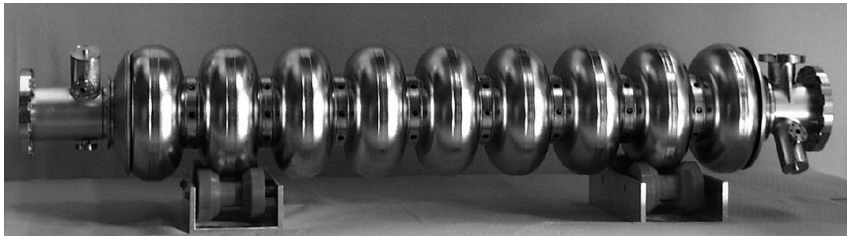




# Superconducting Cavities



CHESS & LEPP



$$Q = 10^{10}$$

$$E = 20\text{MV/m}$$



A bell with this  $Q$   
would ring for a year.

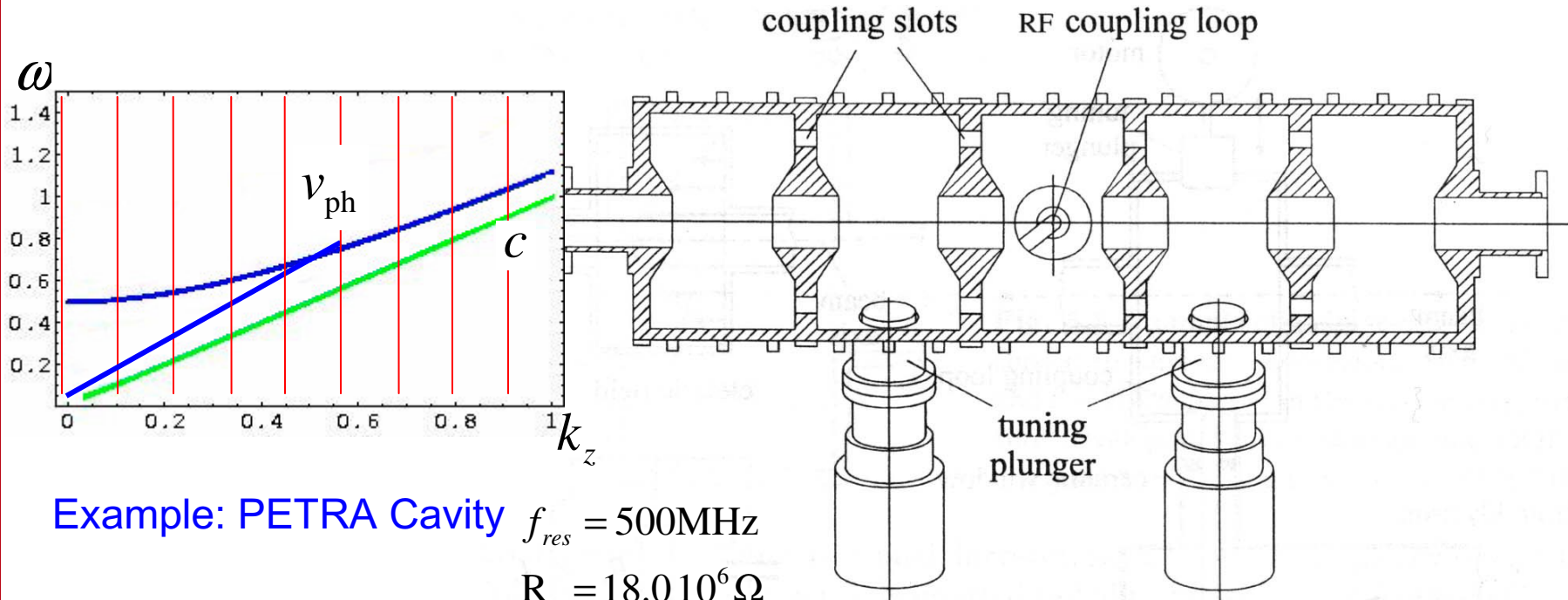
- Very low wall losses.
  - Therefore continuous operation is possible.
- ↓
- Energy recovery becomes possible.

## Normal conducting cavities

- Significant wall losses.
- Cannot operate continuously with appreciable fields.
- Energy recovery was therefore not possible.

# Multicell standing-wave cavities

The field in many cells can be excited by a single power source and a single input coupler in order to have the voltage of several cavities available.



Example: PETRA Cavity  $f_{res} = 500\text{MHz}$   
 $R_s = 18.0 \cdot 10^6 \Omega$   
 $125\text{kW} \rightarrow 2.12\text{MV}$

Without the walls: Long single cavity with too large wave velocity.  $v_{ph} = \frac{\omega}{k}$

Thick walls: shield the particles from regions with decelerating phase.



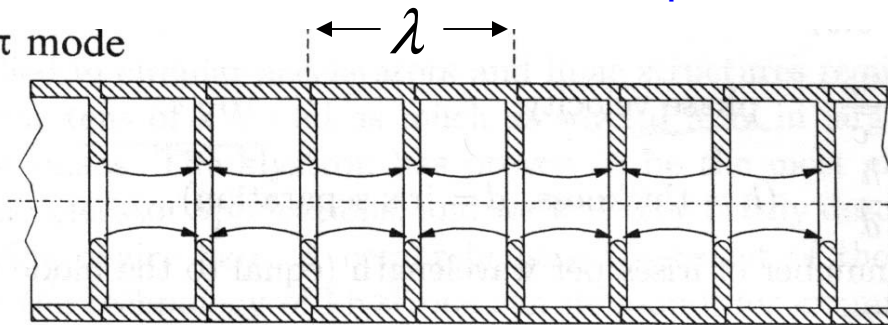
# Modes in Waveguides



CHESS & LEPP

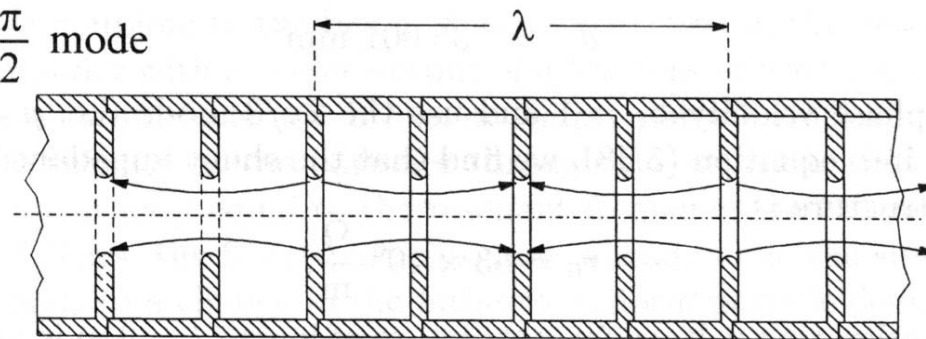
The iris size is chosen to let the phase velocity equal the particle velocity.

$\pi$  mode



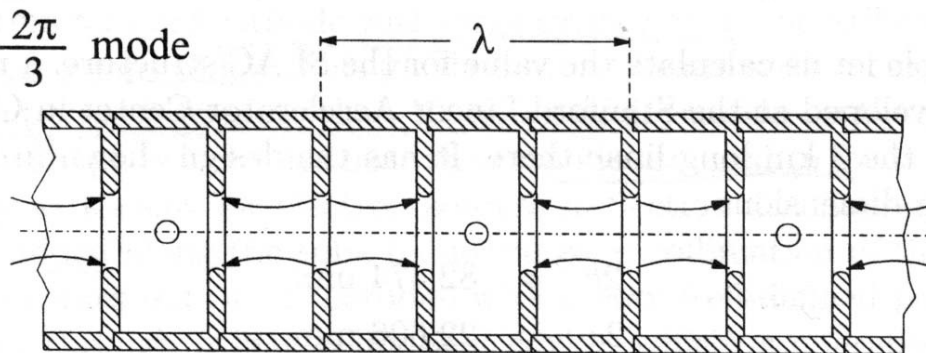
Long initial settling or filling time,  
not good for pulsed operation.

$\frac{\pi}{2}$  mode



Small shunt impedance per length.

$\frac{2\pi}{3}$  mode



Common compromise.

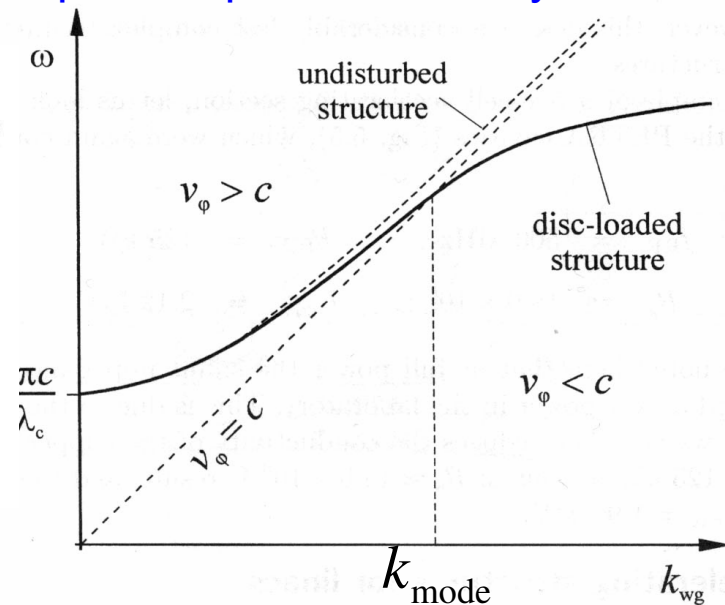
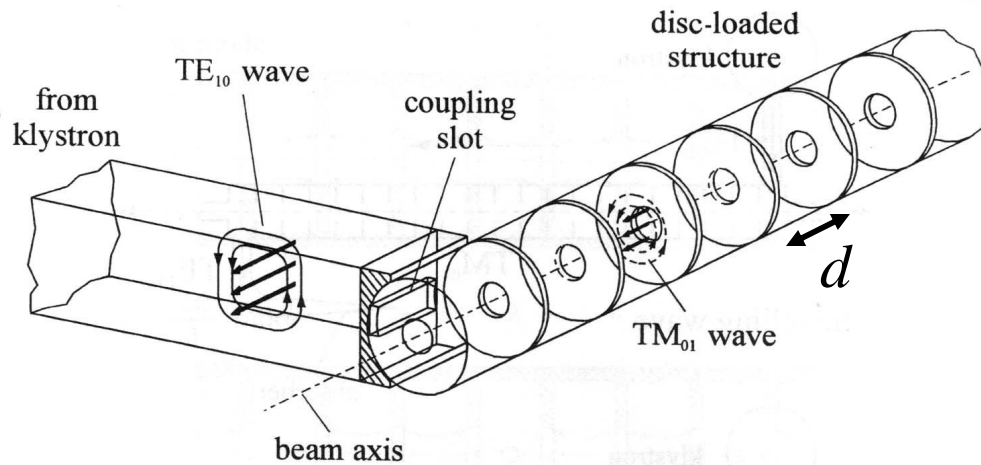


# Disc Loaded Waveguides



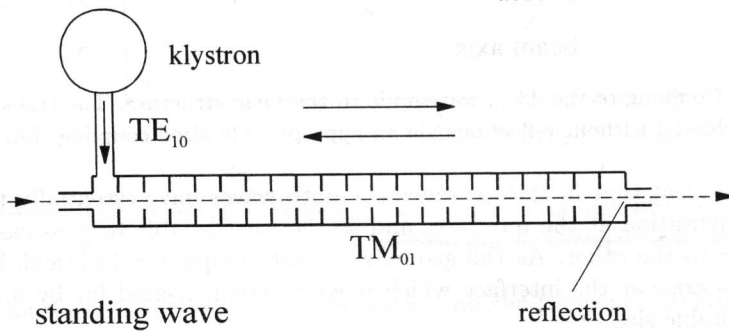
CHESS & LEPP

The iris size is chosen to let the phase velocity equal the particle velocity.

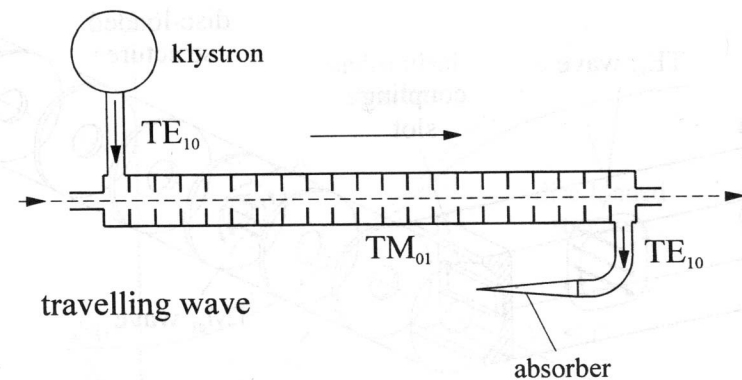


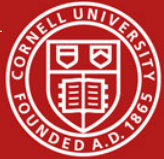
Loss free propagation:  $k = \frac{2\pi}{nd}$

Standing wave cavity.



Traveling wave cavity (wave guide).





(1) Linearization:  $E_r(r, z, t) = -\frac{r}{2} \partial_z E_z(0, z, t) \Rightarrow \vec{\nabla} \cdot \vec{E} = 0$

$$B_\phi(r, z, t) = \frac{1}{c^2} \frac{r}{2} \partial_t E_z(0, z, t) \Rightarrow \vec{\nabla} \times \vec{B} = \frac{1}{c^2} \partial_t \vec{E}$$

(2) Equation of motion:

$$a = \frac{p_x}{p_0}$$

Exact for a traveling wave with  $\omega/k = v$

because of:  $\frac{d}{dz} \cos(\omega t - kz + \phi_0) f(z) = \frac{d}{dz} f(z)$

$$a' = \frac{1}{p_0 v} (F_x - a F_z) = -\frac{q}{p_0 v} \left[ \frac{r}{2} \left( \partial_z + \frac{v}{c^2} \partial_t \right) E_z + a E_z \right]$$

$$= -\frac{q}{p_0 v} \left[ \frac{r}{2} \left( \frac{d}{dz} - \frac{1}{v} \left( 1 - \frac{v^2}{c^2} \right) \partial_t \right) E_z + a E_z \right] \approx -\frac{1}{p_0} \left[ r \frac{1}{2} p_0'' + a p_0' \right]$$

$$u = r \sqrt{p} \quad \text{p denotes } p_0 \text{ for simplicity}$$

$$u' = a \sqrt{p} + r \sqrt{p} \frac{p'}{2p}$$

**Focusing !**

$$u'' \approx -\frac{1}{\sqrt{p}} \left( r \frac{1}{2} p'' + a p' \right) + a \sqrt{p} \frac{p'}{p} + r \left( \frac{p''}{2\sqrt{p}} - \frac{p'}{4\sqrt{p}^3} \right) = -u \left( \frac{p'}{2p} \right)^2$$