Projection amplitudes for linear and circular polarization

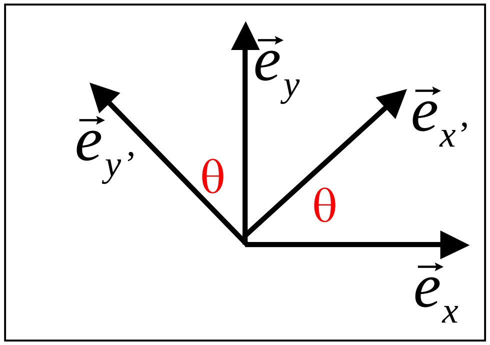
The field of any plane light wave with frequency ν can be decomposed in linear polarization

$$\vec{E} = \mathrm{Re}[(\vec{e}_x A_x + \vec{e}_y A_y) e^{i(kz-\omega t)}]$$

and in circular polarization:

$$\vec{E} = \mathrm{Re}[\{\vec{e}_R B_x + \vec{e}_L B_y\} e^{i(kz-\omega t)}], \qquad \vec{e}_R = \frac{\vec{e}_x - i\vec{e}_y}{\sqrt{2}}, \quad \vec{e}_L = \frac{\vec{e}_x + i\vec{e}_y}{\sqrt{2}}$$

Example: $\vec{E} \propto \vec{e}_{x'}$



$$\mathrm{Re}[\vec{e}_{x'} e^{i(kz-\omega t)}] = \mathrm{Re}[(\vec{e}_x \cos\vartheta + \vec{e}_y \sin\vartheta) e^{i(kz-\omega t)}]$$

Corresponding projection amplitudes

$$< x | x' >= \cos\vartheta, \qquad < y | x' >= \sin\vartheta$$

$$< x | y' >= -\sin\vartheta, \qquad < y | y' >= \cos\vartheta$$

Example of circular polarization: $\mathrm{Re}[\vec{e}_R e^{i(kz-\omega t)}] = \mathrm{Re}[(\frac{1}{\sqrt{2}}\vec{e}_x - i\frac{1}{\sqrt{2}}\vec{e}_y) e^{i(kz-\omega t)}]$

Corresponding **complex** projection amplitudes

$$< x | R >= \tfrac{1}{\sqrt{2}}, \qquad < y | R >= -i\tfrac{1}{\sqrt{2}}$$

$$< x | L >= \tfrac{1}{\sqrt{2}}, \qquad < y | L >= i\tfrac{1}{\sqrt{2}}$$

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Properties of projection amplitudes

Probability to be in a given state: $|<\Psi|\Psi>|^2 = 1$

After the analyzer: $|<L|\Psi>|^2 + |<R|\Psi>|^2 = 1$

$$<L|\Psi>^* <L|\Psi> + <R|\Psi>^* <R|\Psi> = 1$$

After the combiner: $<\Psi|L> <L|\Psi> + <\Psi|R> <R|\Psi> = <\Psi|\Psi> = 1$

Since this has to hold for arbitrary states **Y** and for any complete sets of states like **R** and **L**:

Reflexivity: $<\Psi|\Phi> = <\Phi|\Psi>^*$

Projection amplitudes are sometimes called **Dirac brackets**.

Paul A.M. Dirac
(1902-1984, UK)
Nobel Prize in Physics, 1933

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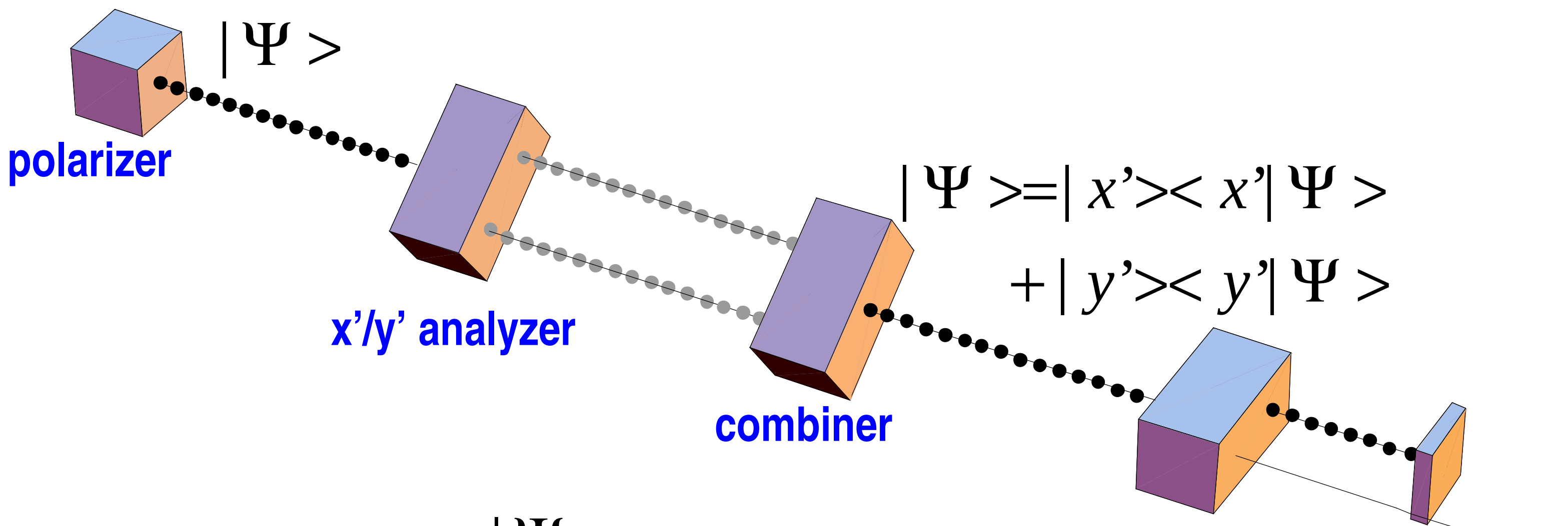
The state vector



Arbitrary initial photon state: $|\Psi>$

Projection amplitude into x-channel: $<x|x'><x'|\Psi>+<x|y'><y'|\Psi>$

Projection amplitude into an arbitrary final state: $<?|x'><x'|\Psi>+<?|y'><y'|\Psi>$

The **state vector** or (brac-) ket-vector: $|\Psi>=|x'><x'|\Psi>+|y'><y'|\Psi>$

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Quantum amplitudes for linear polarization

The field of any plane light wave with frequency ν can be written as

$$\vec{E} = \mathrm{Re}[(\vec{e}_x A_x + \vec{e}_y A_y) e^{i(kz-\omega t)}]$$

The Intensity in the **x** channel after an **x/y** analyzer:

$$I_x \propto 2\left\langle \mathrm{Re}[\vec{e}_x A_x e^{i(kz-\omega t)}]^2 \right\rangle_t$$

$$= 2\left\langle \tfrac{1}{4}[A_x^2 e^{i2(kz-\omega t)} + 2A_x A_x^* + A_x^{*2} e^{-i2(kz-\omega t)}] \right\rangle_t = A_x A_x^*$$

Intensity in the **y** channel: $I_y \propto A_y A_y^*$

The corresponding state vector describes a photon which is in the state $|x>$ with probability $|A_x|^2$ and in the state $|y>$ with probability $|A_y|^2$.

The interferences of the photons correspond to the interferences of the field when the phases and the probability amplitudes are chosen according to the amplitudes and phases of the filed components by using complex quantum amplitudes

$$|\Psi> = |x> A_x + |y> A_y$$

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