Advanced Accelerator Physics and Accelerator Simulation Homework 5

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February 23, 2007

Exercise 1:
What property does the Hamilton function $H(x, a, y, b, \tau, \delta, s)$ have when the motion is mid-plane symmetric?

Exercise 2: Show that the Lorentz-force equation can be derived from the Hamiltonian

$$H = c\sqrt{[\vec{p}_c - q\vec{A}(\vec{r}, t)]^2 + (mc)^2} + q\Phi(\vec{r}, t)$$

where the canonical momentum $\vec{p}_c$ is related to the classical momentum by $\vec{p} = \vec{p}_c - q\vec{A}$.

Exercise 3:
(a) A matrix $\mathbf{M}$ is symplectic when it satisfies $\mathbf{MJM}^T = \mathbf{J}$. Using $\mathbf{J}^{-1} = -\mathbf{J}$ and $\mathbf{J}^T = -\mathbf{J}$, show that the following properties are also satisfied:

$$\mathbf{M}^{-1} = -\mathbf{J}\,\mathbf{M}^T\mathbf{J}\ , \quad \mathbf{M}^T\mathbf{J}\,\mathbf{M} = \mathbf{J}\ . \tag{1}$$

(b) The linear transport map of a quadrupole is given by

$$\begin{pmatrix} x \\ p_x \end{pmatrix} = \begin{pmatrix} \cos(\sqrt{k}s) & \frac{1}{\sqrt{k}}\sin(\sqrt{k}s) \\ -\sqrt{k}\sin(\sqrt{k}s) & \cos(\sqrt{k}s) \end{pmatrix} \begin{pmatrix} x_0 \\ p_{x0} \end{pmatrix} \tag{2}$$

when k is the strength of the quadrupole field. Derive the generating function $F_1(x, x_0, s)$ that represents this map.

Exercise 4:
Show that $\mathbf{M}^T$ is symplectic if and only if $\mathbf{M}$ is symplecitic.