2. Wave properties of particles

02/07/2005

De Broglie (1924): Light waves regularly have quantization conditions (e.g. standing waves) and also show particle character in photons. Electrons have particle character, but also show quantization conditions in Bohr's model of the atom.

Conclusion: Electons could have a similar particle/wave interpretation light.

Assumed oscillation in rest frame: $E_0 = m_0 c^2 = \hbar\omega_0\,,\quad \xi_0 \propto \cos(\omega_0 t_0)$

Moving frame: $ct_0 = \gamma\,(ct - \tfrac{v}{c}x) \rightarrow \xi = \cos(\omega_0\gamma\,(t - \tfrac{v}{c^2}x))$

$$\omega_0\gamma(t - \tfrac{v}{c^2}x) = \omega t - kx \quad \rightarrow \quad k = \tfrac{2\pi}{\lambda} = \omega_0\gamma\tfrac{v}{c^2} = \tfrac{m_0c^2}{\hbar}\gamma\tfrac{v}{c^2} = \tfrac{p}{\hbar}$$

$$\rightarrow \quad \omega = \omega_0\gamma = \tfrac{m_0c^2}{\hbar}\gamma = \tfrac{E}{\hbar}$$

$$p = \hbar k \qquad E = \hbar\omega$$

$$\lambda = \frac{h}{p} \Rightarrow \frac{\lambda}{\lambda_0} = \frac{1}{\beta\gamma},\quad \lambda_0 = \frac{h}{m_0 c}$$

$\lambda 0, \lambda$ for electrons at 10keV : 2pm, 12 pm

Note: 1A is the typical atomic radius.

$\lambda 0, \lambda$ for protons at 10MeV: 1.3fm, 9 fm

Note: 1fm is the radius of the proton.

Prince Louis-V. P. R. de Broglie (1892-1987) Nobel Price 1929

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Group velocity

The phase of the wave $\mathrm{Re}[e^{i(\omega t - kx)}]$ is constant at

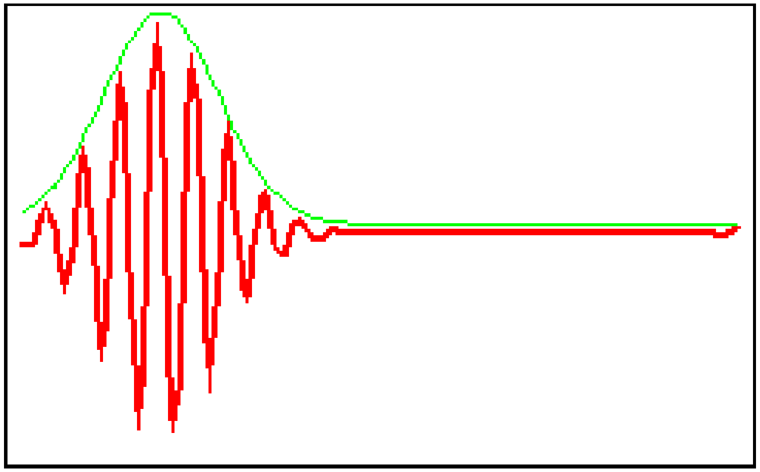
$$\omega t - kx = const. \quad \rightarrow \quad v_{phase} = \frac{dx}{dt} = \frac{\omega}{k}$$

A superposition of waves $\mathrm{Re}[\int_a^b A(k) e^{i(\omega(k)t - kx)} dk]$ has its largest value where the integrand does not oscillate with k so that it does not average out. This is where the exponent does not vary with k:

$$\frac{d}{dk}\omega(k)t - x = 0 \quad \rightarrow \quad v_{group} = \frac{dx}{dt} = \frac{d}{dk}\omega(k)$$

$$E = \sqrt{(pc)^2 + (m_0 c^2)^2} \quad \rightarrow \quad \omega = \sqrt{(kc)^2 + (\frac{m_0 c^2}{\hbar})^2}$$

Group velocity: $v_{group} = \frac{d\omega}{dk} = \frac{kc^2}{\sqrt{(kc)^2 + (m_0 c^2)^2}} = \frac{kc^2}{\omega} = \frac{pc^2}{E} = v_{electron}$



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Davisson-Germer Experiment

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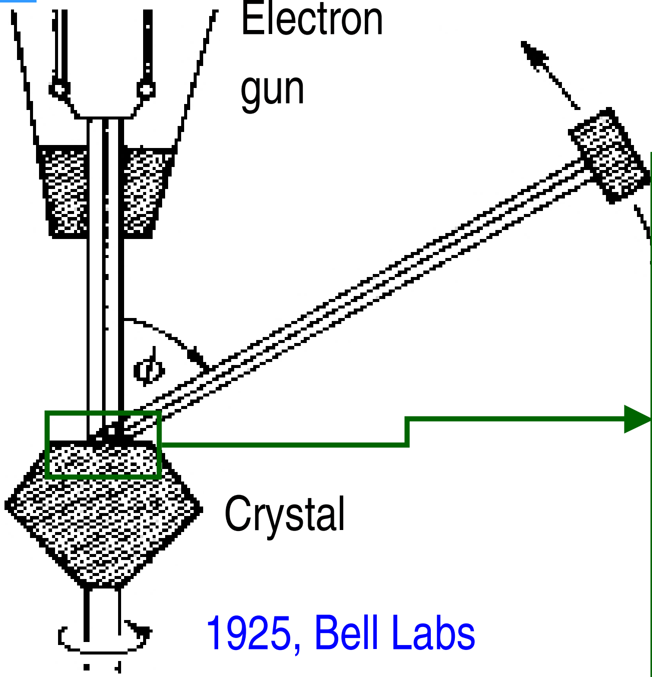
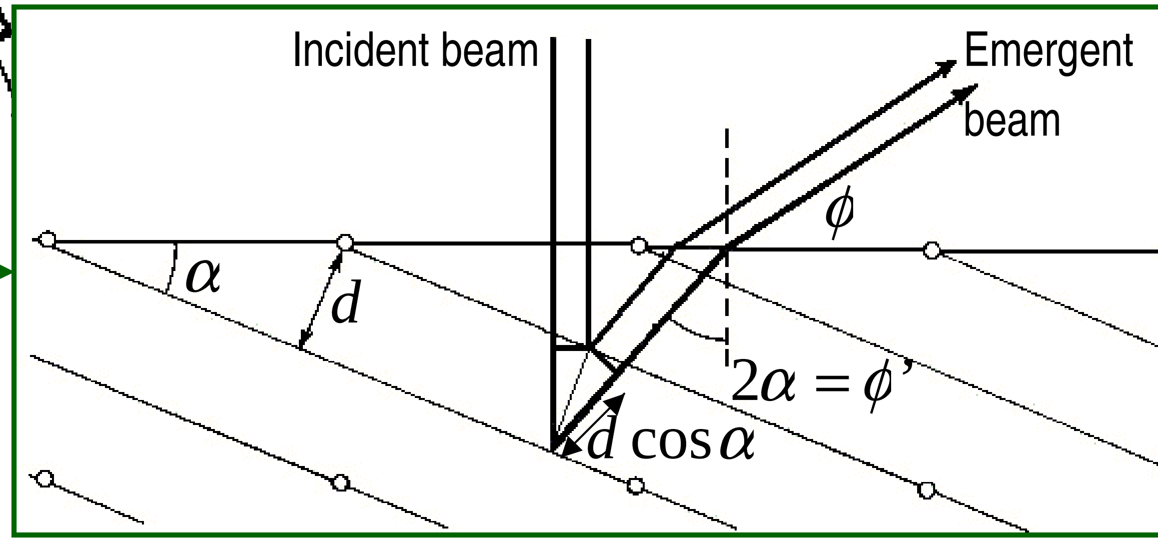


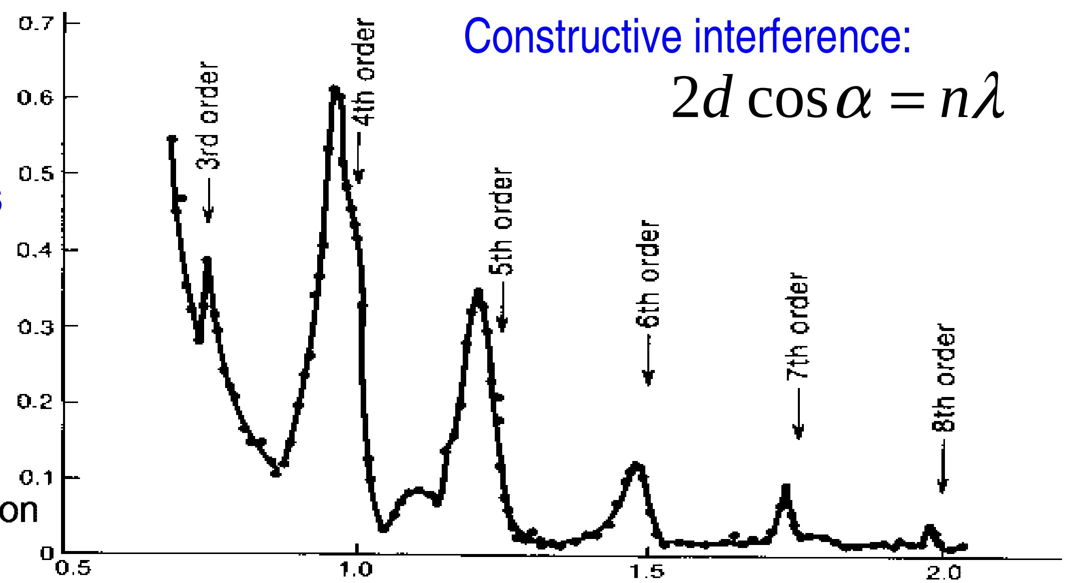


1925, Bell Labs

K=50eV , λ=1.7A

Clinton Davisson
(1881-1958)
Nobel Prize 1937

Reflection as a function of energy



Constructive interference:

$$2d\cos\alpha = n\lambda$$

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