Charged Condensation

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With Rachel A. Rosen, arXiv:0706.2304 [hep-th]; and work in progress

* Bose-Einstein Condensation

A macroscopic state with a non-zero occupation number

* Higgs Condensation

What is Higgs vacuum made of and why it is not decaying?

Higgs vacuum: a macroscopic state with a non-zero occupation number of non-linearly interacting "tachyons". (Nielsen, 78)

* Electrically Charged Condensation

Screening of electric field, i.e., massive gauge boson.

However, does not reduce to the condensation of just free scalar particles: The photon Compton wavelength much greater than the inter-particle separation

$$m_g^{-1} \gg d_{\text{inter-particle}}$$

A simplest model: gauge field A_μ, charged scalar field ϕ with $m_H^2 > 0$, and fermions Ψ^+, Ψ with mass m_J

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + |D_\mu\phi|^2 - m_H^2\phi^*\phi$$

$$+\bar{\Psi}i\gamma^\mu D_\mu\Psi - m_J\bar{\Psi}\Psi + \mu\Psi^+\Psi\,.$$

Chemical potential μ is introduced for the global fermion number carried by Ψ's

Covariant derivatives are $\partial_\mu + ig_\phi A_\mu$ for scalars, and $\partial_\mu + ig_\psi A_\mu$ for fermions. For simplicity we assume that the scalar and fermion charges equal $g_\phi = -g_\psi \equiv -g$.

Introduce gauge invariant variables:

$$\phi = \frac{1}{\sqrt{2}}\sigma\, e^{i\alpha}, \quad B_\mu \equiv A_\mu + \frac{1}{g}\partial_\mu\alpha, \quad \psi = \Psi e^{-i\alpha}$$

Lagrangian in terms of the new variables:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}g^2B_\mu^2\sigma^2 - \frac{1}{2}m_H^2\sigma^2$$

$$+\bar{\psi}i\gamma^\mu D_\mu\psi - m_J\bar{\psi}\psi + \mu\psi^+\psi$$

(Related works: A. Linde 78, J. Kapusta 81)

The Fermions obey the conventional Dirac equation with a nonzero chemical potential. This implies a net fermion number in the system, $\bar{J}_0$. Since the fermions are also electrically charged, they set a background electric charge density. Such charged fermions would repel each other. In our case, however, the charge will be screened.

Consider distance scales that are greater than an average separation between the fermions, so that their spatial distribution could be assumed to be uniform. Then, the background charge density due to the fermions could be approximated as

$$\bar{J}_\mu = \bar{J}_0 \delta_{\mu 0},$$

where $\bar{J}_0$ is a constant. The magnitude of the latter is related to the value of the chemical potential μ. In particular, a self-consistent solution of the equations of motion implies that $\mu - \langle gB_0 \rangle = E_F$, where E_F denotes the Fermi energy of the background fermion sea, and is related to $\bar{J}_0$ as follows,

$$E_F = \sqrt{(3\pi \bar{J}_0/4)^{2/3} + m_J^2}$$

The rest of the equations are:

$$\partial^{\mu} F_{\mu\nu} + g^2 B_{\nu} \sigma^2 = g \bar{J}_{\nu} \,,$$

$$\Box \sigma = g^2 B_{\nu}^2 \sigma - m_H^2 \sigma \,.$$

The Bianchi identity, $\partial^{\nu}(B_{\nu}\sigma^2) = 0$, can also be obtained by varying the action w.r.t. α. For a constant charge density, $\bar{J}_{\mu} = \bar{J}_0 \delta_{\mu 0}$, the theory admits a static solution with constant B_0 and σ:

$$\langle B_0 \rangle = B_{0c} \equiv \frac{m_H}{g} \,, \;\; \langle \sigma \rangle = \sigma_c \equiv \sqrt{\frac{\bar{J}_0}{m_H}} \,.$$

The charge density stored in the condensate,

$$J_0^{\rm scalar} = -i[\phi^* D_0 \phi - (D_0 \phi)^* \phi] = -g \sigma^2 B_0,$$

equals to $-\bar{J}_0$. Hence, the total charge density $J_{\rm total} = \bar{J}_0 + J_0^{\rm scalar} = 0$, vanishes.

The ground state is charge-neutral in its bulk. On the other hand, a nonzero $\langle B_0 \rangle$ suggests that there must be an uncompensated charge on a surface at infinity, as it will be the case (see below).

Important Comments

(i) The expression for the gauge field scales as $1/g$ (non-perturbative). Moreover, it diverges in the limit $m_H \to \infty$. This seeming non-decoupling of the charged scalar field results from a constant background *charge density* in an infinite volume, i.e., from an infinite background charge. When m_H exceeds the fermion mass, the averaging procedure over the background charges should not be applicable in general.

(ii) It is instructive to regularize the problem by considering a finite volume ball of a radius R. A nonzero $\langle B_0 \rangle$ suggests that there must be an uncompensated charge on the surface of the ball, which tends to the value, $Q = m_H R/g$, as $R \to \infty$. Indeed, such a charge Q could give rise to a constant $\langle B_0 \rangle = m_H/g$ in the interior of the ball, where $\langle B_0 \rangle = Q/R$, in analogy with a static potential inside a conducting ball with surface charge Q.

(iii) Upon quantization the charged condensate can be thought of a zero-momentum state with a non-zero occupation number of the charged scalar field quanta. It is useful to consider small temperature T in the system, in which case the de Broglie wavelength of the condensed scalars, should exceeds the average inter-particle separation

$$\lambda_T \sim (1/m_H T)^{1/2} \gtrsim \bar{J}_0^{-1/3}$$

It makes sense to think of the charged condensate, as of any other Bose-Einstein condensate, to be a macroscopically occupied mode. The specifics of our case is that this state carries surface electric charge, which in the bulk of the condensate is balanced by the background charge density of fermions.

The spectrum of perturbations. Introduce small perturbations of gauge and scalar fields, b_μ and τ, as follows:

$$B_\mu = B_{0c}\delta_{\mu 0} + b_\mu(x)\,, \qquad \sigma = \sigma_c + \tau(x)\,.$$

The Lagrangian density for the perturbations reads

$$\mathcal{L}_2 = -\frac{1}{4}f_{\mu\nu}^2 + \tfrac{1}{2}(\partial_\mu \tau)^2$$

$$+\tfrac{1}{2}(g\sigma_c)^2 b_\mu^2 + 2m_H g\sigma_c\, b_0 \tau + \ldots$$

$f_{\mu\nu}$ denotes the field strength for b_μ, and we dropped all the fermionic terms as well as the cubic and quartic interaction terms of b's and τ. The last term is Lorentz-violating.

Spectrum: b_0 is not a dynamical field. It can be integrated out, leaving us with the equations for three polarizations of a massive vector b_j, $j = 1, 2, 3$, and one scalar τ. These constitute four physical degrees of freedom of the theory. The transverse part of the vector b_j obeys the free equation

$$(\Box + g^2\sigma_c^2) b_j^T = 0, \quad b_j^T \equiv b_j - \frac{\partial_j}{\Delta}(\partial_k b_k)\,.$$

The two transverse states of the gauge field carried by b_j^T have the mass

$$m_g^2 = g^2 \sigma_c^2 = g^2 \frac{\bar{J}_0}{m_H} \,.$$

The frequency ω and the three-momentum vector $\mathbf{p}$ of these two states obey the conventional dispersion relation,

$$\omega^2 = \mathbf{p}^2 + m_g^2.$$

The longitudinal mode of the gauge field b_j^L, and the scalar τ, on the other hand, give rise to the following Lorentz-violating dispersion relations (valid for $m_g \neq 0$)

$$\omega_\pm^2 = \mathbf{p}^2 + 2m_H^2 + \frac{1}{2}m_g^2 \pm \sqrt{4\mathbf{p}^2 m_H^2 + (2m_H^2 - \frac{1}{2}m_g^2)^2} \,.$$

The r.h.s. is positive. Both of these modes have masses which can be obtained by putting $\mathbf{p} = 0$. One of them coincides with m_g^2, and the other one, has the mass squared equal to $m_s^2 = 4m_H^2$. Interestingly, the group velocities of the transverse and longitudinal modes of the massive vector boson are different. For $m_H \gg m_g$, and for an arbitrary $\mathbf{p}$, the fastest ones are the transverse modes, they're followed by the scalar, and the longitudinal mode is the slowest.

Regularization : Consider a material ball of a fixed radius R which has a built in constant charge density $g\bar{J}_0$ uniformly distributed over its volume. How does the electric potential of this ball look like when the charged condensate compensates the fermion charge in its interior?

$$B_0(r) = B_{0c} + \delta B_0(r)\,, \quad \sigma(r) = \sigma_c + \delta\sigma(r)\,.$$

Focus on the solutions that in the interior of the ball satisfy $\delta\sigma/\sigma_c \ll 1$ and $\delta B_0/B_{0c} \ll 1$. The equations for δB_0 and $\delta\sigma$:

$$-\nabla^2\delta B_0 + m_g^2\,\delta B_0 = -2\,m_g\,m_H\,\delta\sigma\,,$$
$$-\nabla^2\delta\sigma = 2\,m_g\,m_H\,\delta B_0\,,$$

Explicit solutions: (presented for $m_H \gg m_g$ for simplicity). The solutions in the interior of the ball are

$$\delta B_0(r) = \frac{1}{r}\left[c_1 \sinh(Mr)\cos(Mr) + c_2\cosh(Mr)\sin(Mr)\right]\,,$$
$$\delta\sigma(r) = \frac{1}{r}\left[-c_1\cosh(Mr)\sin(Mr) + c_2\sinh(Mr)\cos(Mr)\right]\,,$$

where $M \equiv \sqrt{m_g\,m_H}$, and c_1 and c_2 are constants to be determined from matching these solutions to the exterior ones.

Outside of the ball we approximate the solutions to be

$$B_0 = \frac{Q}{r}\,, \quad \sigma = k\frac{e^{-m_H(r-R)}}{r}\,,$$

where Q is effective charge which should be determined from the matching conditions, By matching the solutions and their first derivatives at $r = R$, we find c_1, c_2, Q and k.

In the case of physical interest, $MR \gg 1$, the above solutions have a number of interesting properties. The net charge density in the ball,

$$g\bar{J}_{0\,\mathrm{eff}} = g\bar{J}_0 - g^2\sigma(r)^2 B_0(r)$$

is exponentially small in the interior, except in a narrow spherical shell near the surface of width M^{-1}. Thus, the charge is screened in the bulk of the ball, but there remains an unscreened surface charge. In this limit the effective charge of the ball is

$$Q = m_H R/g = g\bar{J}_0 R^3/(m_g R)^2.$$

This system is characterized by the conserved electric charge Q, and conserved fermion number $N = \bar{J}_0 R^3/3$.

In the bulk of the ball the electric field and the electromagnetic energy are negligible. Closer to the boundary, however, the surface energy becomes non-zero due to the varying electric field. The resulting expression scales as

$$\mathrm{Energy}_E \propto \frac{Q^2}{R} \propto \frac{m_H^2 R}{g^2}\,.$$

From our solutions it is also straightforward to get the scaling of the volume energy well within the ball; it reads as $\sim m_H \bar{J}_0 R^3$.

Generalization : Nonzero $\langle gB_0 \rangle$ acts as dynamically induced chemical potential for the perturbations of the scalar. Its value in the ground state, $\langle gB_0 \rangle = m_H$, is consistent with the expectation that the chemical potential be equal to the mass of the scalar in Bose-Einstein condensate.

In general, we could have introduced chemical potential for the charged scalar, μ_s. The above described condensation mechanism would still take place with the result,

$$\langle gB_0 \rangle = m_H + \mu_s,$$

and $\sigma_c^2 = \bar{J}_0/m_H$. The charge density in the condensate in this case would read,

$$-(\mu_s - gB_0)\sigma^2 = -\bar{J}_0,$$

ensuring charge neutrality of the substance in its bulk, but in general there would be a nonzero surface charge, unless

$$\mu_s = -m_H, \quad \langle gB_0 \rangle = 0$$

(the case of focus in J. Kapusta, 81)

Suppose in a laboratory one could prepare a reservoir, or a trap, in which negatively charged electrons and positively charged helium-4 nuclei, with a net negative charge, could be put together. Consider densities of these particles high enough so that the average separation between the particles, $\sim \bar{J}_0^{-1/3}$, is smaller than the size of a helium atom, which we estimate for simplicity to be the Bohr radius $\sim 1/(\alpha_{\rm em} m_e)$ ($\alpha_{\rm em}$ denotes the fine-structure constant, and m_e is the electron mass; we still stay much lower than nuclear densities). As long as

$$\bar{J}_0^{1/3} \gtrsim \alpha_{\rm em} m_e$$

the helium atoms in the substance would not form. Can the charged condensate be formed in this system? Strictly speaking, the calculations of the previous section are not directly applicable to this case, because electrons are lighter than the helium-4 nuclei and averaging over the electron positions to calculate the photon mass may not be a good approximation. The difference in this case would be that the photon mass squared would be determined by $g^2\bar{J}_0/m_e$ instead of $g^2\bar{J}_0/m_H$, which should be applicable when the fermions are heavier than the scalars.

One could introduce small temperature in the above system to see under what conditions the condensation would take place. Once the thermal de Broglie wavelengths of the helium-4 nuclei have overlaps with each other, and as long at the photon Compton wavelength is shorter than the thermal de Broglie wavelength, the system can be treated as a macroscopic mode, or the condensate. The former condition,

$$\lambda_T \sim (1/m_H T)^{1/2} \gtrsim \bar{J}_0^{-1/3},$$

would suggest that $T \lesssim 10^{-1}$ eV $\sim 10^{-5}$ K, while the latter, $1/m_g \lesssim \lambda_T$, would give a stronger bound $T \lesssim 10^{-5}$ eV $\sim 10^{-9}$ K (we use $g^2\bar{J}_0/m_e$ as the photon mass squared). Temperatures reached in experiments on Bose-Einstein condensation of atoms are within this range.

Propagation of light in the bulk of this substance would proceed with a delay caused by the induced photon mass m_g. For simplicity, we have considered above the system of a macroscopic size, but nothing prevents one to look at much smaller systems.

Applications in astrophysics and cosmology? Consider a distribution of N charged fermions and N_s charged scalars with the net electric charge Q.

$$E(R) = m_H N + N\sqrt{p_J^2 + m_J^2} - \frac{GM^2}{R}.$$

The critical radius reads: $R_c \sim B^2/m_J N^{1/3}$ where $B \equiv M_{\rm Pl}/(m_H + m_J)$. The critical energy:

$$E_c = (m_H + m_J)N\left[1 - \left(\frac{m_J}{m_H}\right)\left(\frac{N^{1/3}}{B}\right)^4\right].$$

R_c decreases with increasing N, the bounds:

$$\frac{1}{e^{1/2}}\left(\frac{m_H}{m_J}\right)^{3/4} B^{9/4} \lesssim N \lesssim \min\left\{\left(\frac{m_H}{m_J}\right)^{3/4} B^3; \;\; B^3\right\}.$$

The lower bound is due to the requirement that gravity be dominant in stabilizing this object, and the upper bound is for the relativistic effects to be negligible. These objects are stable as long as the gravitational binding energy exceeds the electrostatic energy of uncompensated charges on its surface. This constraint is taken into account by the above bounds.

In a simple case when the droplet is assumed to be made of electrons and the charged condensate of helium-4 nuclei, N has to be close to the upper bound, $N \sim 10^{57}$. The mass of this object is within an order of magnitude of the mass of the Sun, and its size is $\sim 10^6\ m$. This object has characteristics that are similar to those of neutron stars (except that it will have some surface charge, that we ignored in our considerations). However, propagation of light through such a cold and dense object will have specific characteristics.

Could charged condensate exist on cosmological scales? (GG. and R.A. Rosen, in progress) . The metric takes the FRW form

$$ds^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t) dx_i dx^i.$$

The background with the time-independent $\bar{J}_0$ via an integration constant (GG. and Y. Shang, 06) . Add term $-\partial^\mu \lambda A_\mu + \partial^\mu(\partial_\mu \lambda \beta)$, where λ is a nondynamical Lagrange multiplier. The equations of motion admit a solution with constant $\partial_0 \lambda$ which we identify with $\bar{J}_0 = \partial_0 \lambda$. The stress-tensor of the system takes the form: $T_{\mu\nu} = g^2 B_\mu B_\nu \sigma^2 - g(B_\mu J_\nu + B_\nu J_\mu) - \tilde{g}_{\mu\nu} \left(\frac{1}{2} g^2 B_\alpha^2 \rho^2 - \frac{1}{2} m_H^2 \rho^2 - g B^\beta J_\beta\right) + T^{\mathrm{F}}_{\mu\nu}$. Here $T^{\mathrm{F}}_{\mu\nu}$ is an additional stress-tensor, $T^{\mathrm{F}}_{\mu\nu} = \mathrm{diag}(\rho_{\mathrm{F}}, 0, 0, 0)$. Moreover, in the full stress tensor we will have to include the part due to an ordinary and dark matter, $T^{\mathrm{M}}_{\mu\nu} = \mathrm{diag}(\rho_{\mathrm{M}}, 0, 0, 0)$.

The above stress-tensor gives a negative pressure

$$p = -m_H |\bar{J}_0|.$$

A consistent solution of the corresponding Einstein equations can be obtained. These solutions should be compared with the conventional FRW solution for the cosmological constant $\Lambda \equiv 8\pi G_N m_H \bar{J}_0$, and the matter density equal to $\rho_{\mathrm{M}} + \rho_{\mathrm{F}} - m_H |\bar{J}_0|$. For earlier times $t \ll (\Lambda)^{-1/2}$, we obtain that $\rho_{\mathrm{M}} + \rho_{\mathrm{F}} \sim 1/(24\pi G_N t^2)$ and $a(t) \sim t^{2/3}$, while for later times $t \gtrsim (\Lambda)^{-1/2}$, we obtain the de Sitter asymptotic behavior $\rho_{\mathrm{M}} + \rho_{\mathrm{F}} \sim m_H |\bar{J}_0|$, and $a(t) \sim \exp\left(\sqrt{\frac{8\pi G_N m_H \bar{J}_0}{3}}\, t\right)$. These solutions are consistent with the present day accelerated expansion given that the mater density ρ_{M} today is about 30 percent, $\rho_{\mathrm{F}} \simeq m_H |\bar{J}_0|$ constitutes about 70 percent of $\rho_{\mathrm{critical}} \sim (10^{-4}\ \mathrm{eV})^4$, and ρ_{F} does not grow in past too fast to contradict the data.

The best candidate for the scalar is the $^4He^{++}$ nucleus. Before the recombination, in this case, the Universe would have mostly consisted of the electrons, protons, $^4He^{++}$ nuclei, some small fractions of the other light nuclei, and, importantly of the primordial charge density $g\bar{J}_0$.

It is possible that a tiny fraction (about $\sim 10^{-5}$) of the helium nuclei got condensed to neutralize the background charge density $\bar{J}_0$ according to the mechanism outlined above. If so, a small fraction of the $^4He^{++}$ nuclei today could be residing in the condensate, instead of being in ^4He neutral atoms.

One observational consequence of this scenario is that the photon would acquire the Lorentz-violating mass that would be related to the amount of dark energy. Using the fact that $m_H \bar{J}_0 \simeq \rho_{\rm crit.} \sim (10^{-4}\ {\rm eV})^4$, we obtain:

$$m^2_{\rm phot} \simeq \frac{g^2 \rho_{\rm crit.}}{m^2_H}\,.$$

If the condensing scalar is a $^4He^{++}$ nucleus, then $g = 2e$, $m_H \simeq 4\,{\rm GeV}$, and the photon mass would be $m_{\rm phot} \simeq (10^{-17}–10^{-18})\ eV$. This value is consistent with the laboratory test of the Coulomb law, while the stronger bounds from the galactic magnetic fields are expected to be relaxed because of the vortices a la (Adelberger, Dvali, Gruzinov (2003)) .

In this case, the number density of the condensed $^4He^{++}$ could be estimated as $\bar{J}_0 \simeq (10^{-8}\ {\rm eV})^3$, which is a tiny, $\sim 10^{-5}$ fraction, of the number of helium atoms in the Universe.

Charged Condensate

* An interesting physical medium

Massive photon, distinctive features of its propagation.

* Applications in Astrophysics

Charged compact objects.

*Manifestations in Cosmology

Relation between dark energy and photon mass.