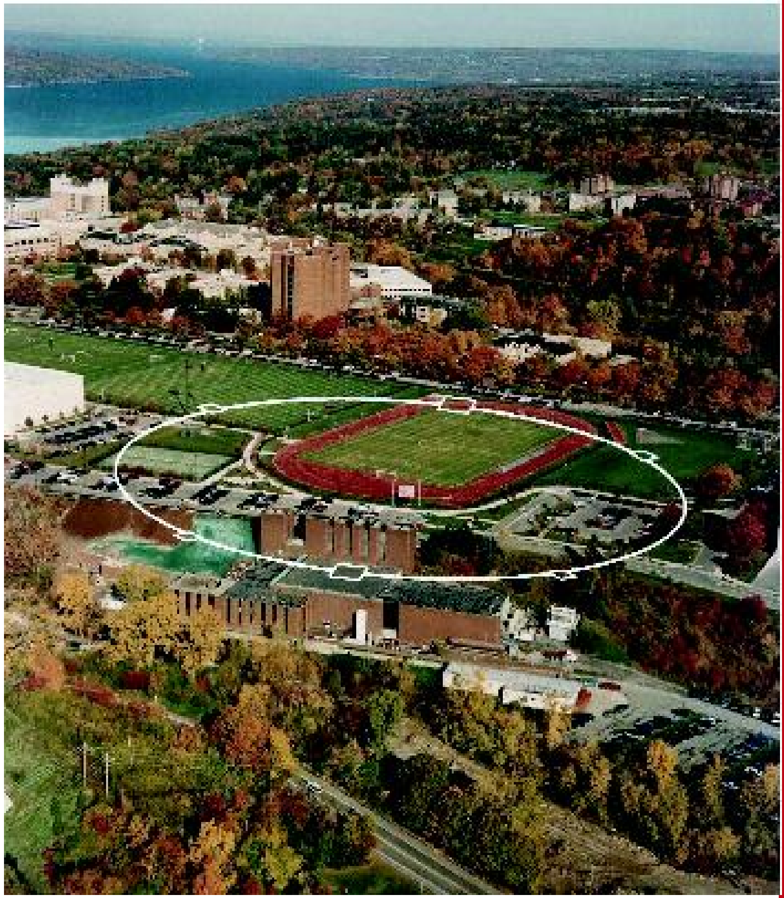
Class Review

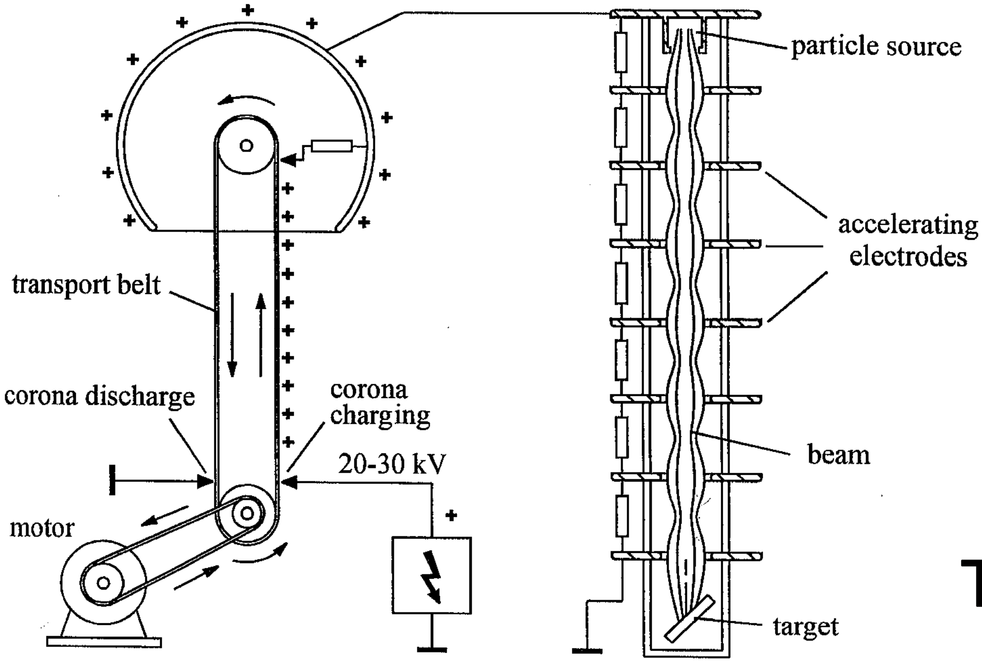
CHESS & LEPP

Content

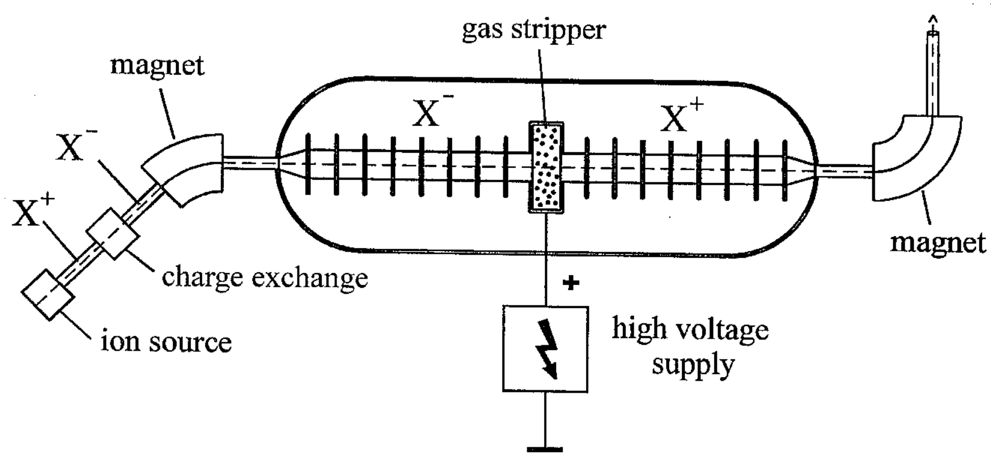
1. A History of Particle Accelerators
2. E & M in Particle Accelerators
3. Linear Beam Optics in Straight Systems
4. Linear Beam Optics in Circular Systems
5. Nonlinear Beam Optics in Straight Systems
6. Nonlinear Beam Optics in Circular Systems
7. Accelerator Measurements
8. RF Systems for Particle Acceleration



The Van de Graaff Accelerators

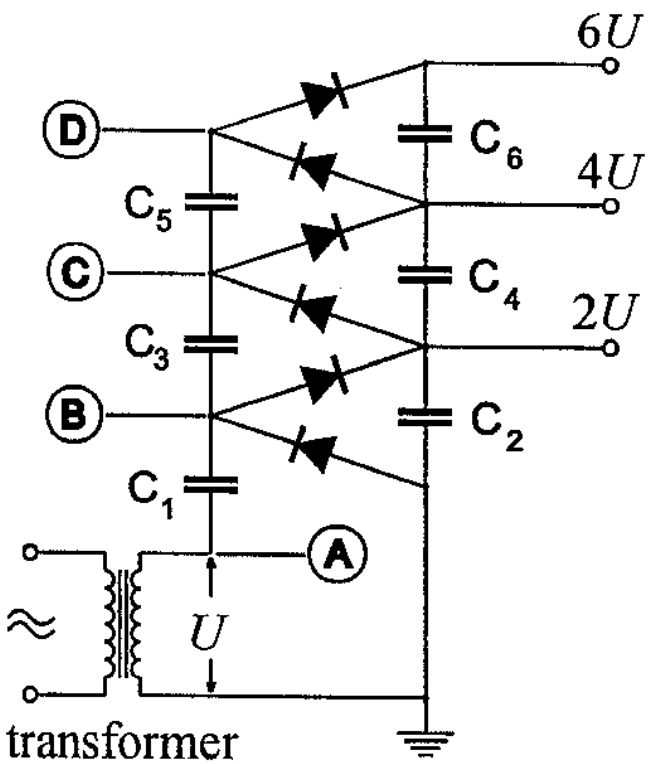


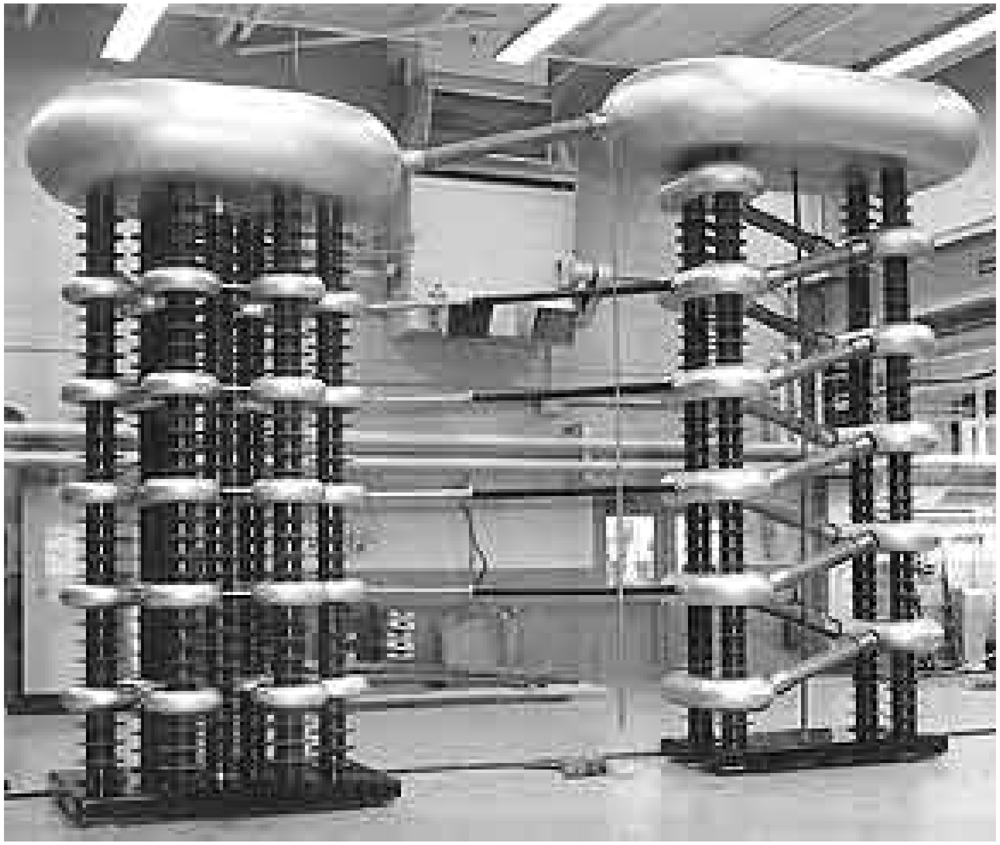
The Tandem Accelerator



The Cockcroft-Walton Accelerator

CHESS & LEPP





Three historic lines of accelerators

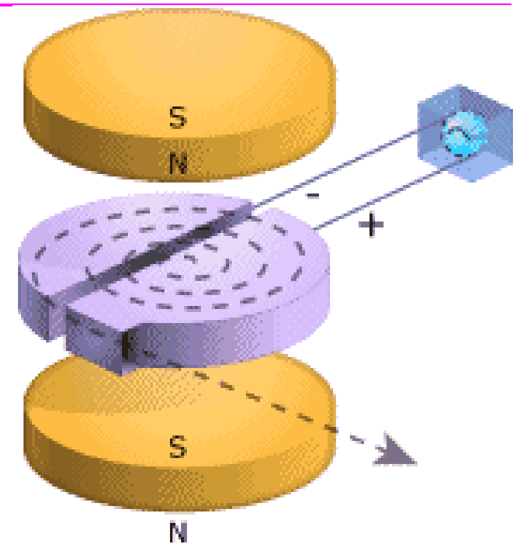
CHESS & LEPP

Resonant Accelerators

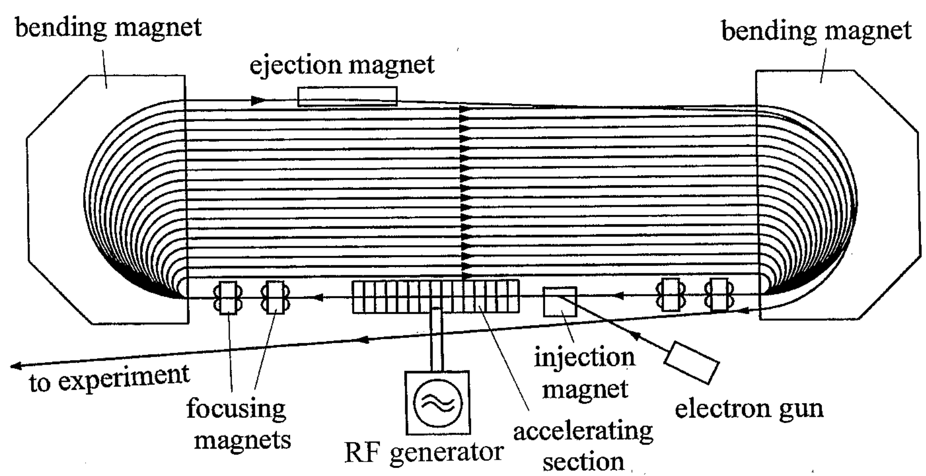
Direct Voltage Accelerators

Transformer Accelerator

The Cyclotron



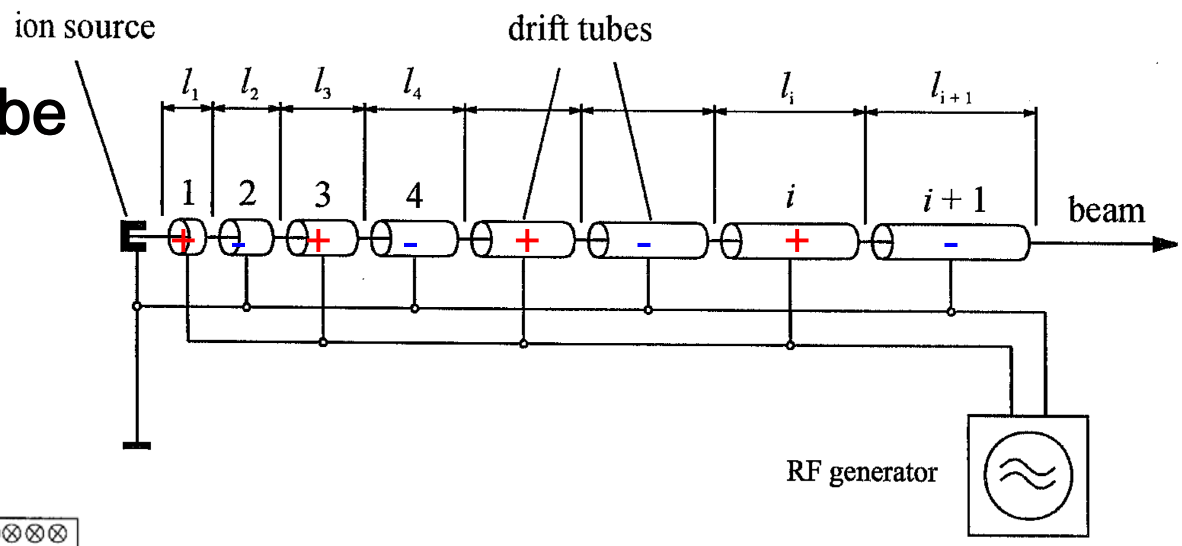
The microtron



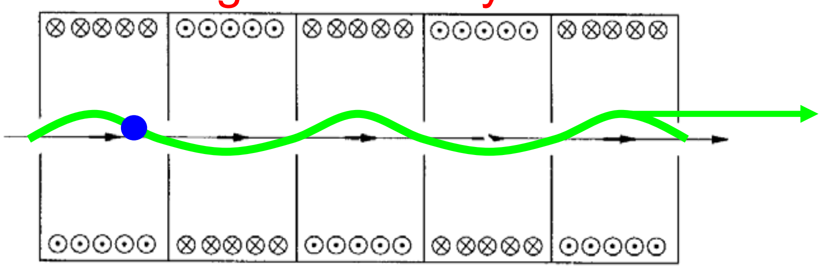
Cornell University

CHESS & LEPP

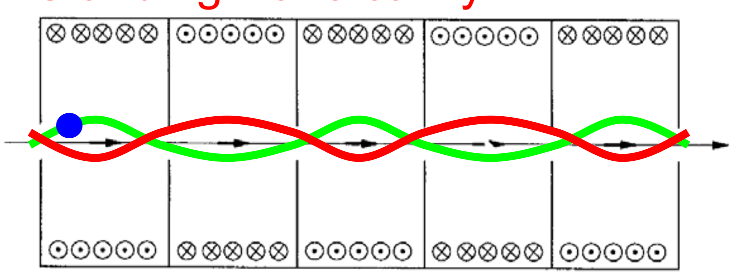
Wideroe or drift tube linear accelerator



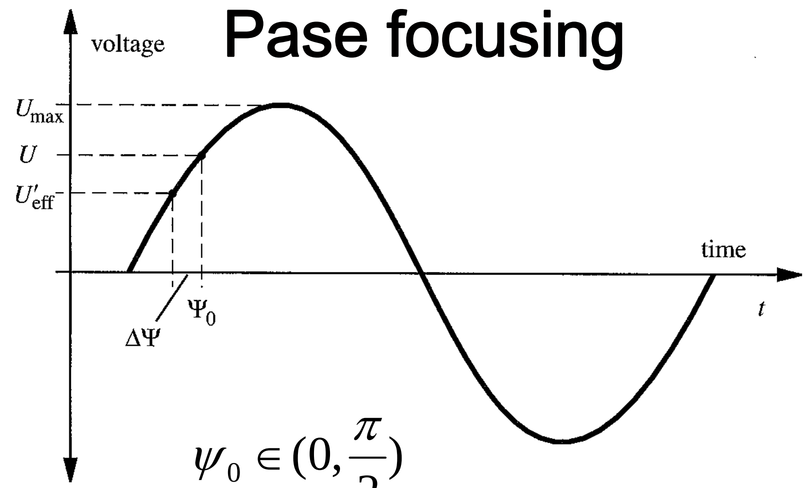
Traveling wave cavity:



Standing wave cavity:



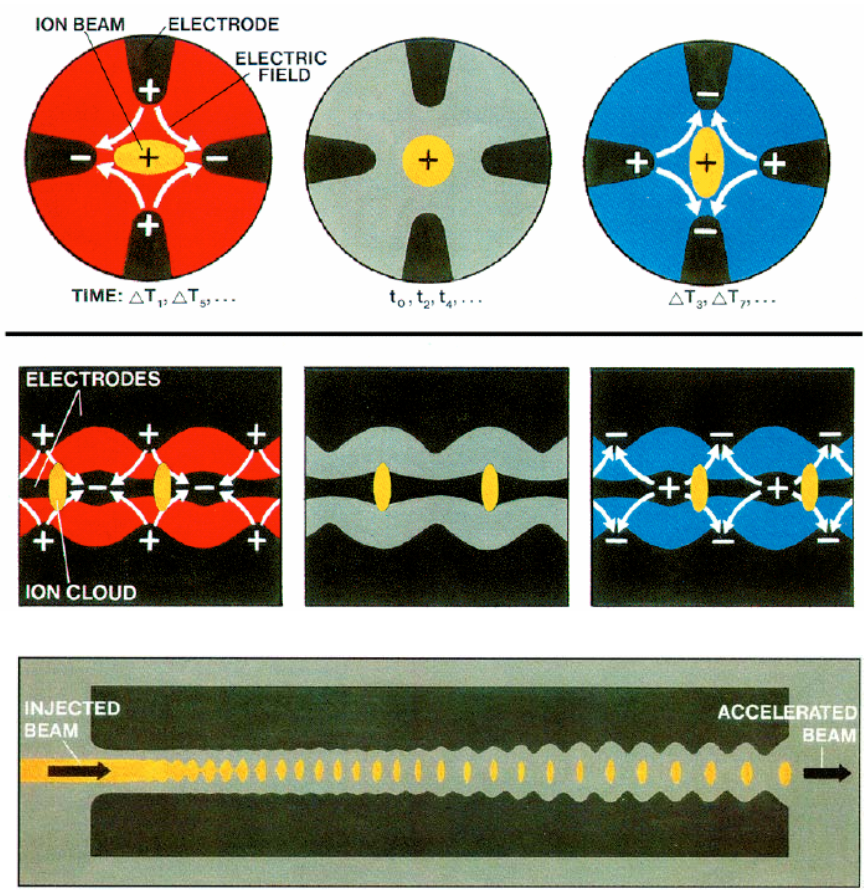
Pase focusing



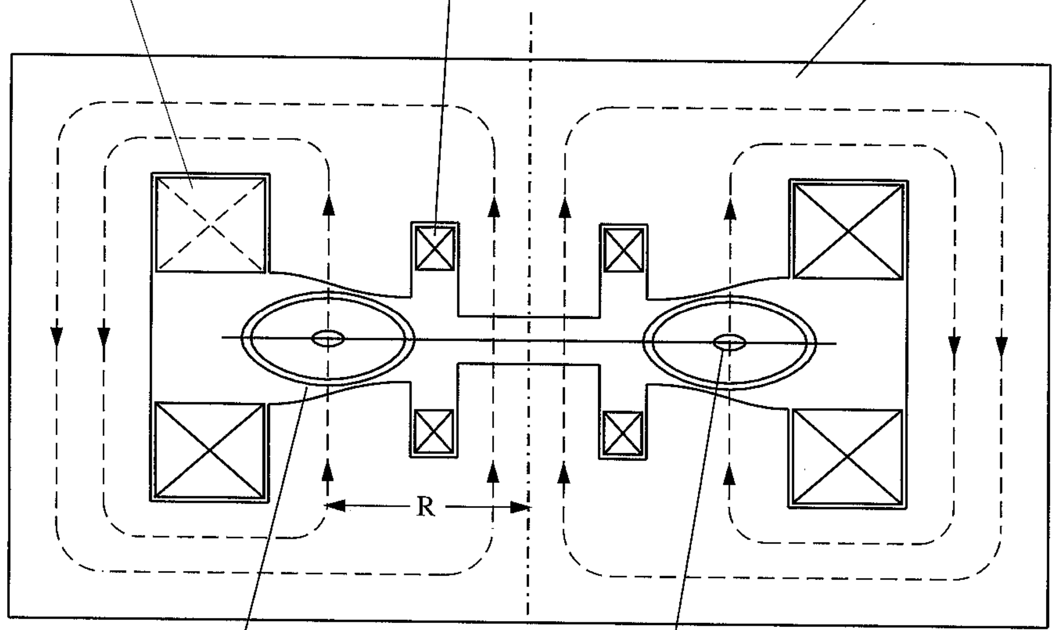
$$\psi_0 \in (0, \frac{\pi}{2})$$

The RF quadrupole (RFQ)

- 1970: Kapchinskii and Teplyakov invent the RFQ

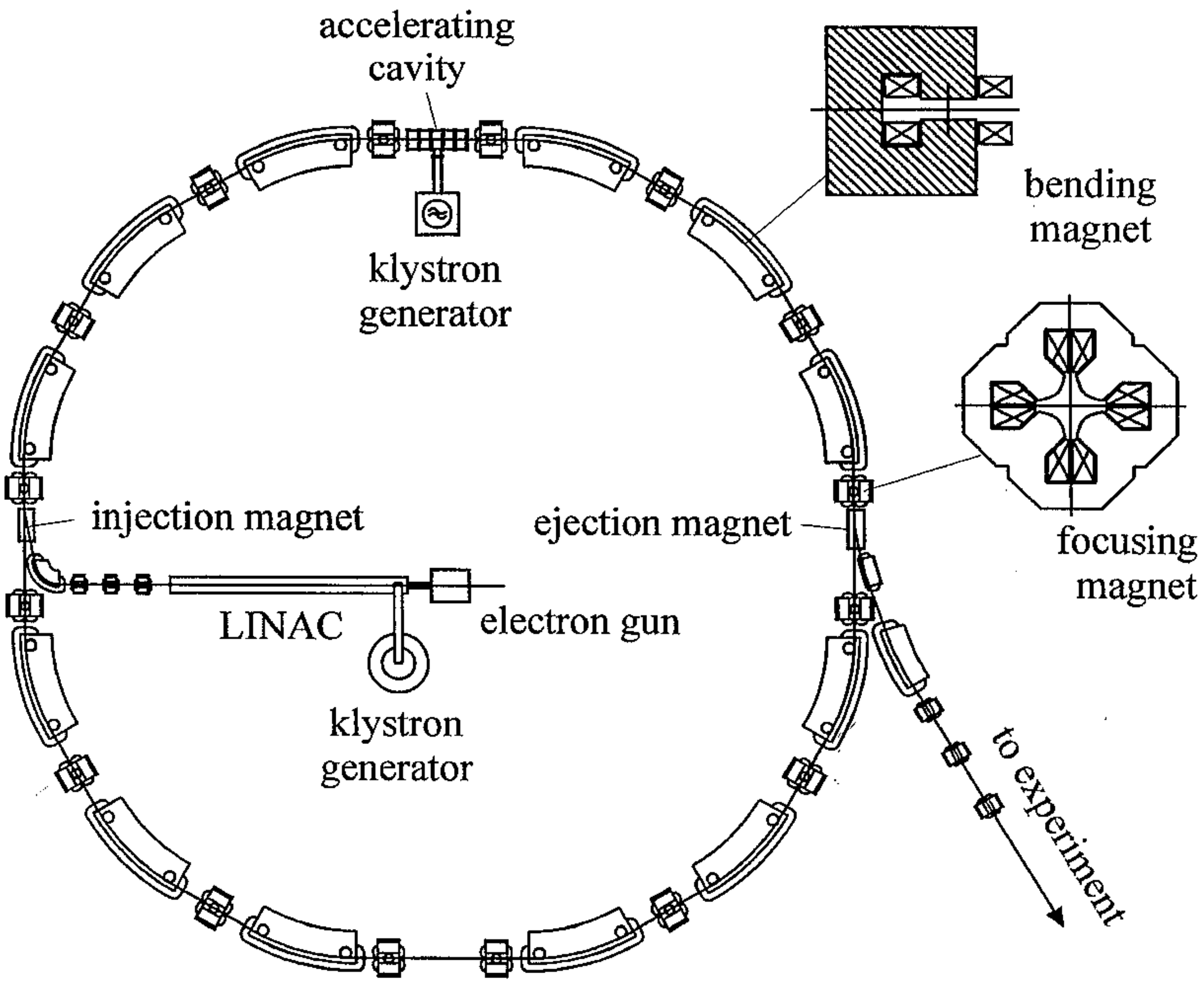


Betatrons



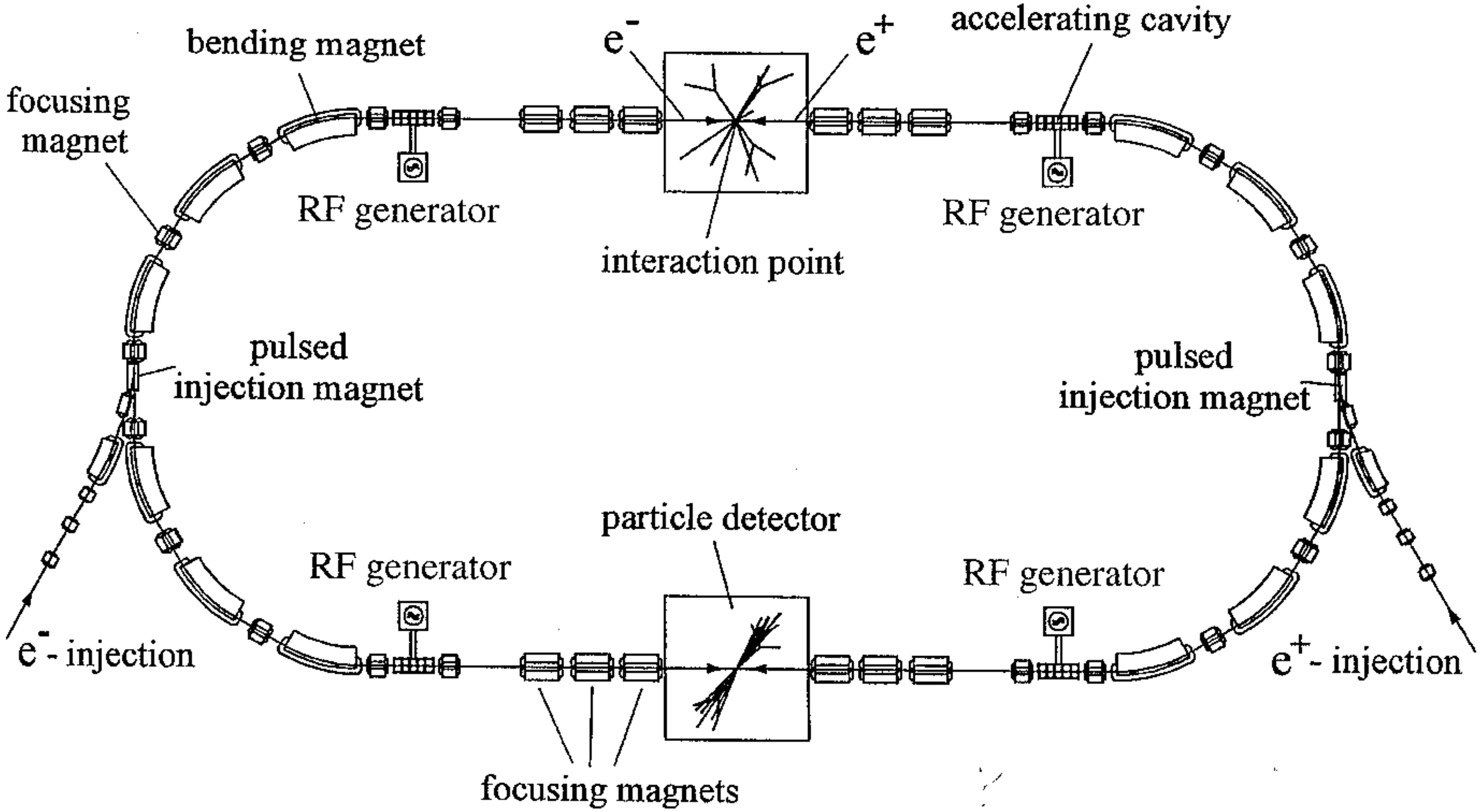
The Synchrotron

CHESS & LEPP



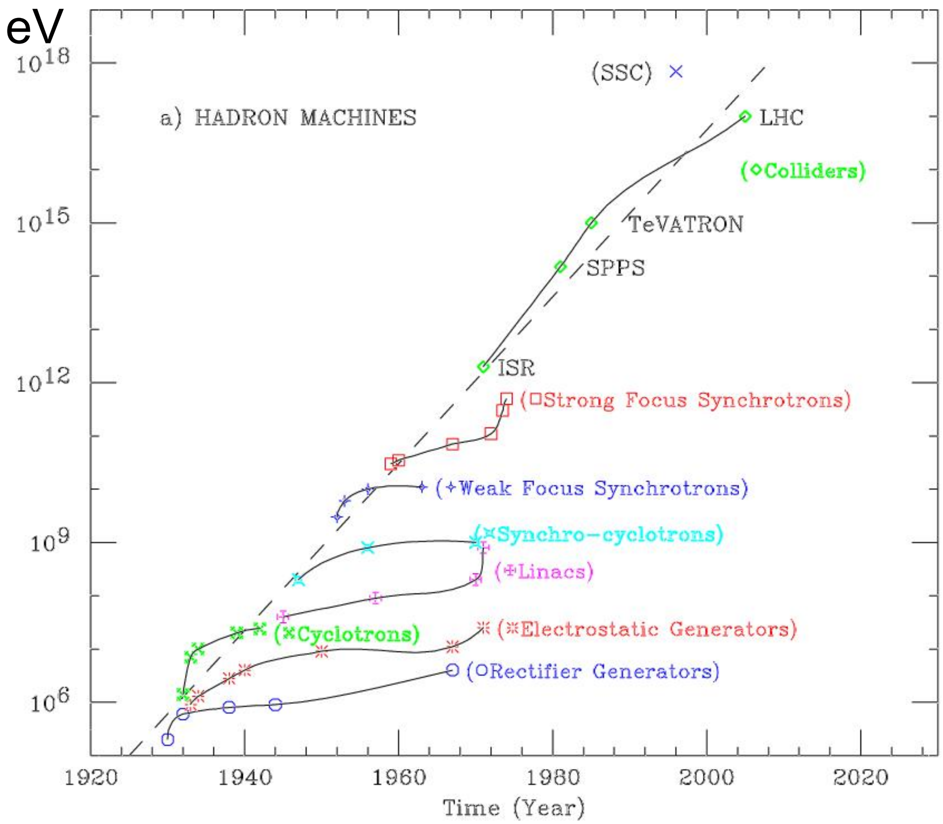
Ellements of a Collider

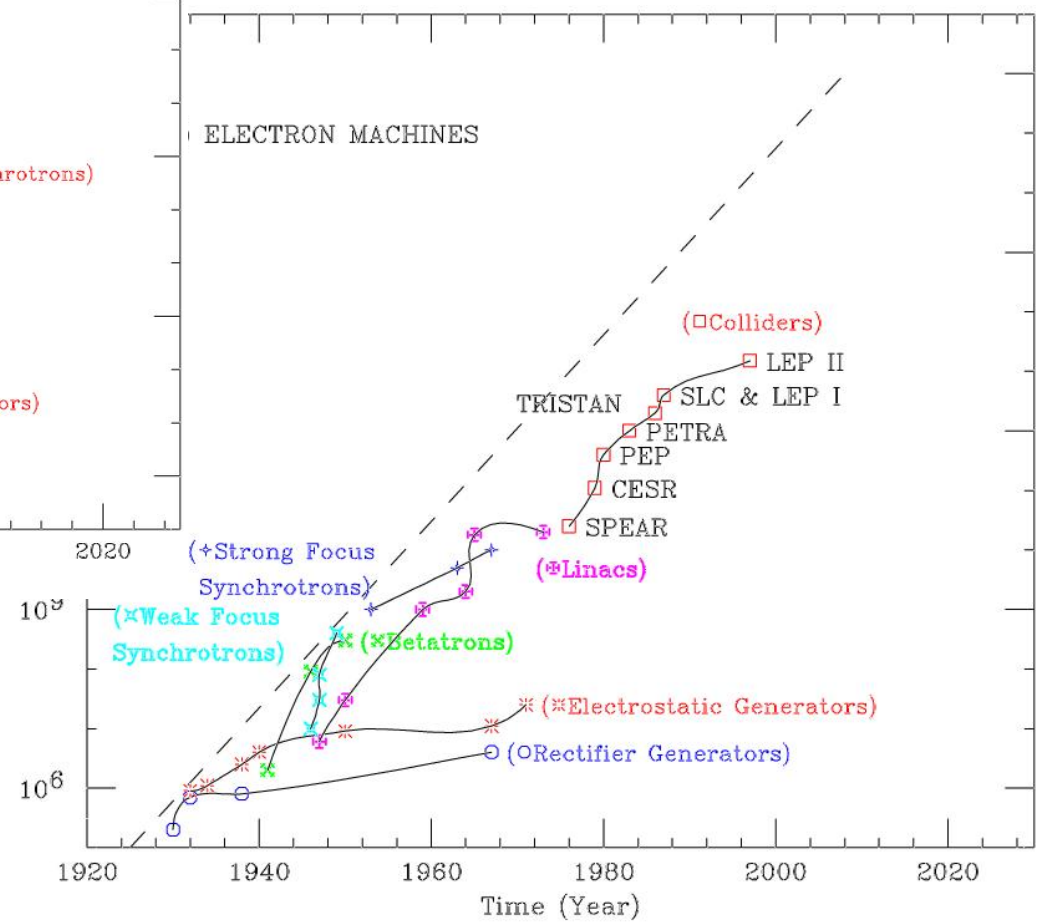
CHESS & LEPP



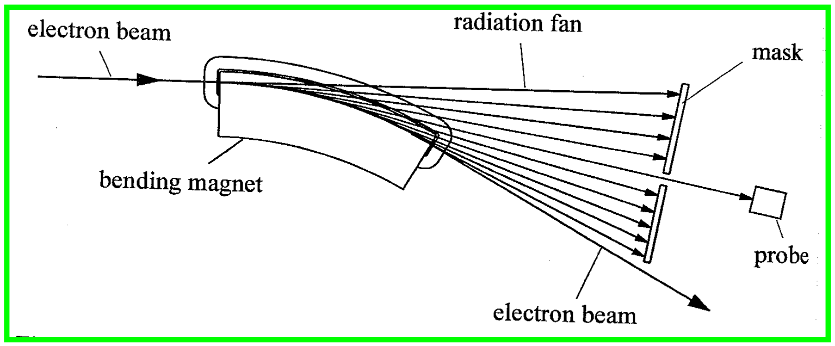
The Livingston Chart

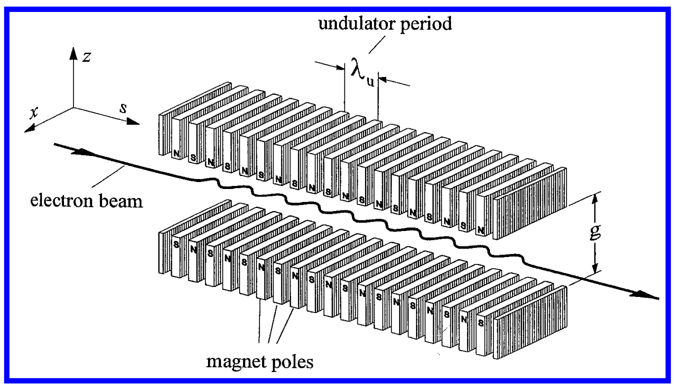
CHESS & LEPP

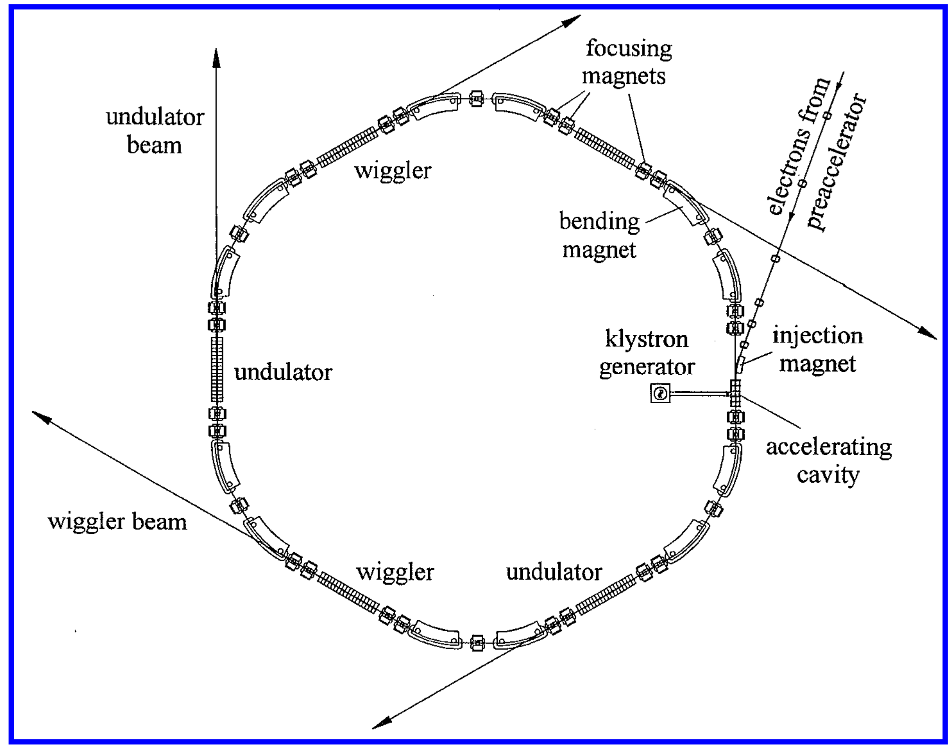




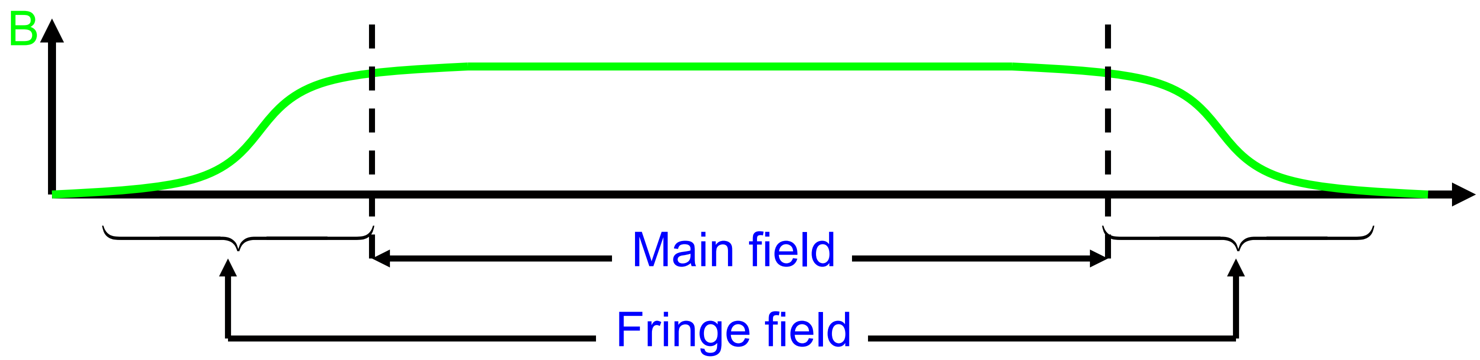
3 Generations of Light Sources







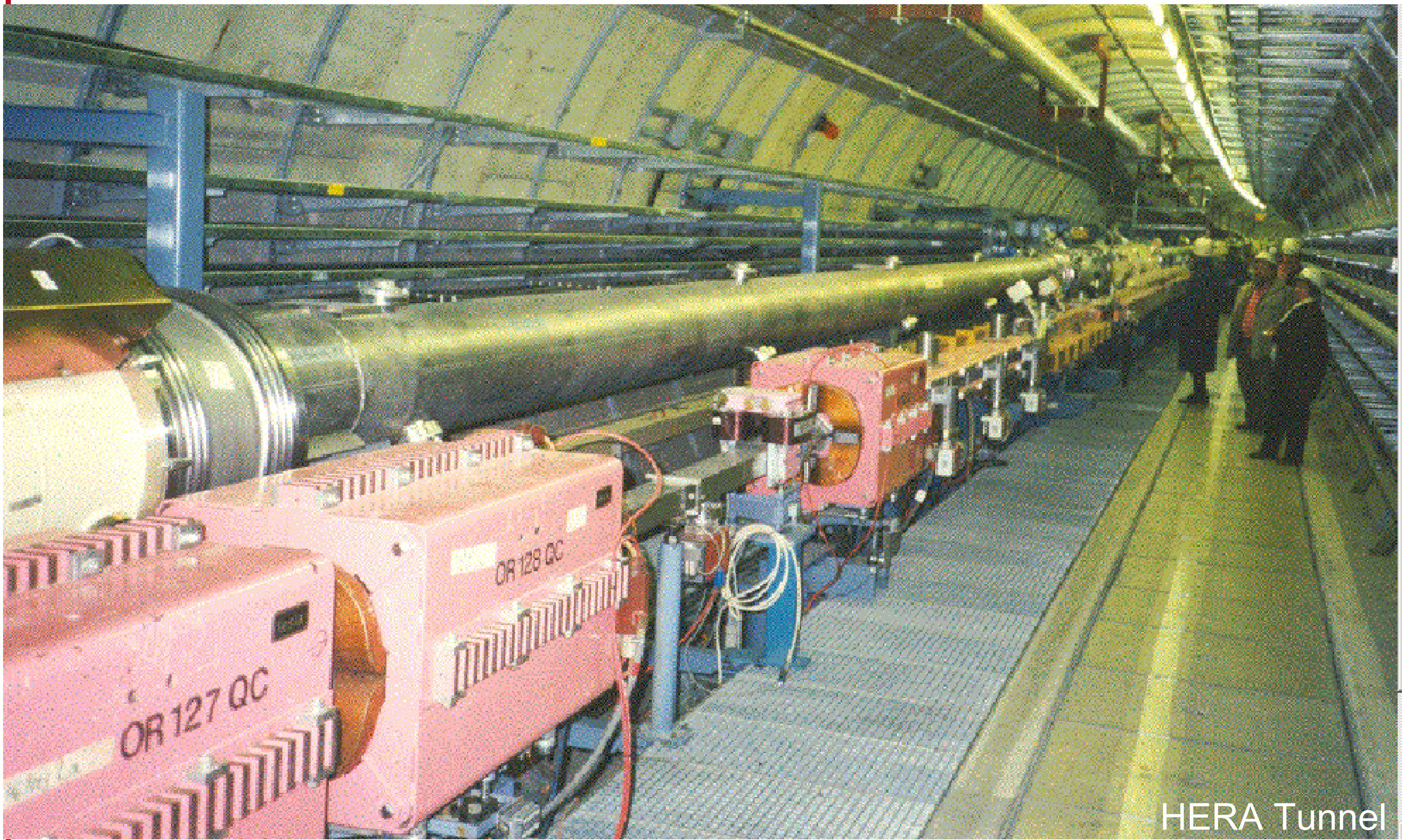
Fringe Fields and Main Fields



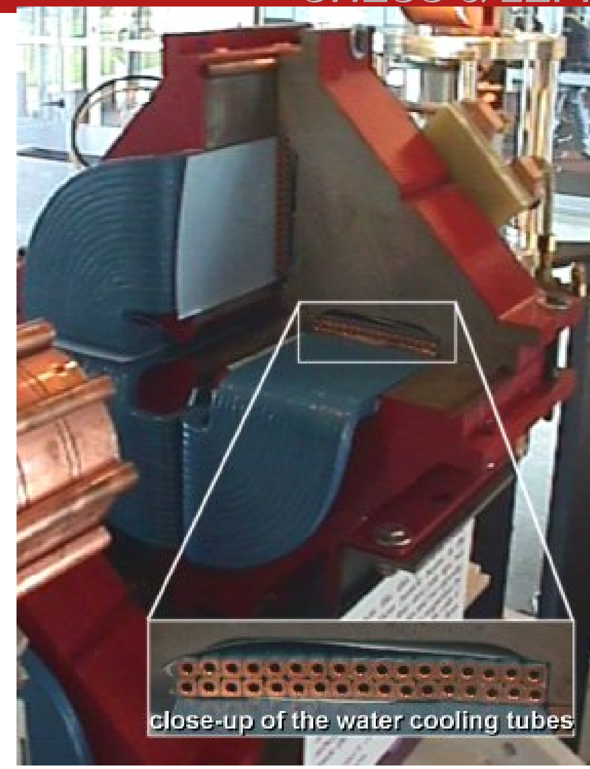
$$\psi(r,\varphi) = \sum_{\nu=1}^{\infty} |\Psi_\nu| \operatorname{Im}\{r^\nu e^{-i\nu(\varphi-\vartheta_\nu)}\} + |\Psi_0| = \sum_{\nu=1}^{\infty} \operatorname{Im}\{\Psi_\nu \overline{w}^\nu\} + |\Psi_0|$$

Where is the vertical Dipole?

CHESS & LEPP



Real Quadrupoles



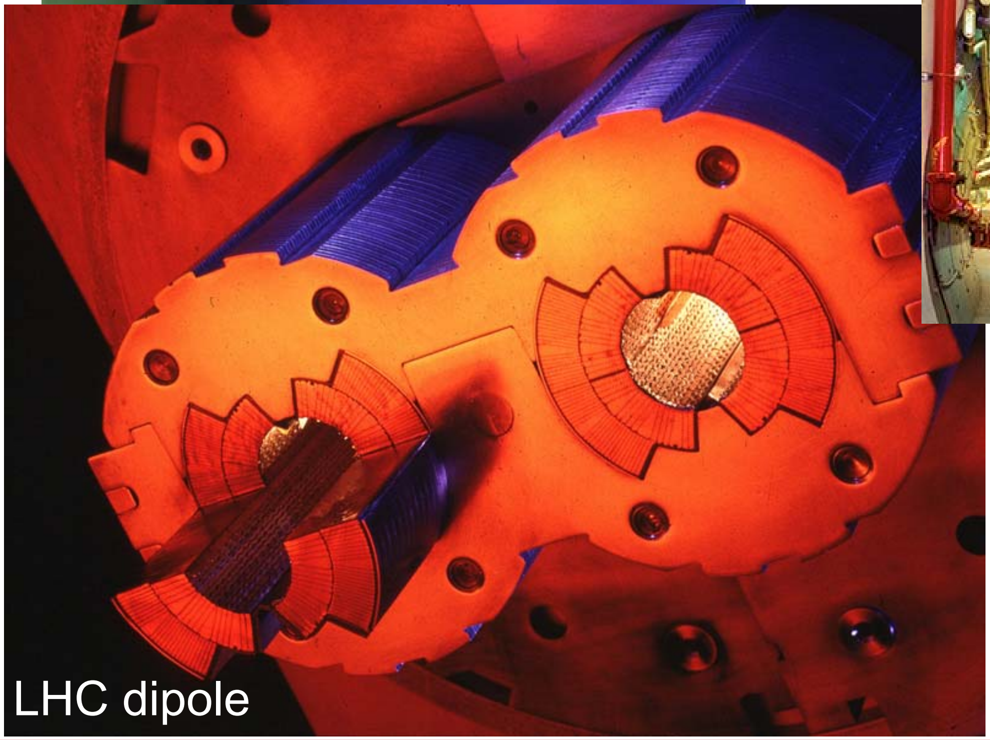
The coils show that this is an upright quadrupole not a rotated or skew quadrupole.

The CESR Tunnel

CHESS & LEPP

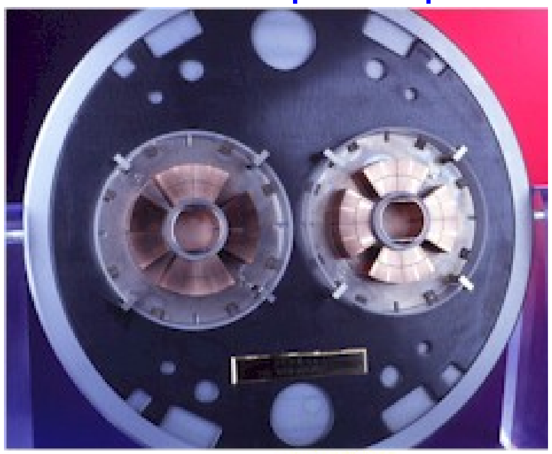
Real Air-coil Multipoles

Quadrupole corrector

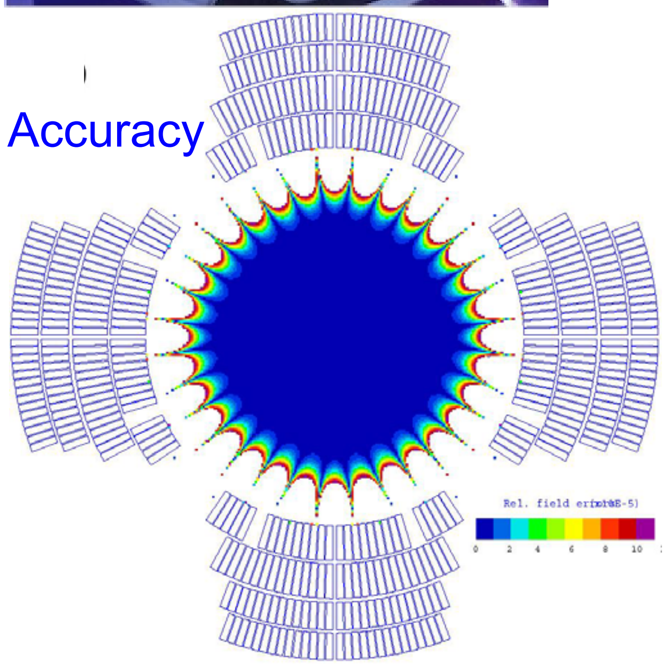


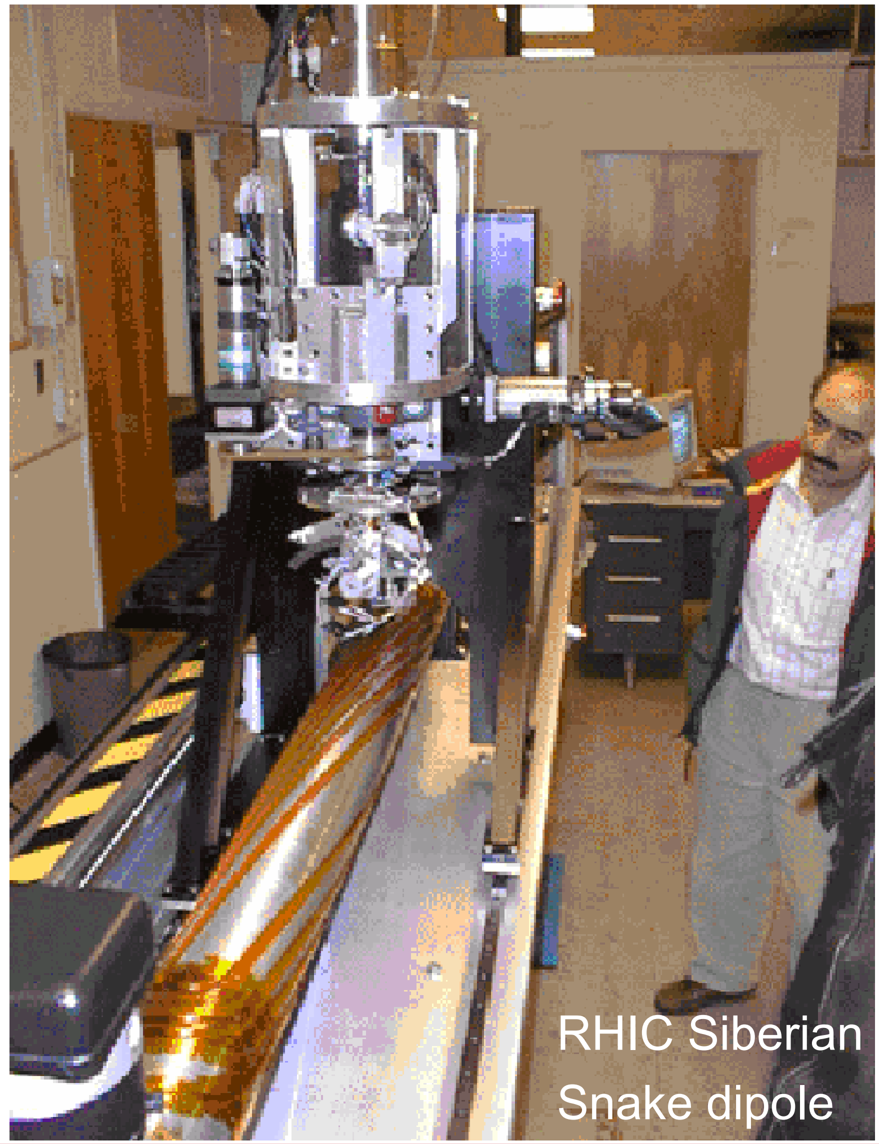
Special SC Air-coil Magnets

LHC double quadrupole



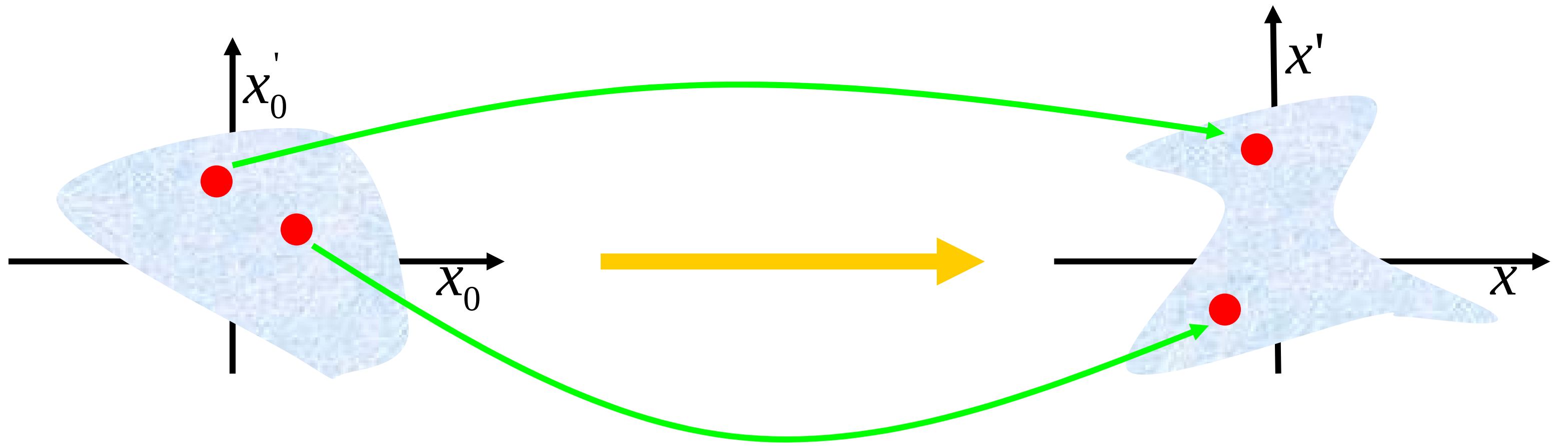
Accuracy





Liovile's Theorem

- A phase space volume does not change when it is transported by Hamiltonian motion. $\vec{z}(s) = \underline{M}(s) \cdot \vec{z}_0$ with $\det[\underline{M}(s)] = +1$



Volume = $V = \iint_V d^n\vec{z} = \iint_{V_0} \left|\frac{\partial \vec{z}}{\partial \vec{z}_0}\right| d^n\vec{z}_0 = \iint_{V_0} |\underline{M}| d^n\vec{z}_0 = \iint_{V_0} d^n\vec{z}_0 = V_0$

Hamiltonian Motion → $V = V_0$

But Hamiltonian requires symplecticity, which is much more than just $\det[\underline{M}(s)] = +1$

Symplectic Representations

Hamiltonian

$$\vec{z}\,' = \underline{J}\,\vec{\partial} H(\vec{z}, s)$$

ODE

$$\vec{z}\,' = \vec{F}\,, \quad \underline{F}\,\underline{J} + \underline{J}\,\underline{F}^T = 0$$

Generating Functions

$$(\vec{p}, \vec{p}_0) = -\underline{J}\,\vec{\partial} F_1(\vec{q}, \vec{q}_0, s)$$

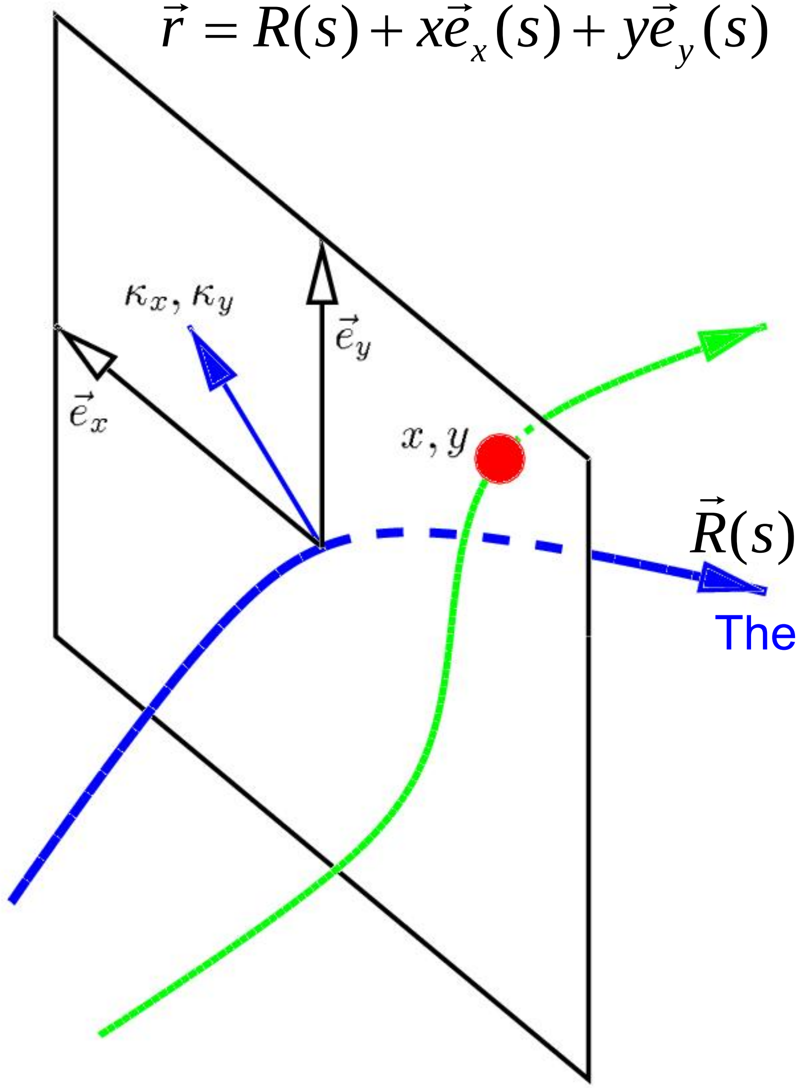
[from notes]

Symplectic transport map

$$\underline{M}\,\underline{J}\,\underline{M}^T = \underline{J}$$

The comoving Coordinate System

$$\vec{r} = \vec{R}(s) + x\vec{e}_x(s) + y\vec{e}_y(s)$$



$$|d\vec{R}| = ds$$

$$\vec{e}_s \equiv \tfrac{d}{ds}\vec{R}(s)$$

The time dependence of a particle's motion is often not as interesting as the trajectory along the accelerator length "s".

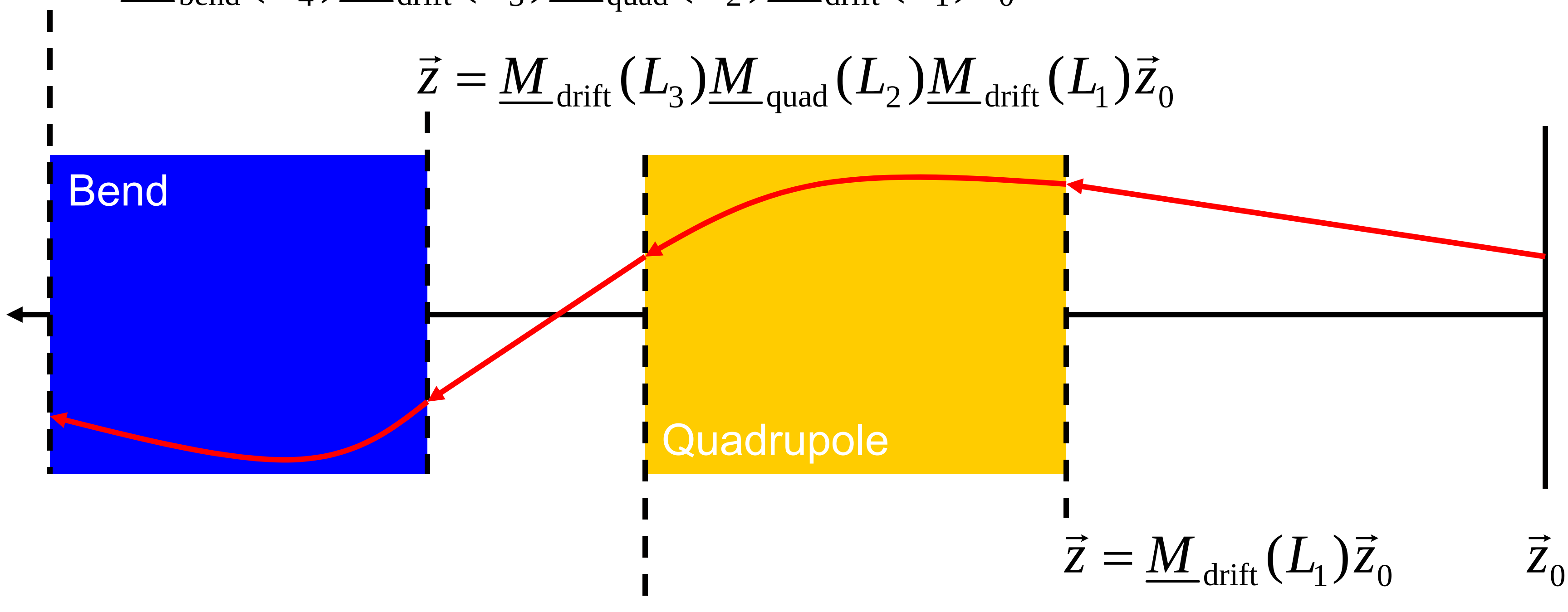
Matrix Solutions

Linear equation of motion: $\vec{z}' = \underline{F}(s)\vec{z}$

Matrix solution of the starting condition $\vec{z}(0) = \vec{z}_0$

$$\vec{z} = \underline{M}_{\text{bend}}(L_4)\underline{M}_{\text{drift}}(L_3)\underline{M}_{\text{quad}}(L_2)\underline{M}_{\text{drift}}(L_1)\vec{z}_0$$

$$\vec{z} = \underline{M}_{\text{drift}}(L_3)\underline{M}_{\text{quad}}(L_2)\underline{M}_{\text{drift}}(L_1)\vec{z}_0$$



$$\vec{z} = \underline{M}_{\text{drift}}(L_1)\vec{z}_0 \qquad \vec{z}_0$$

$$\vec{z} = \underline{M}_{\text{quad}}(L_2)\underline{M}_{\text{drift}}(L_1)\vec{z}_0$$

Variation of Constants

CHESS & LEPP

$$\vec{z}\,' = \vec{f}(\vec{z}, s)$$

$$\vec{z}\,' = \underline{L}(s)\vec{z} + \Delta\vec{f}(\vec{z}, s)$$

Field errors, nonlinear fields, etc can lead to $\Delta\vec{f}(\vec{z}, s)$

$$\vec{z}\,'_H = \underline{L}(s)\vec{z}_H \quad \Rightarrow \quad \vec{z}_H(s) = \underline{M}(s)\vec{z}_{H0} \quad \text{with} \quad \underline{M}'(s)\vec{a} = \underline{L}(s)\underline{M}(s)\vec{a}$$

$$\vec{z}(s) = \underline{M}(s)\vec{a}(s) \quad \Rightarrow \quad \vec{z}\,'(s) = \underline{M}'(s)\vec{a} + \underline{M}(s)\vec{a}\,'(s) = \underline{L}(s)\vec{z} + \Delta\vec{f}(\vec{z}, s)$$

$$\vec{a}(s) = \vec{z}_0 + \int_0^s \underline{M}^{-1}(\hat{s})\Delta\vec{f}(\vec{z}(\hat{s}), \hat{s})\, d\hat{s}$$

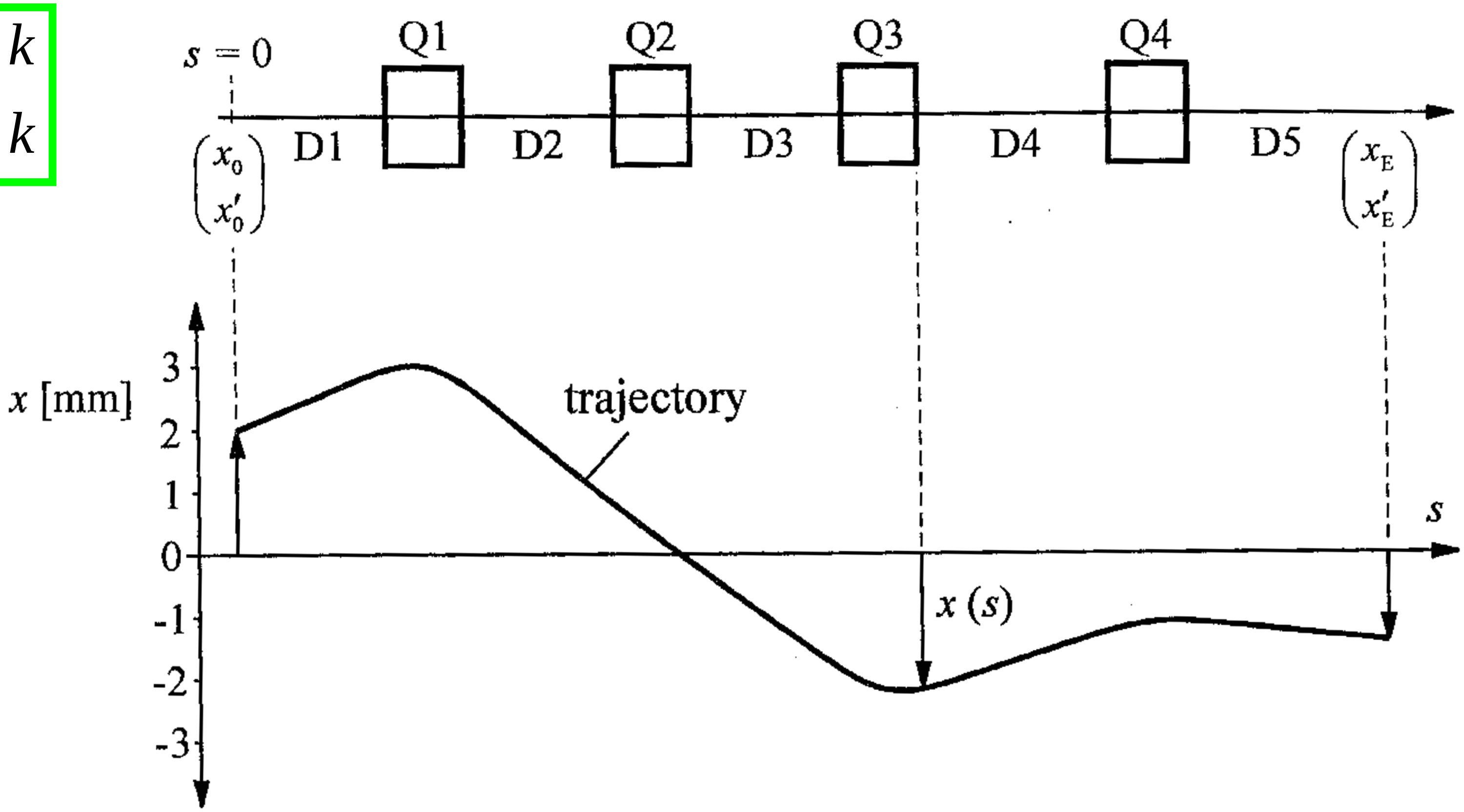
$$\vec{z}(s) = \underline{M}(s)\left\{\vec{z}_0 + \int_0^s \underline{M}^{-1}(\hat{s})\Delta\vec{f}(\vec{z}(\hat{s}), \hat{s})\, d\hat{s}\right\}$$

$$= \vec{z}_H(s) + \int_0^s \underline{M}(s - \hat{s})\Delta\vec{f}(\vec{z}(\hat{s}), \hat{s})\, d\hat{s}$$

Perturbations are propagated from s to s’

Beta Function and Betatron Phase

$$x'' = -x\,k$$
$$y'' = \ \ y\,k$$



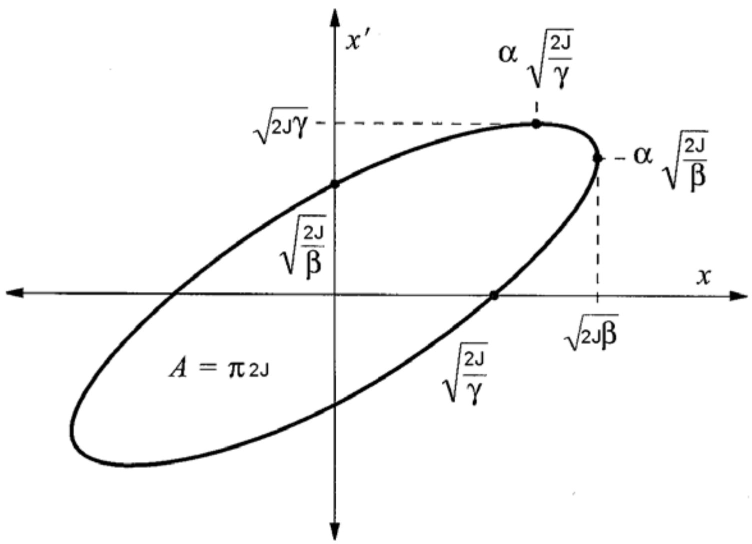
$$x(s) = M_{11}(s)x_0 + M_{12}(s)x_0'$$
$$x(s) = \sqrt{2J\beta(s)}\,\sin(\psi(s)+\phi_0)$$

Twiss Parameters

$$\beta' = -2\alpha$$
$$\alpha' = k\beta - \gamma$$
$$\psi = \int_0^s \frac{1}{\beta(s')} ds'$$

$$(x, x')\begin{pmatrix} \gamma & \alpha \\ \alpha & \beta \end{pmatrix}\begin{pmatrix} x \\ x' \end{pmatrix} = 2J$$

Area: $2\pi\, J$



$$\langle x^2 \rangle = \frac{1}{2\pi\varepsilon}\iint 2J\beta \sin\phi_0^2 e^{-J/\varepsilon} dJ d\phi_0 = \varepsilon\beta \longrightarrow \langle x'^2 \rangle = \varepsilon\gamma$$

$$\langle xx' \rangle = -\frac{1}{2\pi\varepsilon}\iint 2J\alpha \sin\phi_0^2 e^{-J/\varepsilon} dJ d\phi_0 = \varepsilon\alpha$$

$$\varepsilon = \sqrt{\langle x^2 \rangle\langle x'^2 \rangle - \langle xx' \rangle^2}$$ is called the emittance.

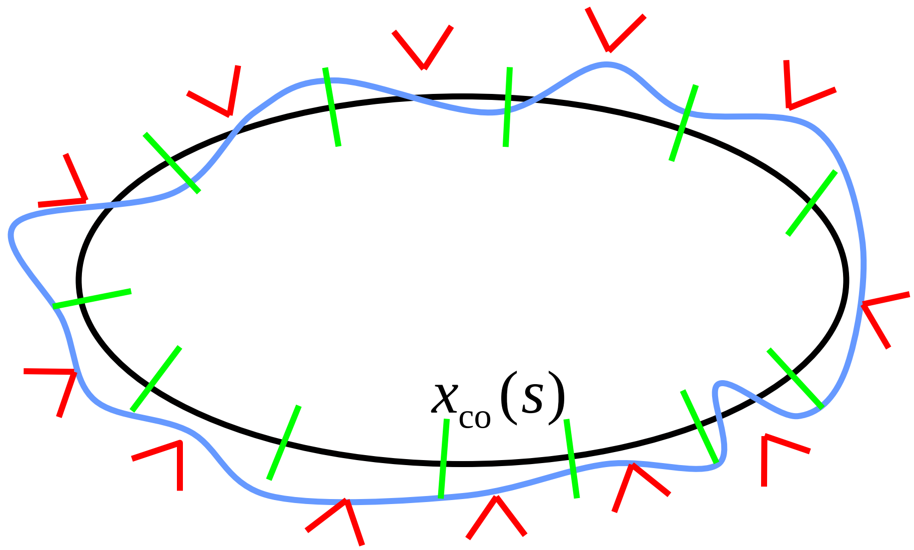
Propagation of Twiss Parameters

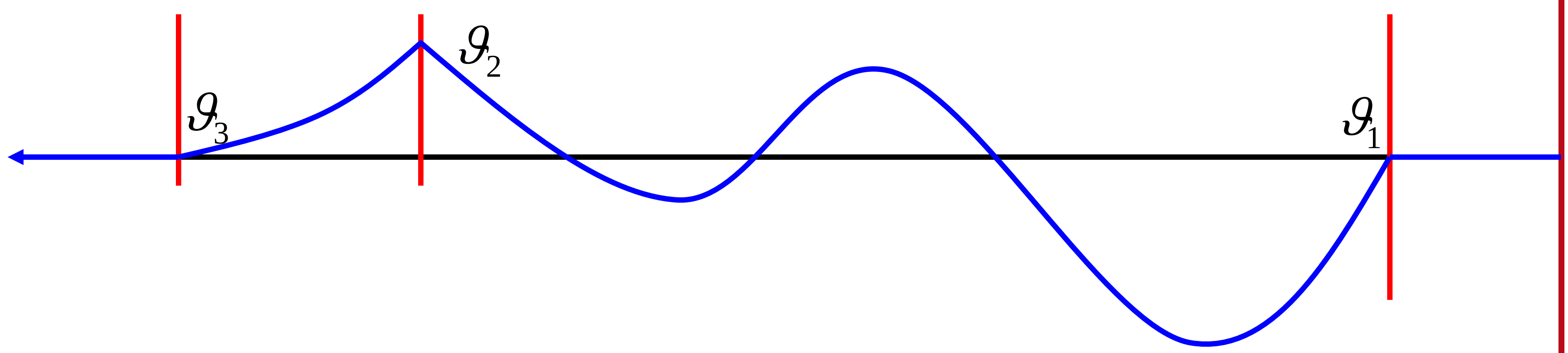
$$\begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \underline{M} \begin{pmatrix} \beta_0 & -\alpha_0 \\ -\alpha_0 & \gamma_0 \end{pmatrix} \underline{M}^T$$

$$\underline{M}(s) = \begin{pmatrix} \sqrt{\beta} & 0 \\ -\frac{\alpha}{\sqrt{\beta}} & \frac{1}{\sqrt{\beta}} \end{pmatrix} \begin{pmatrix} \cos\psi(s) & \sin\psi(s) \\ -\sin\psi(s) & \cos\psi(s) \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{\beta_0}} & 0 \\ \frac{\alpha_0}{\sqrt{\beta_0}} & \sqrt{\beta_0} \end{pmatrix}$$

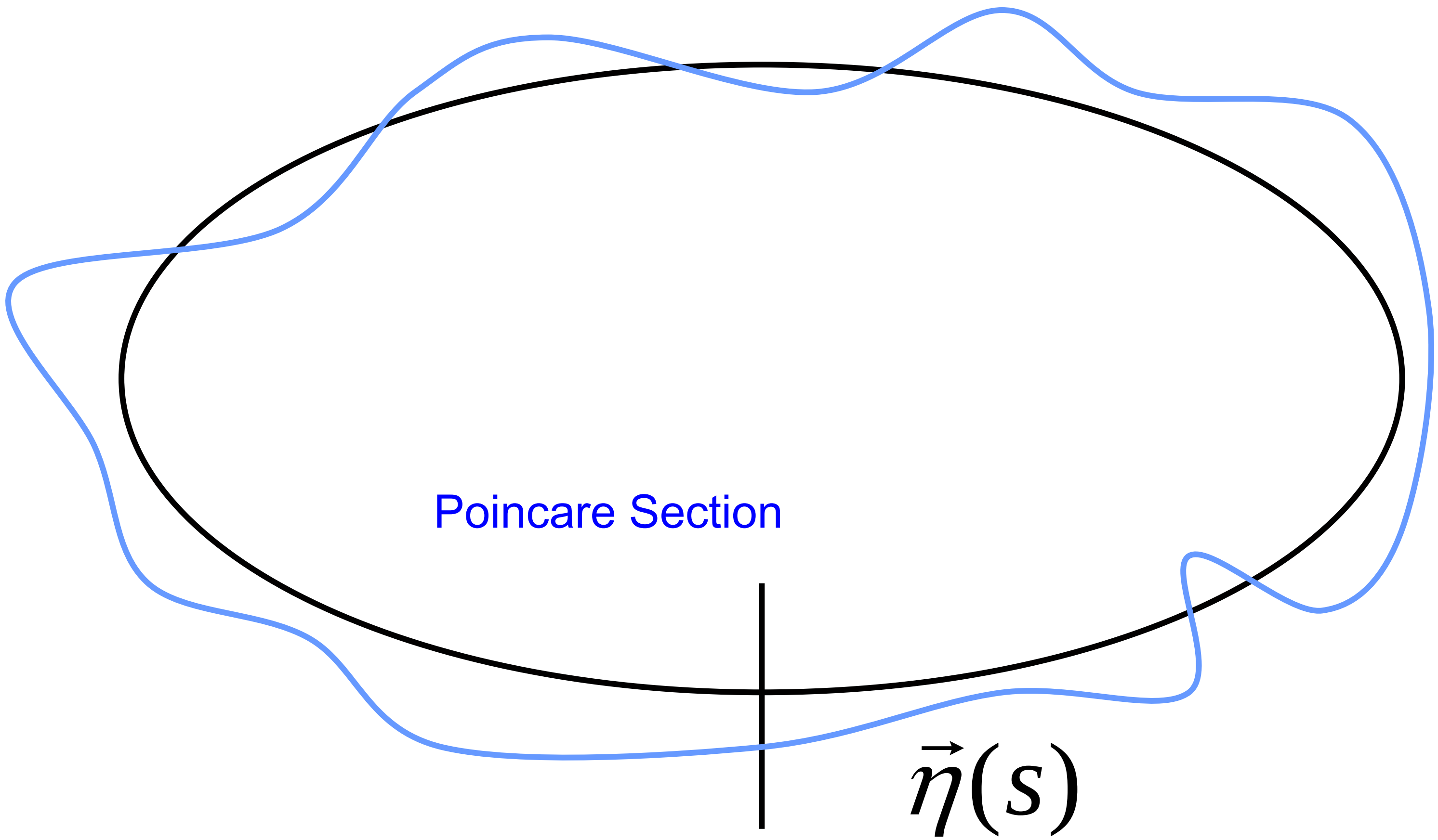
$$= \begin{pmatrix} \sqrt{\frac{\beta}{\beta_0}}[\cos\psi + \alpha_0 \sin\psi] & \sqrt{\beta_0\beta}\sin\psi \\ \sqrt{\frac{1}{\beta_0\beta}}[(\alpha_0 - \alpha)\cos\psi - (1 + \alpha_0\alpha)\sin\psi] & \sqrt{\frac{\beta_0}{\beta}}[\cos\psi - \alpha\sin\psi] \end{pmatrix}$$

Closed Orbit Correction and Bumps

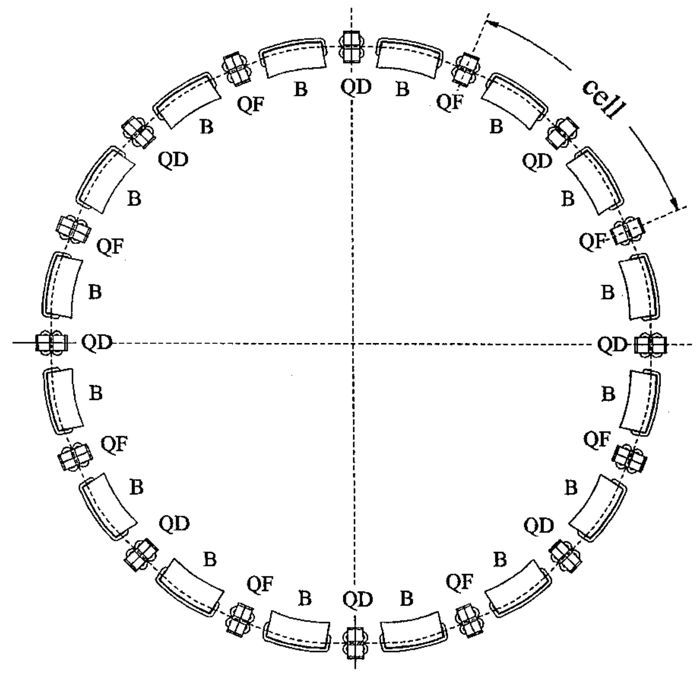


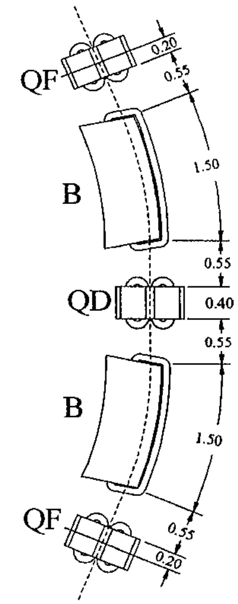


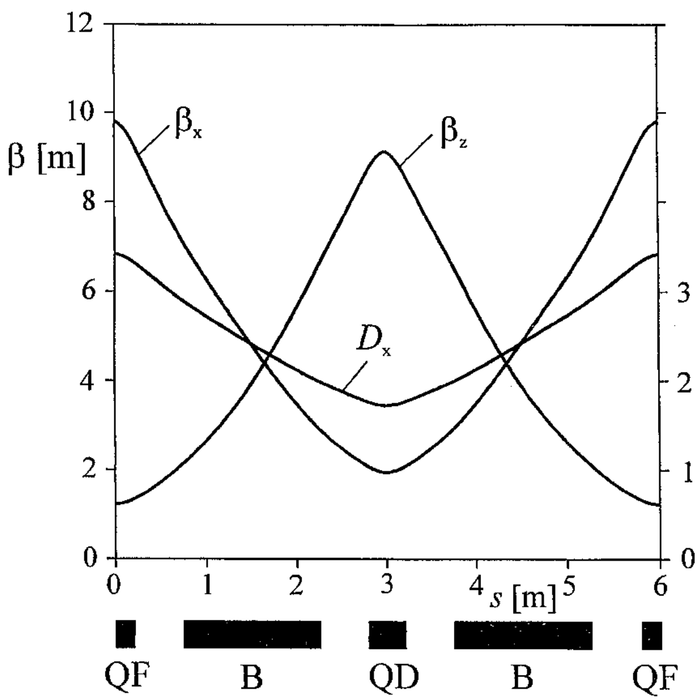
The Periodic Dispersion



The FODO Cell







Quadrupole Error and Tune Shift

One quadrupole error:

$$\Delta\nu = \frac{\beta}{4\pi}\Delta k$$

One quadrupole error:

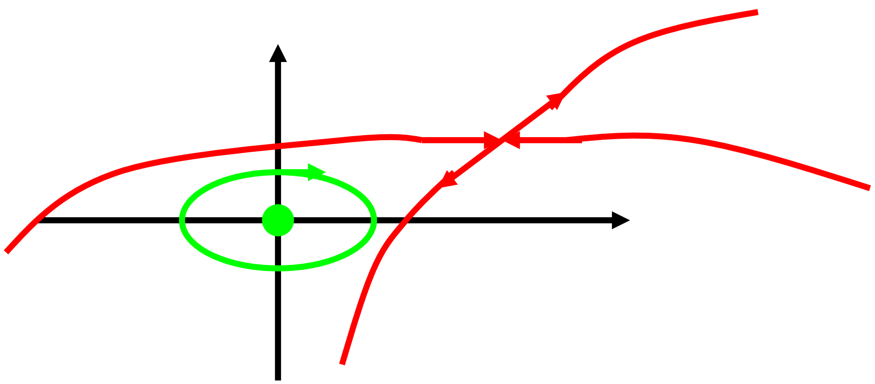
$$\Delta\beta = -\frac{\beta}{2\sin\mu}\Delta\hat{k}\hat{\beta}\cos(2|\hat{\psi}-\psi|-\mu)$$

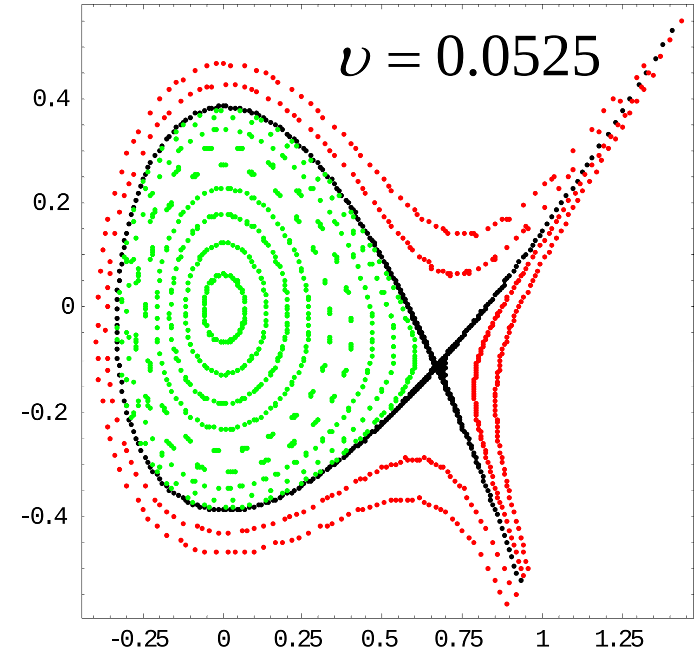
Chromaticity:

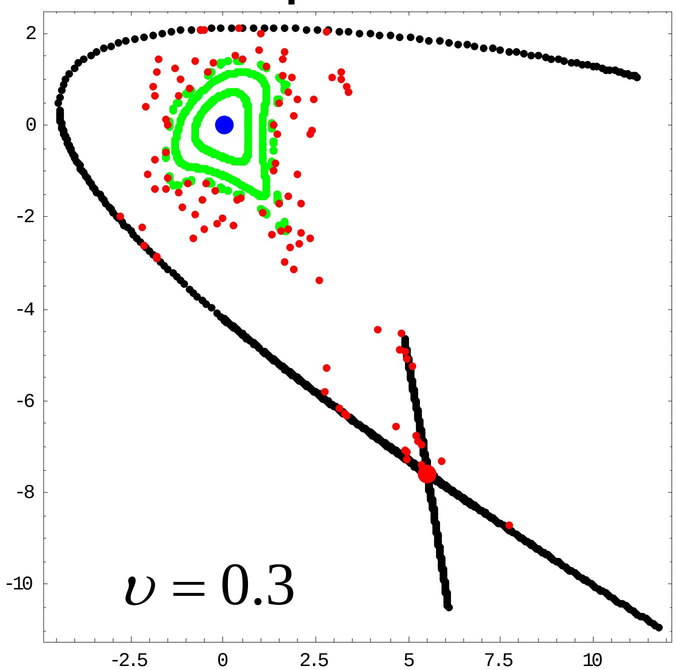
$$\xi_x = \frac{1}{4\pi}\oint\beta_x(-k_1+\eta_x k_2)d\hat{s}$$

The Dynamic Aperture

Stable and unstable fixed points !



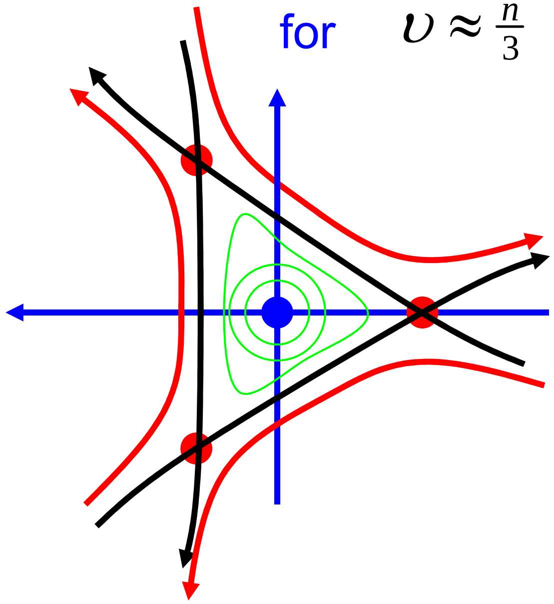




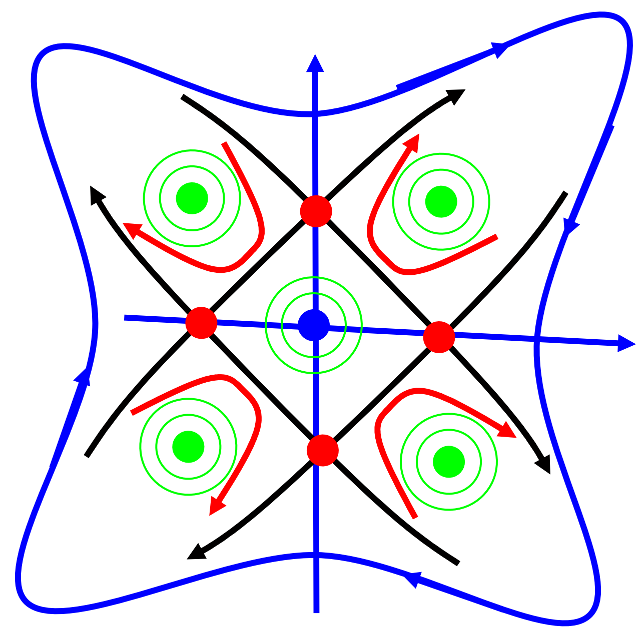
Caotic motion close to fixed points !

Resonances

for $\upsilon \approx \frac{n}{3}$



$\upsilon \approx \frac{n}{4}$

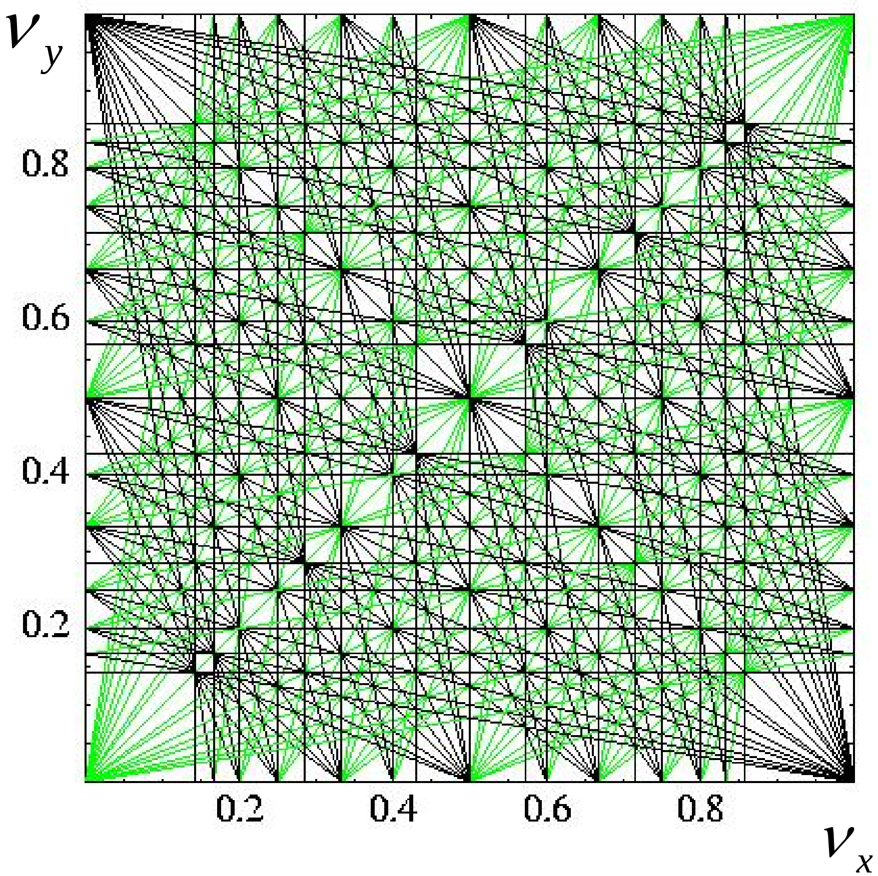


$$\upsilon(J) = \upsilon + \partial_J \langle \Delta H \rangle_{\varphi,\vartheta}(J)$$

$$\boxed{n + m\,\upsilon = 0}$$

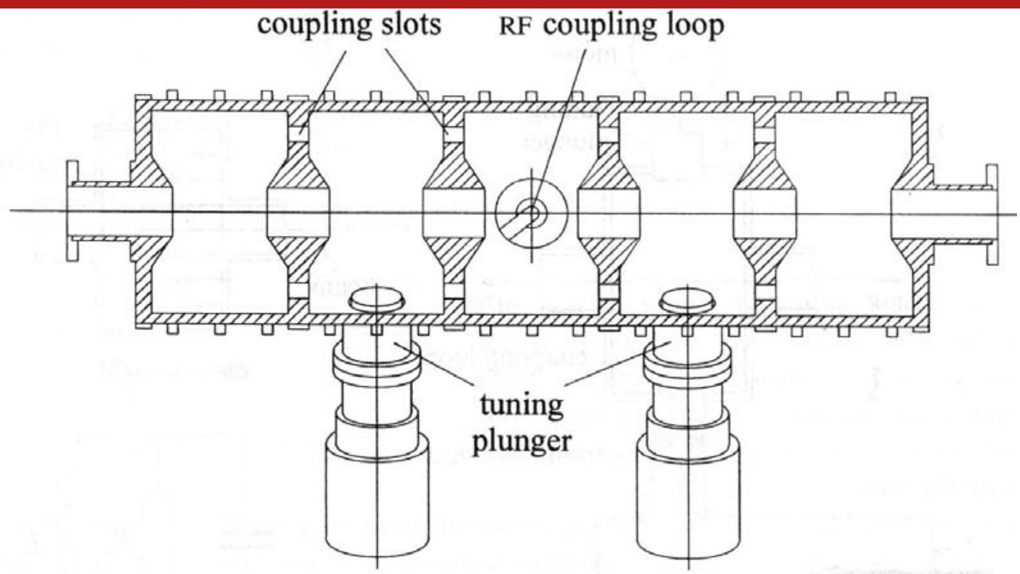
Resonances Diagram

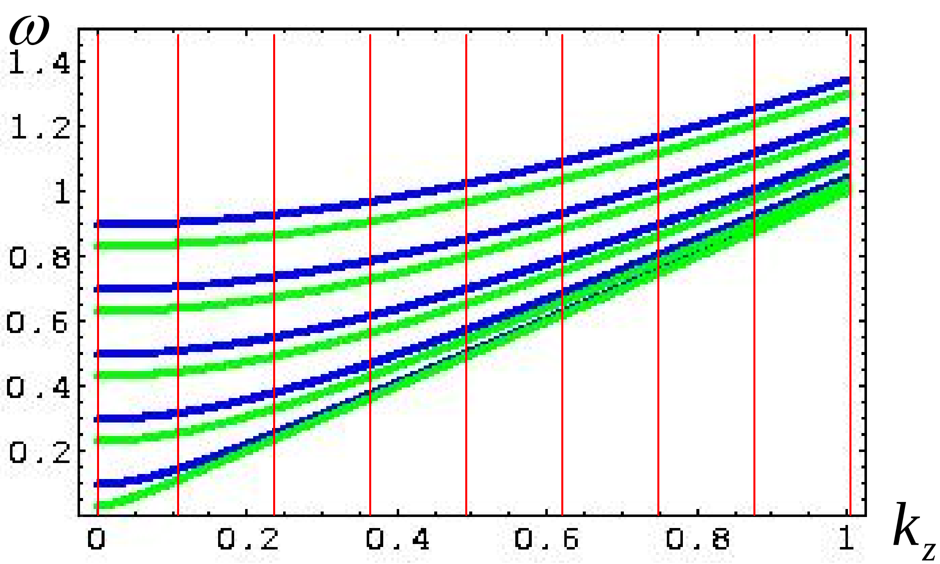
$n + m_x \upsilon_x + m_y \upsilon_y \approx 0$ means that oscillations in y can drive oscillations in x in



Dispersion relation and Cavities

CHESS & LEPP





k_z