Holographic vs Deconstruction technique for a minimal Higgsless model

Stefania De Curtis
INFN, Firenze

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- **5D Higgsless models**
- **Dimensional deconstruction and linear moose models**
- **Unitarity and EW precision test bounds**
- **Fermion delocalization: "ideal" or global cancellation**
- **Holographic approach to a $SU(2)$ Higgsless model**
- **Effective gauge-fermion interaction**
- **Oblique and direct contributions to the EW parameters**
- **Comparison of the two techniques**

based on:
Casalbuoni, D.C., Dominici, PRD hep-ph/0405188
Casalbuoni, D.C., Dolce, Dominici, PRD hep-ph/05022209; hep-ph/0705.2510
Bechi, Casalbuoni, D.C., Dominici, PRD hep-ph/0607314

5D Higgsless Models

- **Symmetry breaking mechanism of a gauge theory in $4 + 1$ dims by boundary conditions on the branes** *(Csáki, Grojean,Murayama,Pilo,Terning; Nomura)*

- **The scale at which partial wave unitarity is lost is delayed with respect to the SM without the Higgs, due the exchange of KK excitations of gauge bosons** *(Chivukula,Dicus,He; Foadi,Gopalakrishna,Schmidt)*

- **Problem: $\epsilon_3(S)$ electroweak parameter is too big if unitarity is fine** *(Barbieri,Pomarol,Rattazzi,Strumia)*

- **Solutions:**
  Brane kinetic terms *(Cacciapaglia,Csáki,Grojean,Terning; Carena,Tait,Wagner; Carena,Ponton,Tait,Wagner; Davoudiasl,Hewett,Lillie,Rizzo)*

  Fermion delocalization *(Cacciapaglia,Csáki,Grojean,Terning; Foadi,Gopalakrishna,Schmidt; Bhattacharya,Csáki,Martin,Shirman,Terning)*

Dimensional deconstruction and moose models

(Arkani-Hamed, Cohen, Georgi)

Theory with gauge symmetry $[G]^{K+1}$ in $3+1$ dims:

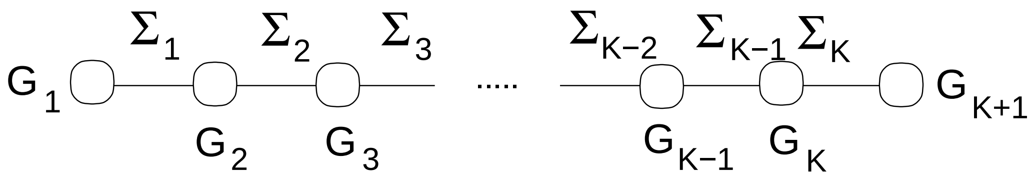
$$A^j = A^{ja} T^a, \quad j = 1, \cdots, K+1;$$

Non linear σ-model fields:

$$\Sigma_i = e^{i\pi_i^a T^a / f_c}\,, \quad \Sigma_i \to U_i \Sigma_i U_{i+1}^\dagger\,, \quad U_i \in G_i, \quad i = 1, 2, \cdots, K$$

Covariant derivatives: $D_\mu \Sigma_i = \partial_\mu \Sigma_i - i g_c A_\mu^i \Sigma_i + i g_c \Sigma_i A_\mu^{i+1}$

$$\mathcal{L}_{moose} = \sum_{i=1}^{K} f_c^2 \mathrm{Tr}[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i] - \frac{1}{2} \sum_{i=1}^{K+1} \mathrm{Tr}[(F_{\mu\nu}^i)^2]$$



Mass spectrum

Mass matrix $\{A_\mu^1, A_\mu^2, \dots, A_\mu^{K+1}\}$:

$$M^2 = g_c^2 f_c^2 \begin{pmatrix} 1 & -1 & 0 & \dots & 0 & 0 \\ -1 & 2 & -1 & \dots & 0 & 0 \\ 0 & -1 & 2 & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 2 & -1 \\ 0 & 0 & 0 & \dots & -1 & 1 \end{pmatrix}$$

Mass eigenvalues:

$$M_k^2 = 4 g_c^2 f_c^2 \sin^2\left(\frac{\pi k}{2(K+1)}\right) \longrightarrow \left(\frac{k}{R}\right)^2 \quad |k| \ll K$$

For $|k| \ll K$ they reproduce the masses of KK excitations for a five dimensional theory with gauge symmetry G, compactification radius R, gauge coupling g_5, lattice spacing a:

$$\pi R = (K+1) a \qquad \frac{a}{g_5^2} = \frac{1}{g_c^2} \qquad a = \frac{1}{g_c f_c}$$

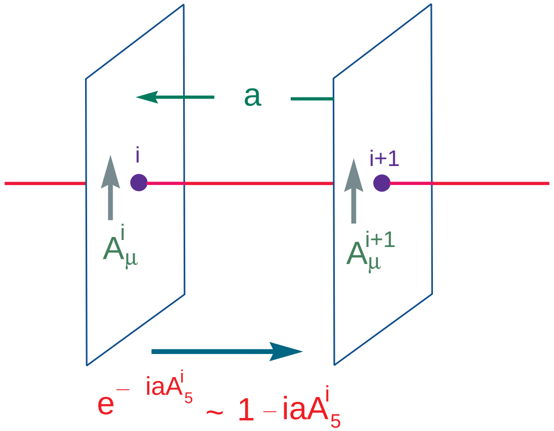
Extra dimension on a lattice

(Hill, Pokorski, Wang; Randall, Shadmi, Weiner; Abe, Kobayashi, Maru, Yoshioka)

Example: theory with abelian gauge symmetry in 5D and flat metric:

$$S = -\frac{1}{2}\int d^4x \int_0^{\pi R} dy \frac{1}{g_5^2}\left[F_{\mu\nu}F^{\mu\nu} - 2F_{\mu 5}F^{\mu 5}\right], \quad F_{\mu 5} = \partial_\mu A_5 - \partial_5 A_\mu$$

gauge transformation $\rightarrow A_5 = 0 \rightarrow A_\mu^{(n)}$ **acquire mass** $M_n = n/R$



$$\Sigma^i \sim e^{-iaA_5^i}, \quad D_\mu \Sigma_i = \partial_\mu \Sigma_i - iA_\mu^{i-1}\Sigma_i + i\Sigma_i A_\mu^j, \quad (F_{\mu 5}^i)^2 \sim \frac{(D_\mu \Sigma_i)^\dagger (D^\mu \Sigma_i)}{a^2}$$

$$S_{moose} \sim -\frac{1}{2}\int d^4x \frac{a}{g_5^2}\sum_j \left[F_{\mu\nu}^j F^{\mu\nu j} - \frac{2}{a^2}(D_\mu \Sigma^j)^\dagger (D^\mu \Sigma^j)\right]$$

Minimal deconstructed Higgsless model
$SU(2)_L \times SU(2)^K \times U(1)$ linear moose

(Casalbuoni, D.C., Dominici; see also: Foadi, Gopalakrishna, Schmidt; Hirn, Stern; Chivukula et al; Georgi)

The transformation properties of the fields are

$$\Sigma_1 \to L \Sigma_1 U_1^\dagger,$$
$$\Sigma_i \to U_{i-1} \Sigma_i U_i^\dagger \ , \quad i = 2, \cdots, K,$$
$$\Sigma_{K+1} \to U_K \Sigma_{K+1} R^\dagger,$$

$$U_i \in G_i \equiv SU(2)_i \qquad A_\mu^i = A_\mu^{ia} \tau^a/2, \quad g_i, \quad i = 1, 2, \cdots, K,$$
$$L \in G_L \equiv SU(2)_L \qquad \tilde{W}_\mu = \tilde{W}_\mu^a \tau^a/2, \quad \tilde{g},$$
$$R \in G_R \equiv SU(2)_R \supset U(1)_Y \qquad \tilde{Y}_\mu = \tilde{\mathcal{Y}}_\mu \tau^3/2, \quad \tilde{g}'$$

$$\mathcal{L} = \sum_{i=1}^{K+1} f_i^2 \mathrm{Tr}[D_\mu \Sigma_i^\dagger D^\mu \Sigma_i] - \frac{1}{2}\sum_{i=1}^{K} \mathrm{Tr}[(F_{\mu\nu}^i)^2] - \frac{1}{2}\mathrm{Tr}[(F_{\mu\nu}(\tilde{W}))^2] - \frac{1}{2}\mathrm{Tr}[(F_{\mu\nu}(\tilde{Y}))^2]$$

Covariant derivatives

$$D_\mu \Sigma_1 = \partial_\mu \Sigma_1 - i\tilde{g}\tilde{W}_\mu \Sigma_1 + i\Sigma_1 g_1 A_\mu^1,$$

$$D_\mu \Sigma_i = \partial_\mu \Sigma_i - i g_{i-1} A_\mu^{i-1} \Sigma_i + i\Sigma_i g_i A_\mu^i, \qquad i = 2, \cdots, K,$$

$$D_\mu \Sigma_{K+1} = \partial_\mu \Sigma_{K+1} - i g_K A_\mu^K \Sigma_{K+1} + i\tilde{g}' \Sigma_{K+1} \tilde{Y}_\mu$$

$f_i = f_c \ \forall i \Rightarrow$ **flat metric in five dims;** **varying** $f_i \Rightarrow$ **warped metric**

Global symmetry: $SU(2)_L \times SU(2)^K \times SU(2)_R$

$$\mathcal{L}_{\mathrm{mass}} = \frac{1}{2}\sum_{i,j=0}^{K+1} (M_2)_{ij} A_\mu^i A^{\mu j}$$

with $A_\mu^0 = \tilde{W}^\mu$, $A_\mu^{K+1} = \tilde{Y}^\mu$, **and**

$$(M_2)_{ij} = g_i^2 (f_i^2 + f_{i+1}^2)\delta_{i,j} - g_i g_{i+1} f_{i+1}^2 \delta_{i,j-1} - g_j g_{j+1} f_{j+1}^2 \delta_{i,j+1}$$

where $g_0 = \tilde{g}$, $g_{K+1} = \tilde{g}'$, $f_0 = f_{K+2} = 0$

Besides the massless photon, the lowest eigenvalues (at the leading order in $O((\tilde{g}/g_i)^2)$) are: $\tilde{M}_W^2 = v^2\tilde{g}^2/4\,, \quad \tilde{M}_Z^2 = \tilde{M}_W^2/\tilde{c}_\theta^2$, where we have identified $\tan\tilde{\theta} = \tilde{g}/\tilde{g}'$ and

$$\frac{4}{v^2} \equiv \frac{1}{f^2} = \sum_{i=1}^{K+1} \frac{1}{{f_i}^2} \qquad v = \text{EW scale} = 246 \text{ GeV}$$

The low-energy limit of the theory is obtained by eliminating the A_i $(i = 1, \cdots K)$ fields with the solution of the e.o.m. for $g_i >> \tilde{g}$ corresponding to heavy masses for A_i:

$$A_i^{\pm} = \frac{1}{g_i}(\tilde{g}\tilde{W}^{\pm} z_i), \quad A_i^3 = \frac{1}{g_i}(\tilde{g}'\tilde{\mathcal{Y}} y_i + \tilde{g}\tilde{W}^3 z_i)$$

with $z_i = \sum_{j=i+1}^{K+1} f^2/f_j^2$ and $y_i = 1 - z_i$. By substituting in $\mathcal{L}^{(2)}$:

$$\mathcal{L}_{eff}^{(2)} = -\frac{1}{4}(1+z_\gamma)\tilde{A}_{\mu\nu}\tilde{A}^{\mu\nu} - \frac{1}{2}(1+z_W)\tilde{W}_{\mu\nu}^{+}\tilde{W}^{-\mu\nu} - \frac{1}{4}(1+z_Z)\tilde{Z}_{\mu\nu}\tilde{Z}^{\mu\nu} + \frac{1}{2}z_{Z\gamma}\,\tilde{A}_{\mu\nu}\tilde{Z}^{\mu\nu}$$

$$z_\gamma = \tilde{s}_\theta^2 \sum_{i=1}^{K} \left(\frac{\tilde{g}}{g_i}\right)^2, \quad z_w = \sum_{i=1}^{K} \left(\frac{\tilde{g}}{g_i}\right)^2 (1 - y_i)^2, \quad z_z = \frac{1}{\tilde{c}_\theta^2} \sum_{i=1}^{K} \left(\frac{\tilde{g}}{g_i}\right)^2 \left(\tilde{c}_\theta^2 - y_i\right)^2$$

$$z_{z\gamma} = -\frac{\tilde{s}_\theta}{\tilde{c}_\theta} \sum_{i=1}^{K} \left(\frac{\tilde{g}}{g_i}\right)^2 \left(\tilde{c}_\theta^2 - y_i\right), \quad M_Z^2 = \tilde{M}_Z^2(1 - z_z), \ \ M_W^2 = \tilde{M}_W^2(1 - z_w)$$

Continuum limit ($K\to\infty$, $(K+1)a=\pi R$)

$SU(2)$ gauge theory in 5D bulk with a generic conformally flat metric $ds^2=e^{-2A(z)}(\eta_{\mu\nu}dx^\mu dx^\nu-dz^2)$ with brane kinetic terms to lower the masses of the lightest states and BC's to break the gauge symm. to $U(1)$

$$S=-\frac{1}{4}\int d^4x\int_0^{\pi R}dz e^{-A(z)}\Big[\frac{1}{g_5^2(z)}[(F^a_{\mu\nu})^2-2(F^a_{\mu 5})^2]+\frac{1}{\tilde g^2}(F^a_{\mu\nu})^2\delta(z)+\frac{1}{\tilde g'^2}(F^3_{\mu\nu})^2\delta(z-\pi R)\Big]$$

Dirichlet BC's: $A^{1,2}_\mu|_{z=\pi R}=0$, and Neumann BC's: $\partial_z A^a_\mu|_{z=0}=0$

- Divide the z-segment in $K+1$ intervals of size a, get a finite set of 4D gauge theories acting at a particular lattice site: A^i_μ and g_{5i} are the gauge fields and gauge coupling constants $i=1,\cdots,K$.
- Introduce the link variables $\Sigma_i=e^{-iaA^i_5}$ $\quad(i=1,..,K+1)$ and obtain the linear moose action

$$S^{moose}_{gauge}=\int d^4x[\sum_{i=1}^{K+1}f_i^2\mathrm{Tr}[D_\mu\Sigma_i^\dagger D^\mu\Sigma_i]-\frac{1}{2}\sum_{i=0}^{K+1}\frac{1}{g_i^2}\mathrm{Tr}(F^i_{\mu\nu})^2]$$

with the indentifications: $ae^{-A_i}/g^2_{5i}=1/g_i^2$, $e^{-A_i}/(ag^2_{5i})=f_i^2$ and $A^0_\mu=\tilde W^a_\mu\tau^a/2$, $A^{K+1}_\mu=\tilde{\mathcal{Y}}\tau^3/2$.

FLAT METRIC: $f_i=f_c$, $g_i=g_c$, $e^{-A_i}=1$, $g^2_{5i}=ag_c^2$

Partial wave unitarity bounds

(Chivukula,He; Papucci; Mück,Nilse,Pilaftis,Rückl; Csáki,Grojean,Murayama,Pilo,Terning)

Unitary gauge for A^i_μ $\Rightarrow$ $\Sigma_i = \exp\left(i f \vec{\pi} \cdot \vec{\tau}/2 f_i^2\right)$ $\Rightarrow$ $\mathcal{L}(\pi, A^i_\mu)$

Equivalence theorem: $\mathcal{A}_{W_L^+ W_L^- \to W_L^+ W_L^-} \sim \mathcal{A}_{\pi^+\pi^-\to\pi^+\pi^-}$ $(\sqrt{s} >> M_W)$

$$\mathcal{A}_{\pi^+\pi^-\to\pi^+\pi^-} \sim -\frac{1}{4} f^4 \sum_{i=1}^{K+1} \frac{u}{f_i^6} + \frac{1}{4} f^4 \sum_{i,j=1}^{K+1} L_{ij} \left(\frac{u-t}{(s-M^2)_{ij}} + \frac{u-s}{(t-M^2)_{ij}} \right)$$

where $$L_{ij} = g_i g_j \left(\frac{1}{f_i^2} + \frac{1}{f_{i+1}^2} \right) \left(\frac{1}{f_j^2} + \frac{1}{f_{j+1}^2} \right)$$

High energy limit: $$\mathcal{A}_{\pi^+\pi^-\to\pi^+\pi^-} \to -\frac{1}{4} f^4 \sum_{i=1}^{K+1} \frac{u}{f_i^6}$$

minimized by $f_i = f_c \quad \forall i$ $$\to -\frac{u}{(K+1)^2 v^2}$$

Unitarity condition from $J=0$ partial wave $|a_0| < 1/2$:
Single channel $\pi^+\pi^- \longrightarrow \pi^+\pi^-$ contribution: $\Lambda_{\rm moose} = (K+1)\Lambda_{\rm HSM}$

But the deconstructed theory has many other longitudinal vector bosons, considering all channels $(\Sigma_i = \exp\left(i\vec{\pi}_i \cdot \vec{\tau}/2f_i\right))$ and using the
Equivalence Theorem: $\mathcal{A}_{A_L^i A_L^i \to A_L^i A_L^i} \sim \mathcal{A}_{\pi^i\pi^i\to\pi^i\pi^i}$ $(\sqrt{s} >> M_{A^i})$

$$\mathcal{A}_{\pi_i\pi_i\to\pi_i\pi_i} \to -\frac{1}{4}\frac{u}{f_i^2}$$

The unitarity limit is determined by the smallest f_i. Taking all equal:

$$\boxed{\Lambda^{TOT}_{\rm moose} = \sqrt{K+1}\,\Lambda_{\rm HSM}}$$

Higgs bosons are not necessary up to $\sqrt{K+1}$ times the scale of unitarity violation in the Higgsless SM (see NDA analysis: $\Lambda_5 = \sqrt{K}\Lambda_4$)

Approximately, by imposing $M_A^{(K+1)} < \Lambda^{TOT}_{moose}$, with $\Lambda_{\rm HSM} = 2\sqrt{2}\pi v$ we get the bound $g_c < 5$ $\longrightarrow$ Electroweak corrections too large

Electroweak Precision Tests

Custodial symmetry: $\epsilon_1^{obl} = \epsilon_2^{obl} = 0$ to $\mathcal{O}((\tilde{g}/g_i)^2)$

Dispersive representation in terms of current-current correlators:

$$\epsilon_3^{obl} \left(= \frac{\tilde{g}^2 S}{16\pi} \right) = -\frac{\tilde{g}^2}{4\pi} \int_0^\infty \frac{ds}{s^2} Im[\Pi_{VV}(s) - \Pi_{AA}(s)]$$

Use vector meson dominance to saturate $Im\Pi_{VV(AA)}$ with vector boson exchanges (to $\mathcal{O}((\tilde{g}/g_i)^2)$):

$$\epsilon_3^{obl} = \frac{\tilde{g}^2}{4} \sum_n \left(\frac{g_{nV}^2}{m_n^4} - \frac{g_{nA}^2}{m_n^4} \right) = \tilde{g}^2 g_1 g_K f_1^2 f_{K+1}^2 (M_2^{-2})_{1K} = \tilde{g}^2 \sum_{i=1}^{K} \frac{(1-y_i)y_i}{g_i^2}$$

where g_{nV}, g_{nA} are vector decay constants, M_2 is the gauge boson squared mass matrix with eigenvalues m_n^2, and $y_i = \sum_{j=1}^{i} f^2/f_j^2$

Since $0 \leq y_i \leq 1 \Rightarrow \epsilon_3 > 0$, whatever the metric

For $f_i = f_c$, $g_i = g_c \longrightarrow \epsilon_3^{obl} = \frac{K(K+2)}{6(K+1)} \frac{\tilde{g}^2}{g_c^2}$

$\epsilon_3^{exp} \sim 10^{-3}$. For $K = 1 \Rightarrow g_c \sim 16\tilde{g} \sim 10$. For large $K \Rightarrow g_c \sim 10\sqrt{K}$
$\Rightarrow$ strongly interacting gauge bosons, unitarity violation

For increasing K and reasonable values of $g_c \Rightarrow \epsilon_3$ too large

Delocalizing fermion interactions

(Casalbuoni,D.C.,Dolce,Dominici; see also: Chivukula,Simmons,He,Kurachi)

Let us build

$$\chi_L^i = \Sigma_i^\dagger \Sigma_{i-1}^\dagger \cdots \Sigma_1^\dagger \psi_L, \qquad \chi_L^i \to U_i \chi_L^i\,, \quad U_i \in SU(2)_i \quad i = 1, \ldots, K$$

New terms describing direct left-handed fermion couplings to A_μ^i, invariant under the $SU(2)_L \times SU(2)^K \times U(1)$ symmetry:

$$\sum_{i=1}^{K} b_i \bar{\chi}_L^i i\gamma^\mu (\partial_\mu + i g_i A_\mu^i + \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu)\chi_L^i$$

b_i dimensionless parameters.

In the unitary gauge ($\Sigma_i \equiv I$) and after a rescaling $\psi_L \to \frac{1}{\sqrt{1+\sum_i b_i}}\psi_L$:

$$\mathcal{L}_{fermions}^{tot} = \bar{\psi}_R i\gamma^\mu \left[\partial_\mu + i\tilde{g}'\frac{\tau^3}{2}\tilde{\mathcal{Y}}_\mu + \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu\right]\psi_R$$

$$+\bar{\psi}_L i\gamma^\mu \left[\partial_\mu + \frac{1}{1+\sum_{i=1}^K b_i}\left(i\tilde{g}\tilde{W}_\mu + i\sum_{i=1}^{K} b_i g_i A_\mu^i\right) + \frac{i}{2}\tilde{g}'(B-L)\tilde{\mathcal{Y}}_\mu\right]\psi_L$$

How can we get b_i from a 5D bulk?

(Foadi,Gopalakrishna,Schmidt; Csáki,Hubitsz,Meade; Bechi,Casalbuoni,D.C.,Dominici)

Consider fermions propagating in the warped 5D bulk with additional brane kinetic terms + BC's: $\psi_R|_0 = 0$, $\psi_L|_{\pi R} = 0$

$$S_{ferm.} = \int d^4x \int_0^{\pi R} dz \Big[e^{-4A(z)} [(\frac{i}{2}\bar{\psi}\Gamma^M D_M \psi + h.c.) - e^{-A(z)} M \bar{\psi}\psi]$$
$$+ e^{-4A(0)} \frac{\delta(z)}{\hat{t}_L^2} i\bar{\psi}_L \gamma^\mu D_\mu \psi_L + \ e^{-4A(\pi R)} \delta(\pi R - z) i\bar{\psi}_R (\frac{1}{\hat{t}_R^2}) \gamma^\mu D_\mu \psi_R \Big]$$

where $\quad D_M\psi = (\partial_M + iT^a A^a_M(z) + iY_L A^3_M(\pi R))\psi \quad$ and $\hat{t}_{L,R}$ set the weight of the brane kinetic terms with respect to the bulk one.

• DISCRETIZE the fifth dimension $\longrightarrow$ the fermions on the j-site with $j = 0, .., K+1$, with a mass term $\quad m_j = (aM_j + 1)/a, \quad j = 1, .., K,$ "hop" from one site to the nearest one due to ∂_z.

• Study the effects of ψ_i $(i = 1, .., K)$ in the low-energy limit that is neglect kinetic terms with respect to mass terms. DECOUPLE the heavy fermions with the solutions of their e.o.m. (consider only the quadratic interactions among fermions)

$$\alpha_j L_j - m_{j+1} L_{j+1} = 0, \qquad j = 0, .., K-1$$
$$\alpha_j R_{j+1} - m_j R_j = 0, \qquad j = 1, .., K$$

where $L_j = \psi_L^j$ e $R_j = \psi_R^j$ $(j = 1, .., K)$, L_0 and R_{K+1} are, up to mixing corrections, the left and right components of the SM fermions, and $\alpha_0 = \hat{t}_L/\sqrt{a}$, $\alpha_j = 1/a$ $(j = 1, .., K-1)$, $\alpha_K = \hat{t}_R/\sqrt{a}$, are the "hopping" strengths.

- **PLUG** the solutions in the gauge-fermion interaction, get direct SM fermion couplings to A_μ^i + SM fermion mass term (normalized fields):

$$S_{ferm}^b = \int d^4x \sum_{j=1}^K \frac{b_j^L}{1+\sum_{i=1}^K b_i^L} \; i\bar{L}_0 \gamma^\mu (\partial_\mu + i g_j T^a A_\mu^{aj} + i\tilde{g}' Y_L A_\mu^{K+1}) L_0$$
$$+ \sum_{j=1}^K \frac{b_j^R}{1+\sum_{i=1}^K b_i^R} \; i\bar{R}_{K+1} \gamma^\mu (\partial_\mu + i g_j T^3 A_\mu^{3j} + i\tilde{g}' Y_L A_\mu^{K+1}) R_{K+1}$$
$$+ \sum_{j=1}^K \frac{b_j^R}{1+\sum_{i=1}^K b_i^R} \frac{g_j}{\sqrt{2}} (\bar{R}_{K+1} \gamma^\mu A_\mu^{+j} R_{K+1} + h.c.) - m^f (\bar{L}_0^f R_{K+1}^f + h.c.)$$

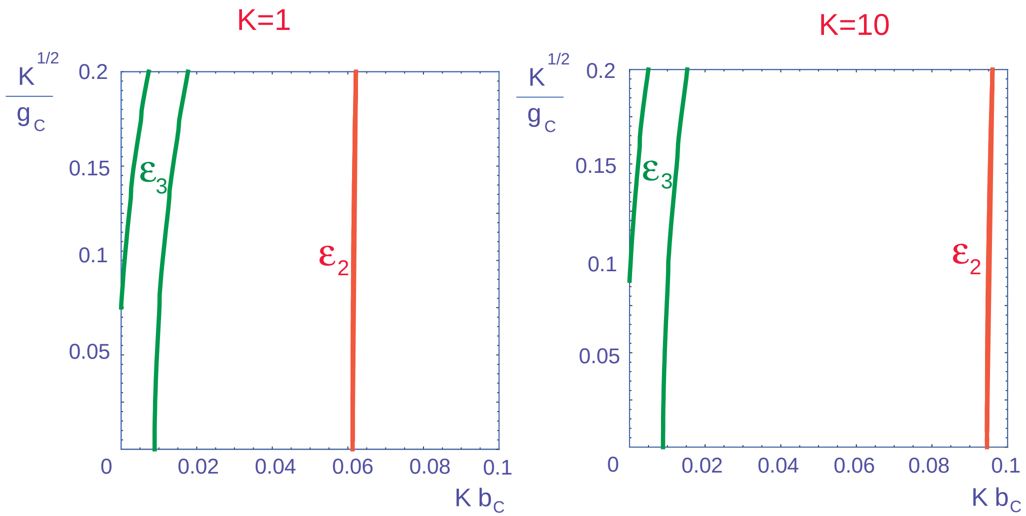
with $\quad b_j^L = (\frac{\alpha_0}{m_j} \prod_{i=1}^{j-1} \frac{\alpha_i}{m_i})^2 \geqslant 0, \quad b_j^R = (\frac{\alpha_K}{m_K} \prod_{i=j}^{K-1} \frac{\alpha_i}{m_i})^2 \geqslant 0 \qquad (\alpha_K \ll \alpha_0)$

$$m^f = m_j \sqrt{\frac{b_j^L}{(1+\sum_{i=1}^K b_i^L)}} \sqrt{\frac{b_j^R}{(1+\sum_{i=1}^K b_i^R)}} \qquad \forall j = 1, .., K$$

Bounds from ϵ_i parameters

(Altarelli,Barbieri,Caravaglios; Burgess et al; Anichini,Casalbuoni D.C.; Casalbuoni,D.C.,Dolce,Dominici)

Take the low-energy limit by eliminating the A_i fields with their e.o.m.
Evaluate the corrections to the relevant physical quantities ($b_i^R = 0$)
Simplest model: $g_i \equiv g_c$, $b_i^L \equiv b_c$, $f_i \equiv f_c$, $\forall i$



95%CL bound from ϵ_1: $b_c < 0.14 \ \ (0.025)$ **for** $K = 1 \ (10)$**, independent on** g_c

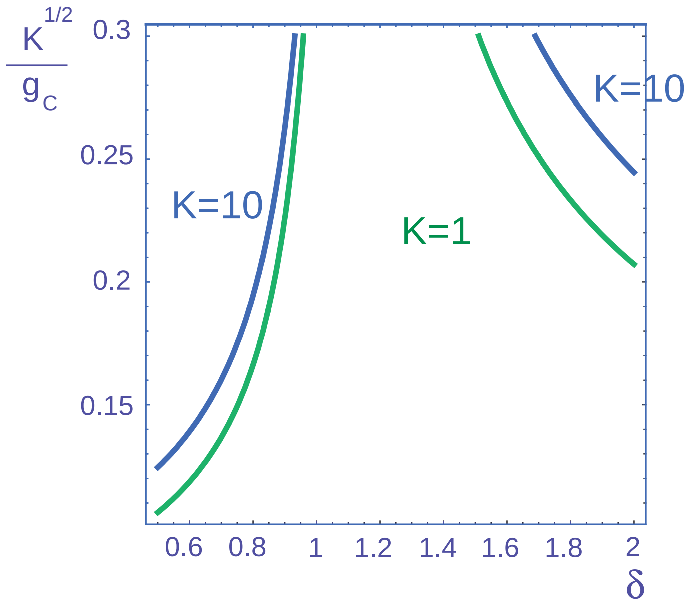
To $\mathcal{O}(\tilde{g}/g_i)^2$, $\mathcal{O}(b_i^L)$, neglecting $\mathcal{O}(b_i^L/g_i)^2$ and effective four-fermion couplings ($\mathcal{O}(b_i^2)$):

$$\epsilon_1 \simeq 0, \quad \epsilon_2 \simeq 0 \quad \epsilon_3 \simeq \sum_{i=1}^{K} y_i\left(\frac{\tilde{g}^2}{g_i^2}(1-y_i) - b_i^L\right) \qquad \left(y_i = \sum_{j=1}^{i} \frac{f^2}{f_j^2}\right)$$

Local cancellation: $b_i^L = \delta \frac{\tilde{g}^2}{g_i^2}(1 - y_i)$

for $\delta = 1$ there is no contribution to ϵ_3 from new physics

Take: $g_i \equiv g_c, f_i \equiv f_c, \quad \forall i$



95% CL bounds on the parameter space $(\delta, \sqrt{K}/g_c)$ from the experimental value of ϵ_3. The allowed region is between the corresponding lines. ϵ_1, ϵ_2 give very loose bounds.

By fine-tuning every direct fermion coupling in each site to compensate the gauge bosons contribution to ϵ_3, a sizeable region of the parameter space is left.

Which is the continuum limit for the direct fermionic couplings when local cancellation is required?

To explore the ways to realize the compensation between fermion and gauge sector contributions let $K \to \infty$ with $(K+1)a \to \pi R$

$$\lim_{a\to 0} \frac{b_j^{L,R}}{a} = b^{L,R}(z), \quad \lim_{a\to 0} a f_j^2 = f^2(z), \quad \lim_{a\to 0} a g_j^2 = g_5^2(z) e^{A(z)}$$

Conformally flat metric: $ds^2 = e^{-2A(z)}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$

Use the relations between the moose and the 5D theory

$$\frac{1}{f^2} = \int_0^{\pi R} \frac{dz}{f^2(z)} = \int_0^{\pi R} dz\; g_5^2(z) e^{A(z)}$$

From the expressions of the $b_i^{L,R}$ couplings generated by decoupling the heavy modes in the bulk:

$$b^L(z) = \hat{t}_L^2 \exp[-2\int_0^z dt M e^{-A(t)}], \quad b^R(z) = \hat{t}_R^2 \exp[-2\int_z^{\pi R} dt M e^{-A(t)}]$$

And, for the fermion mass, neglecting terms $\mathcal{O}(b^2)$:

$$m^f = \sqrt{b^L(z) b^R(z)} = \hat{t}_R \sqrt{b^L(\pi R)}$$

- **Local cancellation** of ϵ_3 requires:

$$\lim_{a\to 0}\frac{1}{a}\frac{\tilde g^2}{g_i^2}(1-y_i)=\lim_{a\to 0}\frac{b_i^L}{a}$$

$$b^L(z)=\hat t_L^2\exp[-2\int_0^z dtMe^{-A(t)}]=\tilde g^2 f^2\int_z^{\pi R}dt\frac{f^2(z)}{f^2(t)}\ \forall z\ \Rightarrow\ b^L(\pi R)=0\Rightarrow m^f=0$$

Example: $f_j=f_c$ and $g_j=g_c$ in order to satisfy the ideal delocalization one must require $e^{A(z)}=1-\frac{z}{\pi R}$ a singular metric on the right brane $\Rightarrow$ the right handed fermions are on one horizon of the metric

- **Global cancellation**:(*Foadi,Gopalakrishna,Schmidt*)

$$\epsilon_3=\int_0^{\pi R}dz\frac{z}{\pi R}\Big[\frac{\tilde g^2}{g_5^2(z)}(1-\frac{z}{\pi R})-\hat t_L^2 e^{-2\int_0^z dtMe^{-A(t)}}\Big]=0$$

FLAT METRIC: $e^{-A(z)}=1$, $g_5(z)=g_5$, we get $\lambda^2/6=t_L^2\hat A$ with $t_L=\hat t_L\sqrt{\pi R}$, $\lambda=\frac{\tilde g}{g_5}\sqrt{\pi R}$, $\hat A=\frac{1}{4\hat M^2}-\frac{1}{2\hat M}(1+\frac{1}{2\hat M})e^{-2\hat M}$ and $\hat M=\pi RM$.

RS METRIC for $k>>(\pi R)^{-1},M,$ and $g_5(z)=g_5$,
$\epsilon_3=0\Rightarrow t_L\sim\pi R\sqrt{3/8}M_W$, a result very close to the flat case. Ex: for $R^{-1}\sim 1$ TeV, we have $t_L\sim 0.15$. A smaller t_L requires a higher R^{-1} which implies a partial wave unitarity violation at a lower scale.

Holographic approach to Higgsless model

(Luty,Porrati,Rattazzi;Barbieri,Pomarol,Rattazzi; Burdman,Nomura)

Holography: useful technique avoiding the KK decomposition

Bulk dynamics projected on the branes through the bulk eqs. of motion with fixed boundary value to the source or interpolating field

$$\Phi(x,y)|_{\Sigma} = \phi_{\Sigma}(x)$$

The resulting (4D) holographic theory is

$$\mathcal{S}_{\Phi}^{holo} = \mathcal{S}_{\Phi_\Sigma}^{kin} + \int \frac{d^4p}{(2\pi)^4}\phi_\Sigma(p)\Pi_{\phi_\Sigma\phi_\Sigma}(p^2)\phi_\Sigma(p)$$

$\Pi_{\phi_\Sigma\phi_\Sigma}(p^2)$ bulk contribution to the vacuum polarization

When a dual AdS/CFT interpretation is possible, holography is necessary to compute correlators of the dual 4D theory $\Rightarrow$ the boundary values of the 5D fields are the source for the 4D operators

$$\mathcal{Z} = \int \mathcal{D}\phi_{CFT} e^{-S_{CFT}[\phi_{CFT}] - \int_{-\infty}^{\infty} d^4x \phi_\Sigma \mathcal{O}} = \int_{\phi_\Sigma} \mathcal{D}\phi e^{-S_\Phi[\Phi]} \equiv e^{i\mathcal{S}_{\Phi}^{holo}[\phi_\Sigma]} = \mathcal{Z}[\phi_\Sigma]$$

BUT the holographic procedure is a technical tool which can be applied to any 5D theory even if no dual 4D exists. For some applications it is much simpler than the standard effective theory for the zero-modes.

Holographic gauge sector

(Barbieri,Pomarol,Rattazzi)

Bulk gauge theory: 5D $SU(2)$ **Unitary gauge: $A_5^a = 0$** **Flat metric**

$$S_{YM}^{(2)} = -\frac{1}{g_5^2}\int d^4x \mathrm{Tr}[A^\mu \partial^5 A_\mu]|_0^{\pi R} - \frac{1}{2g_5^2}\int d^4x \int_0^{\pi R} dy \left\{-2\mathrm{Tr}[A^\mu \partial_5^2 A_\mu] + \mathrm{Tr}[A^{\mu\nu}A_{\mu\nu}]\right\}$$

The linearized bulk eqs of motion for the transverse component (from the longitudinal suppression factor $(m_f/m_A)^2$ to the electroweak corrections):

$$(\partial_y^2 + p^2)A_\mu^t(p,y) = 0 \quad \Rightarrow \quad S_{YM}^{(2)Holog} = -\frac{1}{g_5^2}\int \frac{d^4p}{(2\pi)^4} Tr[A_\mu(p,y)\partial_5 A^\mu(p,y)]_0^{\pi R}$$

Require the SM gauge content at the end points $\longrightarrow$ Add brane kinetic terms $\Rightarrow$ standard kinetic contribution + BC's from mass boundary terms

$$S_{YM}^{brane} = \int d^4x \int_0^{\pi R} dy \Big\{ \frac{\delta(y)}{2\tilde{g}^2}\Big[-\mathrm{Tr}[F^{\mu\nu}F_{\mu\nu}] + c_1^2 Tr[(A^\mu - \tilde{g}\tilde{W}^\mu)(A^\mu - \tilde{g}\tilde{W}_\mu)]\Big]$$
$$+ \frac{\delta(y-\pi R)}{4\tilde{g}'^2}\Big[-F^{3\mu\nu}F^3_{\mu\nu} + c_2^2[(A^{3\mu} - \tilde{g}'\tilde{\mathcal{Y}}^\mu)(A^3_\mu - \tilde{g}'\tilde{\mathcal{Y}}_\mu) + A^{1,2\mu}A^{1,2}_\mu]\Big]\Big\}$$

BC's in the limit $c_{1,2} \to \infty$ fix the boundary values of A_μ to the standard gauge fields $\tilde{W}_\mu = \tilde{W}_\mu^a T_a$ and $\tilde{\mathcal{Y}}_\mu$:

$$A_\mu^\pm(x,y)|_{y=0} \equiv \tilde{g}\tilde{W}_\mu^\pm(x), \qquad A_\mu^\pm(x,y)|_{y=\pi R} \equiv 0$$
$$A_\mu^3(x,y)|_{y=0} \equiv \tilde{g}\tilde{W}_\mu^3(x), \qquad A_\mu^3(x,y)|_{y=\pi R} \equiv \tilde{g}'\tilde{\mathcal{Y}}_\mu(x)$$

Generic solutions in terms of interpolating functions $h(p,y)$

$$A_\mu^\pm(p,y) = \tilde{g}h_\pm(p,y)\tilde{W}_\mu^\pm(p)\ , \quad A_\mu^3(p,y) = \tilde{g}h_W(p,y)\tilde{W}_\mu^3(p) + \tilde{g}'h_Y(p,y)\tilde{\mathcal{Y}}_\mu(p)$$

solutions of: $(\partial_y^2 + p^2)h(p,y) = 0 \quad + \ \text{BC's}$

$$h_Y(p,y) = \frac{\sin[py]}{\sin[p\pi R]}\ , \quad h_\pm(p,y) = h_W(p,y) = \frac{\sin[p(\pi R - y)]}{\sin[p\pi R]}$$

The LO behavior in p of the functions $h_{Y,W}(p,y)$ reproduces the same solutions obtained by the heavy mode elimination by e.o.m. in the 4D deconstructed model

By comparing with the generic extension of the quadratic SM gauge Lagrangian, we get the bulk contributions to the two-point functions

$$\Pi_{WY}(p^2) = -\frac{1}{2g_5^2}[h_Y h'_W + h_W h'_Y]_0^{\pi R}, \quad \Pi_{YY}(p^2) = -\frac{1}{2g_5^2}[h_Y h'_Y]_0^{\pi R}$$

$$\Pi_{WW}(p^2) = \Pi_{\pm\mp}(p^2) = -\frac{1}{2g_5^2}[h_W h'_W]_0^{\pi R} \longrightarrow SU(2)_{cust}$$

Electroweak parameters (only oblique corrections)

The holographic form of the oblique corrections to the ϵ parameters, given in terms of the vacuum polarization amplitudes, is:

$$\epsilon_1^{obl} = \epsilon_2^{obl} = 0 \,, \qquad \epsilon_3^{obl} = -\frac{\tilde{g}^2}{2g_5^2}\frac{d}{dp^2}[h_Y h'_W + h_W h'_Y]_{0,p^2=0}^{\pi R} \,.$$

Alternatively $\Rightarrow$ find the extra contribution to the Π_{WY} by substituting the solutions of the bulk equation of motions for the gauge fields

$$\epsilon_3^{obl} = \frac{\tilde{g}^2}{g_5^2}\int_0^{\pi R} dy[h_Y h_W]_{p^2=0} = \frac{\tilde{g}^2}{g_5^2}\frac{\pi R}{6}$$

Relation with the deconstructed model: $\epsilon_3^{obl} = \tilde{g}^2 \sum_{i=1}^{K} \frac{y_i}{g_i^2}(1-y_i)$

$$g_5^2 \longrightarrow a g_j^2$$
$$h_Y(0,y) = y/\pi R \longrightarrow y_i = \sum_{j=1}^{i} f^2/f_j^2$$
$$h_W(0,y) = 1 - y/\pi R \longrightarrow z_i = 1 - y_i$$

Additional electroweak parameters *(Barbieri,Pomarol,Rattazzi,Strumia)* as W, Y, W, are suppressed by a factor $M_W^2 R^2$ with respect to $\hat{S}$, while $V = 0$ due to the custodial symmetry of the model

Holographic fermionic sector

(Contino,Pomarol; Panico,Serone,Wulzer; Foadi,Schmidt)

Bulk action for 5D Dirac field in unitary gauge $A_5 = 0$

$$S^{bulk}_{ferm} = \frac{1}{\hat{g}_5^2} \int d^4x \int_0^{\pi R} dy \left\{ \bar{\Psi} i \gamma^\mu D_\mu \Psi + \frac{1}{2} [\bar{\Psi} \gamma^5 \partial_5 \Psi - \partial_5 \bar{\Psi} \gamma^5 \Psi] - M \bar{\Psi} \Psi \right\}$$

$$D_\mu \Psi(x,y) = \Big(\partial_\mu + i T_a A^a_\mu(x,y) + \frac{i}{2}(B-L) A^3_\mu(x,\pi R) \Big) \Psi(x,y)$$

Bulk eqs. of motion ($\Psi = \psi_L + \psi_R, \quad \gamma_5 \psi_{L,R} = \mp \psi_{L,R}$):

$(\partial_5^2 + \omega^2)\psi_{L,R} = 0$ where $\omega = \sqrt{p^2 - M^2}$

Add the brane action (q_L and q_R interpolating fields):

$$\begin{aligned} S^{brane}_{ferm} &= \int d^4x \int_0^{\pi R} dy \delta(y) \left[\bar{q}_L i \gamma^\mu D_\mu q_L + \frac{1}{\hat{g}_5^2} \left(\mathtt{t}_L (\bar{\psi}_R q_L + \bar{q}_L \psi_R) - \frac{1}{2} \bar{\Psi} \Psi \right) \right] \\ &+ \delta(y - \pi R) \left[\bar{q}_R i \gamma^\mu D_\mu q_R + \frac{1}{\hat{g}_5^2} \left(\mathtt{t}_R (\bar{q}_R \psi_L + \bar{\psi}_L q_R) - \frac{1}{2} \bar{\Psi} \Psi \right) \right] \end{aligned}$$

In agreement with the gauge symmetries on the branes, we have

$$D_\mu q_L|_{y=0} = \left(\partial_\mu + i\tilde{g}T^a\tilde{W}^a_\mu + i\frac{\tilde{g}'}{2}(B-L)\tilde{\mathcal{Y}}_\mu\right)q_L$$

$$D_\mu q_R|_{y=\pi R} = \left(\partial_\mu + i\tilde{g}'T^3\tilde{\mathcal{Y}}_\mu + i\frac{\tilde{g}'}{2}(B-L)\tilde{\mathcal{Y}}_\mu\right)q_R$$

BC's from variational analysis

$$\psi_L(p,0) \equiv \mathtt{t}_L q_L(p)\ , \qquad \psi_R(p,\pi R) \equiv \mathtt{t}_R q_R(p)$$

Solutions:

$$\begin{aligned}\psi_L(p,y) &= f_L(p,y)\mathtt{t}_L q_L(p) + \not{p}\pi R\tilde{f}_L(p,y)\mathtt{t}_R q_R(p)\\ \psi_R(p,y) &= f_R(p,y)\mathtt{t}_R q_R(p) + \not{p}\pi R\tilde{f}_R(p,y)\mathtt{t}_L q_L(p)\end{aligned}$$

Low energy behavior:

$$\begin{aligned}\psi_L(p,y) &\sim \mathtt{t}_L q_L(p)e^{-My} + \mathtt{t}_R\ \not{p}q_R(p)\frac{\sinh[My]}{M}e^{-M\pi R}\\ \psi_R(p,y) &\sim \mathtt{t}_R q_R(p)e^{M(y-\pi R)} + \mathtt{t}_L\ \not{p}q_L(p)\frac{\sinh[M(y-\pi R)]}{M}e^{-M\pi R}\end{aligned}$$

Fermion kinetic terms from S_{ferm}^{brane}:

$$\mathcal{L}_{ferm}^{kin} = \bar{q}_L \not{p} \left(1 + \mathtt{t}_L^2 \frac{\pi R}{\hat{g}_5^2} \tilde{f}_R(0,0)\right) q_L + \bar{q}_R \not{p} \left(1 + \mathtt{t}_R^2 \frac{\pi R}{\hat{g}_5^2} \tilde{f}_L(0,\pi R)\right) q_R$$

Normalization of $q_{L,R}$ (use the properties of f and $\tilde{f}$):

$$q_{L,R} \to \frac{q_{L,R}}{\sqrt{1 + \int_0^{\pi R} dy\ b_{L,R}(y)}} \quad \text{with} \quad b_{L,R}(y) = \mathtt{t}_{L,R}^2 \frac{f_{L,R}^2(0,y)}{\hat{g}_5^2}$$

relation between the holographic procedure and the continuum limit of the 4D deconstructed model $\int dy \to a \sum_{i=1}^{K}$, $b_L(y) \to b_L^i/a$, $b_R(y) \to b_R^i/a$ with $b_{L,R}^i$ generated by the elimination of the bulk fermions.

Fermion mass term

$$\mathcal{L}_{ferm}^{mass} = \frac{1}{2} \frac{\mathtt{t}_R \mathtt{t}_L}{\hat{g}_5^2} \left[\frac{f_R(0,0) + f_L(0,\pi R)}{\sqrt{1 + \int_0^{\pi R} \frac{dy}{\hat{g}_5^2} b_L(y)}\sqrt{1 + \int_0^{\pi R} \frac{dy}{\hat{g}_5^2} b_R(y)}}\right] (\bar{q}_L q_R + \bar{q}_R q_L)$$

For $\mathtt{t}_{L,R}^2 \pi R/\hat{g}_5^2 \ll 1$ the 4D fermion mass is $m_q = \frac{\mathtt{t}_L \mathtt{t}_R}{\hat{g}_5^2} \exp(-M\pi R)$

The interaction

$$S^{Int} = -\int \frac{d^4p}{(2\pi)^4} \int_0^{\pi R} \frac{dy}{\hat{g}_5^2} \bar{\Psi}(p,y)\gamma^\mu \left[A_\mu(p,y) + \frac{\tilde{g}'}{2}(B-L)\tilde{\mathcal{Y}}_\mu(p) \right] \Psi(p,y)$$

Use the solution of the bulk e.o.m. for fermions and for gauge fields:

$$A_\mu^a(p,y) = \tilde{g} h_W(p,y) \tilde{W}_\mu^a(p) + \tilde{g}' \delta^{a3} h_Y(p,y) \tilde{\mathcal{Y}}_\mu(p)$$

After fermion normalization, we get the lowest order effective interaction:

$$\begin{aligned} \mathcal{L}_{ferm} &= -\tilde{e} Q \tilde{A}_\mu \bar{q}\gamma^\mu q - \frac{\tilde{e}}{\tilde{s}_\theta \tilde{c}_\theta} \tilde{Z}_\mu \bar{q}\gamma^\mu \left\{ T^3 \frac{1-\gamma_5}{2}\left(1 - \frac{\mathtt{b}_L}{2}\right) - T^3 \frac{1+\gamma_5}{2}\frac{\mathtt{b}_R}{2} - \tilde{s}_\theta^2 Q \right\} q \\ &- \left[\frac{\tilde{e}}{\tilde{s}_\theta \sqrt{2}} \tilde{W}_\mu^- \bar{q}_d \gamma^\mu \left\{ \frac{1-\gamma_5}{2}\left(1 - \frac{\mathtt{b}_L}{2}\right) - \frac{1+\gamma_5}{2}\frac{\mathtt{b}_R}{2} \right\} q_u + h.c. \right] \end{aligned}$$

where $q = q_L + q_R$, $Q = T^3 + \frac{B-L}{2}$ and the corrections to the electroweak currents are given by

$$\mathtt{b}_L = \frac{2\int_0^{\pi R} dy\ b_L(y) h_Y(0,y)}{1 + \int_0^{\pi R} dy\ b_L(y)}, \quad \mathtt{b}_R = \frac{2\int_0^{\pi R} dy\ b_R(y) h_Y(0,y)}{1 + \int_0^{\pi R} dy\ b_R(y)}$$

In the following neglect $\mathtt{b}_R$ strongly bounded by $b \to s\gamma$ (take $\mathtt{t}_R << \mathtt{t}_L$)

Direct contribution to the ϵ parameters

(Altarelli,Barbieri,Caravaglios; Burgess et al; Anichini,Casalbuoni,D.C.; Casalbuoni,D.C.,Dolce, Dominici)

Due to vertex corrections $\longrightarrow \epsilon$ parameters from a general formulation involving the renormalization of the EW observables Δr_W, $\Delta\rho$ and Δk.

At the leading order in $\mathtt{t}_L^2 \pi R/\hat{g}_5^2$ and in the limit $\tilde{g}^2\pi R/g_5^2 \ll 1$ (corresponding to $\tilde{g}^2/g_i^2 \ll 1$ in the deconstructed version) we get:

$$\epsilon_1 \sim \epsilon_2 \sim 0, \qquad \epsilon_3 \sim \int_0^{\pi R} dy h_Y(0,y)\left\{\frac{g^2}{g_5^2} h_W(0,y) - b_L(y)\right\}$$

The ideal fermionic delocalization for vanishing ϵ_3 is given by

$$b_L(y) = \mathtt{t}_L^2 \frac{f_L^2(0,y)}{\hat{g}_5^2} = \frac{\tilde{g}^2}{g_5^2} h_W(0,y) \quad \forall y \in [0, \pi R]$$

But, using the explicit solution for $h_W(0,y)$ we must require $f_L(0,y) \propto \sqrt{1 - \frac{y}{\pi R}}$ which is not a solution of the e.o.m. for the Dirac bulk field $\longrightarrow$ the ideal delocalization is not allowed $\Rightarrow$ same result in the 4D deconstructed model *(Bechi,Casalbuoni,D.C.,Dominici)*

Vanishing ϵ_3 ONLY via global cancellation by linking the parameters of the gauge and fermion sectors

Holographic gauge sector: warped scenario

$SU(2)$ **bulk gauge theory**: RS metric $ds^2 = \frac{1}{(kz)^2}(dx^2 - dz^2)$

The bulk e.o.m. for the transverse components of the gauge field are:

$$(D_5^2 + p^2)A^t(p,z) = 0 \text{ with } D_5^2 = z\partial_5(\frac{1}{z}\partial_5)$$

Branes kinetic terms: standard kinetic contribution + **BC's** from mass boundary terms $\longrightarrow$ same boundary values for the gauge fields up to the redefinition $\tilde{g} \to \tilde{g}\sqrt{kL_0}$, $\tilde{g}' \to \tilde{g}'\sqrt{kL_1}$, ($L_0$, L_1 are the brane locations)

$$\mathcal{L}_{YM}^{(2)hol+brane} = -\frac{\tilde{g}'}{2g_5^2}\Big[\frac{1}{kz}\tilde{\mathcal{Y}}^\mu(p)\partial_5 A_\mu^3(p,z)\Big]_{z=L_1} + \frac{\tilde{g}}{2g_5^2}\Big[\frac{1}{kz}\tilde{W}^{a\mu}(p)\partial_5 A_\mu^a(p,z)\Big]_{z=L_0}$$
$$+ \frac{p^2}{2}\tilde{W}_\mu^a(p)\tilde{W}^{a\mu}(p) + \frac{p^2}{2}\tilde{\mathcal{Y}}_\mu(p)\tilde{\mathcal{Y}}^\mu(p)$$

Electroweak parameters (only oblique corrections):

$$\epsilon_1^{obl} = 0\ , \qquad \epsilon_2^{obl} = 0\ , \qquad \epsilon_3^{obl} = -\frac{\tilde{g}^2}{2g_5^2}\frac{d}{dp^2}\Big[\frac{1}{kz}(h_Y h_W' + h_W h_Y')\Big]_{L_0,p^2=0}^{L_1}$$

The delocalization functions $h_{Y,W}(p,z)$, solutions of the e.o.m. are given in terms of Bessel functions J_1 and Y_1, substituting:

$$\epsilon_3^{obl} = \frac{\tilde{g}^2}{4kg_5^2}\frac{L_1^4 - L_0^4 - 4L_0^2L_1^2\log[L_1/L_0]}{(L_1^2 - L_0^2)^2}$$

Fermions in warped scenario

Bulk e.o.m. $\not{p}\psi_{L,R}(p,z) \pm \left(\partial_5 \mp \frac{c\pm 2}{z}\right)\psi_{R,L}(p,z) = 0$, with $c = \frac{M}{k}$

Brane action terms as in the flat scenario $\longrightarrow$ BC's:

$$\psi_L(p, L_0) \equiv \mathtt{t}_L q_L(p) \ , \qquad \psi_R(p, L_1) \equiv \mathtt{t}_R q_R(p)$$

The solutions for $f_{L,R}$ and $\tilde{f}_{L,R}$ are in terms of Bessel functions.

For $p^2 = 0$ we get: $f_L(0,z) = (\frac{z}{L_0})^{2-c}$ and $f_R(0,z) = (\frac{z}{L_1})^{2+c}$

Following the same procedure as in the flat case, we get the same effective interaction with:

$$b_{L,R}(z) = \mathtt{t}^2_{L,R}\frac{f^2_{L,R}(0,z)}{\hat{g}_5^2(kz)^4} = \frac{\mathtt{t}^2_{L,R}}{\hat{g}_5^2}\left(\frac{1}{kL_{0,1}}\right)^4\left(\frac{L_{0,1}}{z}\right)^{2c}$$

Neglecting the $\mathtt{t}_R$ contribution:

$$\epsilon_3 \sim \int_{L_0}^{L_1} dz h_Y(0,z)\left\{\frac{1}{kz}\frac{\tilde{g}^2}{g_5^2}h_W(0,z) - b_L(z)\right\}$$

Ideal delocalization $\longrightarrow$ $h_W(0,z) = \frac{L_1^2 - z^2}{L_1^2 - L_0^2} = \left(\frac{g_5^2}{\tilde{g}^2}\right) kz b_L(z)$

Satisfied by $c = -\frac{1}{2}$, $L_0 = (\mathtt{t}_L^2 g_5^2/(\tilde{g}^2\hat{g}_5^2))^{1/3}/k$, and $L_1 = 0$ that is in a SINGULAR METRIC on $z = L_1$ (same result in 4D decostructed model)

Conclusions

- The holographic prescription applied to 5D YM and 5D Dirac theories is an alternative approach to the deconstruction analysis of the Higgsless models for studying low-energy effective Lagrangians

- The holographic technique here is equivalent to the elimination of the fields of the internal sites of the moose in terms of the light fields, the interpolating SM fields

- This equivalence has been shown in a minimal Higgsless model based on the symmetry $SU(2)$ broken by BC's in the limit $\tilde{g}^2 \pi R / g_5^2 \ll 1$ which corresponds in the deconstructed theory to the limit $\tilde{g}^2 / g_i^2 \ll 1$ where g_i is the coupling constant of the gauge group of the i-th site

- We have shown that an ideal delocalization of the fermions along the extra dimension is not allowed by the bulk equations of motion, whatever the metric. However a global cancellation of the ϵ_3 parameter is possible with fine-tuning

Local $U(1)_{B-L}$ interaction

Bulk gauge symmetry $SU(2) \otimes U(1)_{B-L}$

$$S_{B-L}^{hol+int} = -\frac{1}{2g_5'^2}\int \frac{d^4p}{(2\pi)^4}[B_\mu(p,y)\partial_5 B^\mu(p,y)]_0^{\pi R}$$
$$-\int \frac{d^4p}{(2\pi)^4}\int_0^{\pi R}\frac{dy}{\hat{g}_5^2}\bar{\Psi}(p,y)\gamma^\mu\left[A_\mu(p,y)+\frac{1}{2}(B-L)B_\mu(p,y)\right]\Psi(p,y)$$

with $B_\mu(p,y)$ solution of the bulk e.o.m.

$B_\mu(p,y) = \tilde{g}' h_{B-L}(p,y)\tilde{\mathcal{Y}}_\mu(p)$, with BC's $B_\mu(x,y)|_{y=0,\pi R} \equiv \tilde{g}'\tilde{\mathcal{Y}}_\mu(x)$

from which $h_{B-L}(0,y)=1 \longrightarrow$ flat profile

For $g_5' \gg \hat{g}_5 \longrightarrow$ neglect the holographic $U(1)_{B-L}$ contribution

Neglecting the $O(p^2)$ terms we get the effective interaction

$$S_{B-L}^{int} = -\int \frac{d^4p}{(2\pi)^4}\int_0^{\pi R}\frac{dy}{\hat{g}_5^2}\bar{\Psi}(p,y)\gamma^\mu\Big[\tilde{g}' h_Y(0,y)\tilde{Y}_\mu$$
$$+\tilde{g}h_W(0,y)\tilde{W}_\mu + \frac{\tilde{g}'}{2}(B-L)\tilde{\mathcal{Y}}_\mu\Big]\Psi(p,y)$$

No additional oblique corrections to ϵ_3 from the extra $B-L$ factor but phenomenological implications to be studied

Top quark problem

(Foadi,Schmidt; Bechi,Casalbuoni,D.C., Dominici)

Allow a microscopically broken Lorentz invariance: $R_f = R/\mathtt{k}$

$$S_{top}^{bulk} = \frac{1}{\hat{g}_5^2} \int d^4x \int_0^{\pi R} dy \left\{ \bar{\Psi} i\gamma^\mu D_\mu \Psi + \frac{\mathtt{k}}{2} [\bar{\Psi}\gamma^5 \partial_5 \Psi - \partial_5 \bar{\Psi}\gamma^5 \Psi] - M\bar{\Psi}\Psi \right\}$$

$$\begin{aligned} S_{ferm}^{brane} &= \int d^4x \int_0^{\pi R} dy \delta(y) \left[\bar{q}_L i\gamma^\mu D_\mu q_L + \frac{\mathtt{k}}{\hat{g}_5^2} \left(\mathtt{t}_L (\bar{\psi}_R q_L + \bar{q}_L \psi_R) - \frac{1}{2}\bar{\Psi}\Psi \right) \right] \\ &+ \delta(y - \pi R) \left[\bar{q}_R i\gamma^\mu D_\mu q_R + \frac{\mathtt{k}}{\hat{g}_5^2} \left(\mathtt{t}_R (\bar{q}_R \psi_L + \bar{\psi}_L q_R) - \frac{1}{2}\bar{\Psi}\Psi \right) \right] \end{aligned}$$

$$\psi_{L,R}(p,y) = \mathtt{t}_{L,R} f_{L,R}(p,y) q_{L,R}(p) + \mathtt{t}_{R,L} \frac{\not{p}}{\mathtt{k}} \pi R \tilde{f}_{L,R}(p,y) q_{R,L}(p)$$

Interaction unchanged; **rescaled mass** $m_q = \mathtt{k}\frac{\mathtt{t}_L \mathtt{t}_R}{\hat{g}_5^2} \exp(-M\pi R)$

Different $\mathtt{t}_R$ **for different flavors** $\Rightarrow$ $\mathtt{t}_R^t/\mathtt{t}_R^b = m^t/m^b \sim 40$

Exp. Bound on $\mathtt{k}_R^{CC} =\leq 4 \times 10^{-3}$. **From the** tbW **coupling** $\Rightarrow$

$\mathtt{k}_R^{CC} = \frac{1}{2}\sqrt{\mathtt{b}_R^t \mathtt{b}_R^b}$. **Possible choice (good for** ϵ_3**):** $\mathtt{t}_L = 0.01$, $\mathtt{t}_R^t = 0.4$

Assuming $R^{-1} \sim 1$ **TeV the top mass value can be obtained for** $\mathtt{k} \sim 10 \Rightarrow$
KK fermions ten times heavier than KK gauge bosons.

Effective current-current interaction

First solve the free e.o.m. for the bulk free fermion fields:

$$\begin{aligned}\psi_L(p,y) &= f_L(p,y)\mathtt{t}_L q_L(p) + \not{p}\pi R\tilde{f}_L(p,y)\mathtt{t}_R q_R(p)\\ \psi_R(p,y) &= f_R(p,y)\mathtt{t}_R q_R(p) + \not{p}\pi R\tilde{f}_R(p,y)\mathtt{t}_L q_L(p)\end{aligned}$$

then solve the bulk e.o.m. for the gauge fields including the gauge-fermion interaction term (taken at LO in p and for $\mathtt{t}_R << \mathtt{t}_L$):

$(\partial_y^2 + p^2)A^a_\mu(p,y) = g_5^2 J^a_{L\mu}(p,y)$, with $J^a_{L\mu}(p,y) \sim -b_L(y)\bar{q}_L(p)\gamma_\mu T^a q_L(p)$

and $b_L(y) = \mathtt{t}_L^2 \frac{f_L^2(0,y)}{\hat{g}_5^2}$

The general solution is:

$$A^a_\mu(p,y) = h_W(p,y)\Big[\tilde{g}\tilde{W}^a_\mu(p) - V^a_\mu(p,0)\Big] + h_Y(p,y)\delta^{a3}\tilde{g}'\tilde{\mathcal{Y}}_\mu(p) + V^a_\mu(p,y)$$

with

$$\begin{aligned}V^a_\mu(p,y) &= g_5^2\Big[\sin(py)\int_y^{\pi R} dy' J^a_{L\mu}(p,y')\frac{\cos(py')}{p}\\ &\quad - \cos(py)\int_y^{\pi R} dy' J^a_{L\mu}(p,y')\frac{\sin(py')}{p}\Big]\end{aligned}$$

By substituting in $S_{YM}^{(2)Holog}$ and in S^{int} we get an effective current-current interaction

$$\mathcal{L}^{4f} = \beta \sum_{a=1}^{3} \left(\bar{q}_L(p)\gamma_\mu T^a q_L(p)\right)^2$$

with

$$\beta = -\pi R g_5^2 \{(\bar{\mathtt{b}}_L(\pi R) - \mathtt{b}_L)^2 - \int_0^{\pi R} \frac{dy}{\pi R} \bar{\mathtt{b}}_L^2(y)\}$$

$$\mathtt{b}_L = \frac{2\int_0^{\pi R} dy\ b_L(y) h_Y(0,y)}{1 + \int_0^{\pi R} dy\ b_L(y)}, \quad \bar{\mathtt{b}}_L(y) = \frac{2\int_0^{y} dy'\ b_L(y')}{1 + \int_0^{\pi R} dy\ b_L(y)}$$

in agreement with the continuum limit of the result obtained in the 4D deconstructed model *(Casalbuoni,D.C.,Dolce,Dominici)* after identification

$$\pi R \longrightarrow (K+1)/a$$
$$\int_0^{\pi R} dy \longrightarrow a \sum_{i=1}^{K}$$
$$g_5^2 \longrightarrow 1/(a f_j^2)$$
$$h_Y(0,y) = y/\pi R \longrightarrow y_i = \sum_{j=1}^{i} f^2/f_j^2$$
$$b_L(y) \longrightarrow b_L^i/a$$

Holographic approach versus KK expansion in normal modes

Expand the bulk fields in terms of KK eigenfunctions

$$A^a_\mu(p,y) = \sum_n f^a_n(y) A^{a(n)}_\mu(p)$$

impose Neumann conditions $A^3_\mu|_{0,\pi R} = 0$ and Neumann condition on the $y=0$ brane and Dirichlet condition on $y=\pi R$ brane for the charged components of the bulk field, at leading order in $\tilde{g}^2\pi R/g_5^2$

$$\begin{aligned}
A^\pm_\mu(p,0) &\sim f^\pm_0(0) W^\pm_\mu(p) \sim \frac{\tilde{e}}{\tilde{s}_\theta}\tilde{W}^\pm_\mu(p),\\
A^\pm_\mu(p,\pi R) &\sim f^\pm_0(\pi R) W^\pm_\mu(p) \equiv 0,\\
A^3_\mu(p,0) &\sim f^3_0(0) A_\mu(p) + f^3_1(0) Z_\mu(p) \sim \tilde{e}\tilde{A}_\mu(p) + \tilde{e}\frac{\tilde{c}_\theta}{\tilde{s}_\theta}\tilde{Z}_\mu(p) = \tilde{g}\tilde{W}^3_\mu(p)\,,\\
A^3_\mu(p,\pi R) &\sim f^3_0(\pi R) A_\mu(p) + f^3_1(\pi R) Z_\mu(p) \sim \tilde{e}\tilde{A}_\mu(p) - \tilde{e}\frac{\tilde{s}_\theta}{\tilde{c}_\theta}\tilde{Z}_\mu(p) = \tilde{g}'\tilde{\mathcal{Y}}_\mu(p)
\end{aligned}$$

where we have introduced the SM neutral fields through the standard rotation and $\tilde{e} = \tilde{g}\tilde{s}_\theta$ and $\tilde{s}_\theta = \tilde{g}/\tilde{g}'$