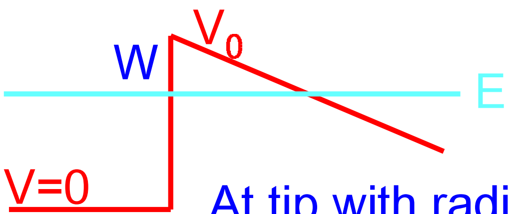
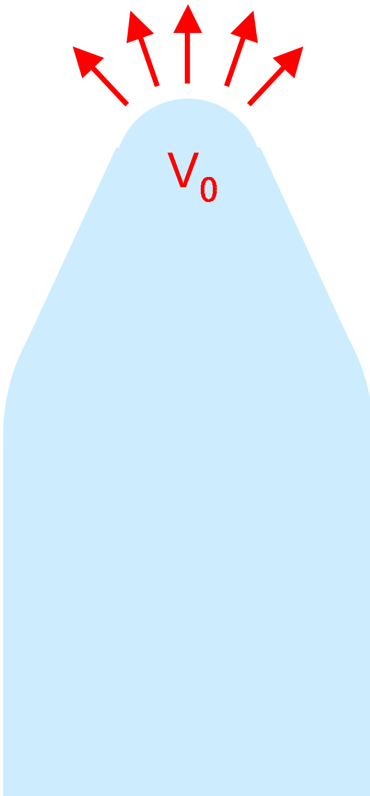
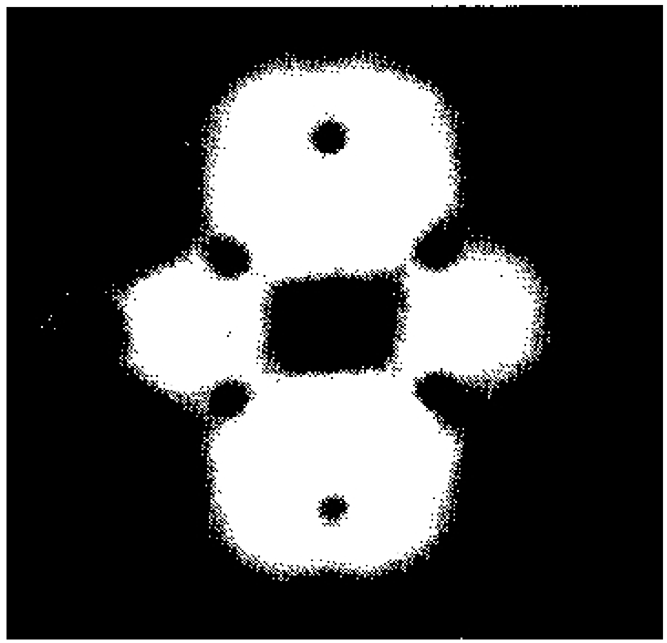
04/11/2005

Field emission microscopy

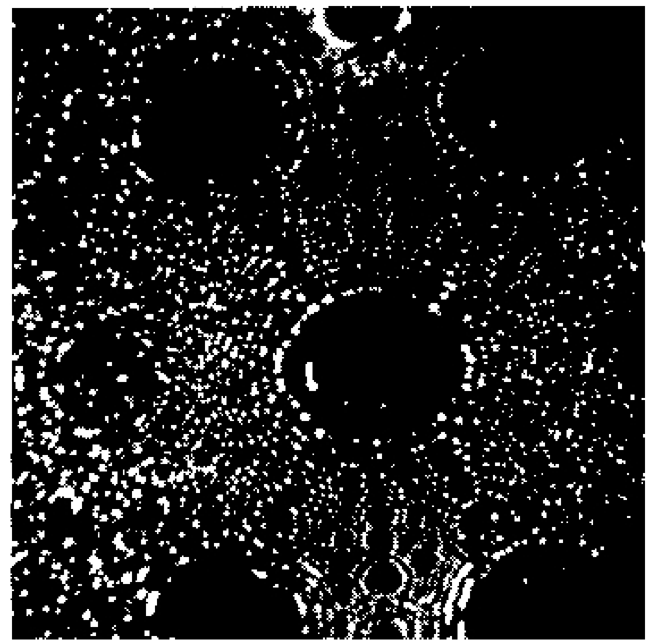


At tip with radius R = 500nm and V_0 = 500V has $\mathbf{E_{field}}$ = V/R = 1GV/m





Electron field-emission microscopic image

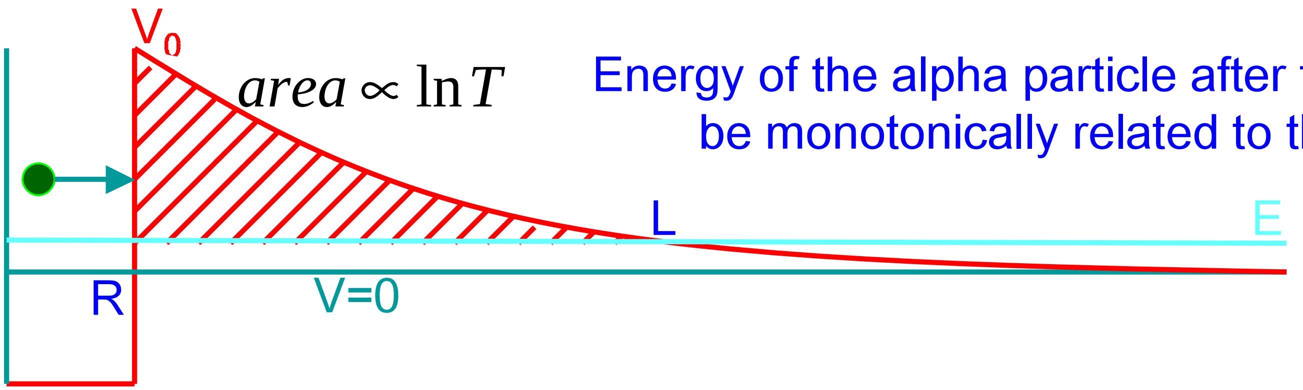


Helium ion field-emission microscopic image (He emission of a previously applied Helium)

Georg.Hoffstaetter@Cornell.edu

The alpha decay

The transmission probability T for an alpha particle traveling from the inside towards the potential well that keeps the nucleus together determines the lifetime for alpha decay.



Energy of the alpha particle after the decay has to be monotonically related to the lifetime of the alpha decay.

$$T \approx \exp[-2\int_R^L \frac{\sqrt{2m[V(r)-E]}}{\hbar} dr]$$

Georg.Hoffstaetter@Cornell.edu

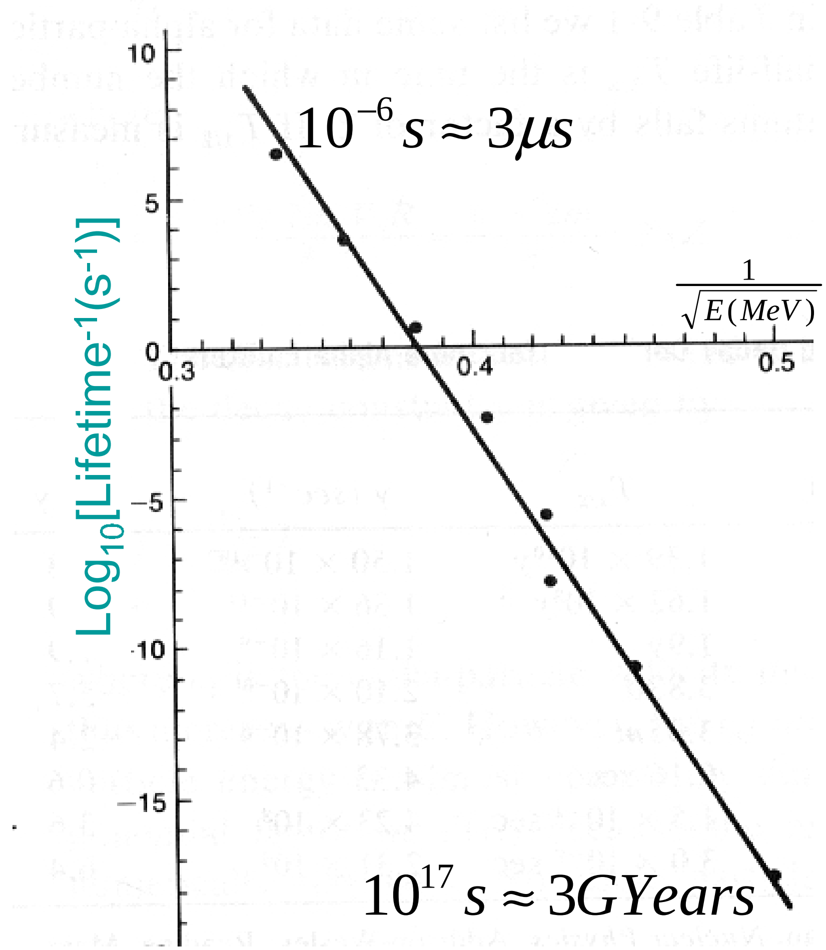
Scaling alpha decay times with Energy

The correct scaling of the alpha decay times with the alpha energies after the decay was one of the early successes of Quantum Mechanics, and of barrier penetration in 1928 (George Gamow).

$$\ln T \propto A - \frac{C}{\sqrt{E}}$$

$$C = \frac{\pi\sqrt{2m}}{\hbar} V_0 R = \frac{\pi\sqrt{2m}}{\hbar} 2(Z-2)e^2$$

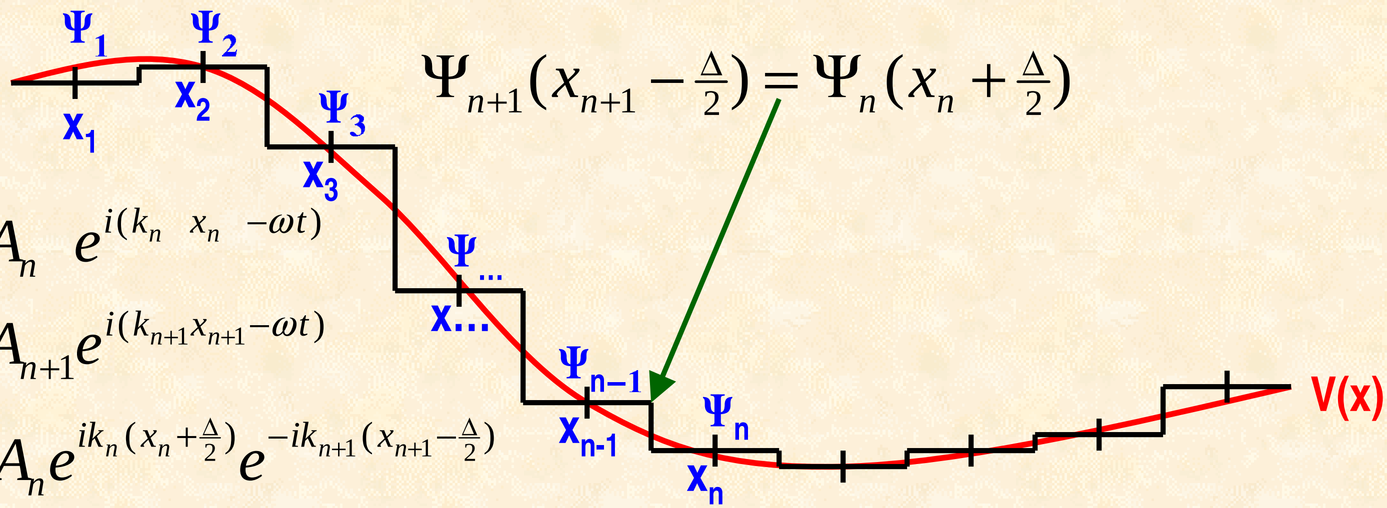
$$\text{Lifetime} \propto T^{-1}$$



Georg.Hoffstaetter@Cornell.edu

The Schrödinger equation in three dimensions

Review of one dimension:



$$\Psi_{n+1}(x_{n+1}-\tfrac{\Delta}{2}) = \Psi_n(x_n+\tfrac{\Delta}{2})$$

$$\Psi_n = A_n\ e^{i(k_n\ x_n\ -\omega t)}$$

$$\Psi_{n+1} = A_{n+1}e^{i(k_{n+1}x_{n+1}-\omega t)}$$

$$A_{n+1} = A_n e^{ik_n(x_n+\frac{\Delta}{2})}e^{-ik_{n+1}(x_{n+1}-\frac{\Delta}{2})}$$

$$\Psi_{n+1} = A_n e^{i[k_n(x_n+\frac{\Delta}{2})-k_{n+1}(x_{n+1}-\frac{\Delta}{2})+k_{n+1}x_{n+1}]\}}e^{-i\omega t} = \Psi_n e^{i[k_n\frac{\Delta}{2}+k_{n-1}\frac{\Delta}{2}]}$$

$$= \Psi_n e^{i\frac{k_{n+1}+k_n}{2}\Delta} \approx \Psi_n + ik_n\Psi_n\Delta \quad \rightarrow \quad \frac{\partial}{\partial x}\Psi(x,t) = ik\Psi$$

Conclusion: whenever $k\Psi$ needs to be computed, one can use $-i\frac{\partial}{\partial x}\Psi$

Dispersion relation $\hbar\omega = \frac{\hbar^2}{2m}k^2 + V(x)$ then leads to

$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi + V(x)\Psi$$

Georg.Hoffstaetter@Cornell.edu

From the dispersion relation to Schrödinger's equation

The wave vector $\vec{k}$ changes with position $\vec{x}$ according to: $\frac{\hbar^2}{2m}\vec{k}^2 + V(\vec{x}) = E$

for a particle with fixed constant energy **E**, where ω does not change.

Time independent Schrödinger's equation

For a stationary state: $\Psi(\vec{x},t) = \Phi(\vec{x})e^{-i\omega t} = \Phi(\vec{x})e^{-i\frac{E}{\hbar}t}$

$$\frac{\hbar^2}{2m}(k_x^2 + k_y^2 + k_z^2)\Phi + V(\vec{x})\Phi = E\Phi$$

$$-\frac{\hbar^2}{2m}(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})\Phi + V(\vec{x})\Phi = E\Phi$$

Time dependent Schrödinger's equation

If $\boldsymbol{\Psi}_{\omega 1}$ and $\boldsymbol{\Psi}_{\omega 2}$ are solutions of for different energies $\mathbf{E_1}$ **and** $\mathbf{E_2}$,

then $\boldsymbol{\Psi} = \boldsymbol{\Psi}_{\omega 1} + \boldsymbol{\Psi}_{\omega 2}$ is a solution of

$$-\frac{\hbar^2}{2m}(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})\Psi(x,t) + V(x)\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t)$$

As in one dimension this holds for an arbitrary linear combination of stationary states.

Georg.Hoffstaetter@Cornell.edu