Homework 488/688

Advanced Topics in Accelerator Physics (Hoffstaetter)

Due Date: Monday, 9/20/04 - 14:45 in 132 Rockefeller Hall

Exercise 1: Show that the Lorentz-force equation can be derived from the Hamiltonian $H = c\sqrt{[\vec{p}_c - q\vec{A}(\vec{r},t)]^2 + (mc)^2} + q\Phi(\vec{r},t)$, where the canonical momentum $\vec{p}_c$ is related to the classical momentum by $\vec{p} = \vec{p}_c - q\vec{A}$.

Exercise 2: Transform the Lorentz-force equation $\vec{p} = m\gamma\dot{\vec{r}}$ with $\gamma = \frac{1}{\sqrt{1-(\frac{\dot{\vec{r}}}{c})^2}}$ and $\dot{\vec{p}} = \vec{F}(\vec{p},\vec{r})$ so that s is the independent variable. Note that for simplicity it is assumed that the force does not depend on t. Derive $\vec{G}$ for the resulting equation $\vec{p}_c' = \vec{G}(\vec{p}_c,\vec{r},s)$. Use a straight coordinate system so that $\frac{ds}{dt} = \frac{p_s}{m\gamma}$.

Exercise 3: Use exercise 2 and $\vec{F} = q(\frac{\vec{p}}{m\gamma} \times \vec{B} + \vec{E})$, again assuming a time independent force to compute the equation of motion for the position and momentum components $\vec{r}_\perp$ and $\vec{p}_\perp$ perpendicular to the s-direction. Show that the Hamiltonian for these equations with s as independent variable agrees with $-p_s$, as derived in class.

Exercise 4: Show that $\mathbf{M}^T$ is symplectic if and only if $\mathbf{M}$ is symplecitic.

Exercise 5: Derive the equation of motion for Twiss parameters, $\alpha' + \gamma = K(s)\beta$ with $K = [\kappa(s)^2 + k(s)]$ from the linearized equation of motion $x'' = -K(s)x$. Use $x = \sqrt{2J\beta(s)}\sin(\Psi(s) + \Phi)$, $\alpha = -\frac{1}{2}\beta'$ and $\Psi(s)' = \frac{1}{\beta}$.