



$$\Psi_{n+1}(x_{n+1}-\tfrac{\Delta}{2})=\Psi_n(x_n+\tfrac{\Delta}{2})$$

$$\Psi_n = A_n\, e^{i(k_n x_n - \omega t)}$$

$$\Psi_{n+1} = A_{n+1} e^{i(k_{n+1}x_{n+1}-\omega t)}$$

$$A_{n+1} = A_n e^{ik_n(x_n+\frac{\Delta}{2})} e^{-ik_{n+1}(x_{n+1}-\frac{\Delta}{2})}$$

$$\Psi_{n+1} = A_n e^{i[k_n(x_n+\frac{\Delta}{2})-k_{n+1}(x_{n+1}-\frac{\Delta}{2})+k_{n+1}x_{n+1}]\}} e^{-i\omega t} = \Psi_n e^{i[k_n\frac{\Delta}{2}+k_{n-1}\frac{\Delta}{2}]}$$

$$=\Psi_n e^{i\frac{k_{n+1}+k_n}{2}\Delta} \approx \Psi_n + ik_n\Psi_n\Delta \quad\rightarrow\quad \frac{\partial}{\partial x}\Psi(x,t) = ik\Psi$$

Conclusion: whenever $k\Psi$ needs to be computed, one can use $-i\frac{\partial}{\partial x}\Psi$

$$\langle p_x\rangle = \sum_{\text{all } j}\hbar k_j\left|\Psi_j\right|^2 dx = \sum_{\text{all } j}\Psi_j^*\left(-i\hbar\frac{\partial}{\partial x}\Psi(x_j)\right)dx = \int_{-\infty}^{\infty}\Psi^*\left(-i\hbar\frac{\partial}{\partial x}\right)\Psi dx$$

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Spread of measurements

$$\Delta x^2 = \int_{-\infty}^{\infty} \Psi^*(x,t)(x-\langle x\rangle)^2 \Psi(x,t)dx = \left\langle x^2 - 2x\langle x\rangle + \langle x\rangle^2 \right\rangle = \left\langle x^2\right\rangle - \langle x\rangle^2$$

$$\Delta p_x^2 = \int_{-\infty}^{\infty} \Psi^*(x,t)(\hat{p}_x-\langle p_x\rangle)^2 \Psi(x,t)dx = \left\langle p_x{}^2\right\rangle - \langle p_x\rangle^2, \quad \hat{p}_x = -i\hbar \tfrac{\partial}{\partial x}$$

$$\Delta E^2 = \int_{-\infty}^{\infty} \Psi^*(x,t)(\hat{E}-\langle E\rangle)^2 \Psi(x,t)dx = \left\langle E^2\right\rangle - \langle E\rangle^2, \quad \hat{E} = V(x) - \tfrac{\hbar^2}{2m}\tfrac{\partial^2}{\partial x^2}$$

For an arbitrary physical quantity A that depends on x and px, a corresponding operator can be formed. The spread of measurements of A is then given by:

$$\Delta A = \int_{-\infty}^{\infty} \Psi^*(x,t)(\hat{A}-\langle A\rangle)^2 \Psi(x,t)dx = \left\langle A^2\right\rangle - \langle A\rangle^2$$

The spread of the measurements is 0 when the wave function is an eigenfunction of the operator.

$$\int_{-\infty}^{\infty} \Psi_a^*(x,t)(\hat{A}-\langle A\rangle)^2 \Psi_a(x,t)dx = 0 \quad for \quad \hat{A}\Psi_a(x,t) = a\Psi_a(x,t)$$

$$\text{since} \quad \langle A\rangle = a \quad \text{and} \quad \left\langle A^2\right\rangle = a^2$$

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