Local SUSY-breaking minima in $N_f = N_c$ SQCD?

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Outline

1. ISS review, $N_f = N_c + 1$
2. ISS $N_f = N_c$ conjecture
3. Another deformation, saddle point
4. Phenomenological consequences
5. Conclusions

ISS review

Itriligator, Seiberg, Shih, 2006

- framework $SU(N_c)$ SQCD; $N_c + 1 \leq N_f < \frac{3}{2}N_c$, $W_{\rm tree} = (m_Q)_{ij}\bar{Q}_i Q_j$ and $\mathrm{rank}[m_Q] > N_c$
- SUSY-breaking local minimum near the origin
- magnetic dual: SUSY is broken by rank condition
- possesses SUSY vacuum

Demand $m_Q \ll \Lambda$ to get

- calculablity
- SUSY vacuum far from the orgin $\Longrightarrow$ metastability

$N_f = N_c$ *conjecture*

ISS approach:

- take $N_f = N_c + 1$, local SUSY-breaking minimum exists
- consider the limit $(m_Q)_{N_f,N_f} \to \infty$
- conjecture: $N_f = N_c$ has a similar vacuum,

$N_f = N_c$ *conjecture*

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but $N_f = N_c$ is very different

- $N_F > N_c$: $\frac{m_Q}{\Lambda} \ll 1 \Rightarrow$ non-calculable terms (Kähler) are under control
- $N_f = N_c$: m_Q/Λ **small, but Kähler is not under control**

ISS minimum for $N_f = N_c + 1$

- $\hat{\Lambda}$ - confining scale

$$W = \frac{1}{\hat{\Lambda}^{2N_c-1}} \left(B M \bar{B} + \mathrm{Tr}(m_Q M) + \ldots \right)$$

- take: (m_Q) diagonal $N_F \times N_F$ matrix and order

$$(m_Q)_1 \sim \ldots \sim (m_Q)_{N_c} \sim m_Q < (m_Q)_{N_c+1}$$

SUSY-breaking local minimum

$$M = 0 \quad B = -\bar{B} \propto (0, \ldots, 0, (m_Q)_{N_f})$$

- tree level masses $m^2_{\text{tree}} \sim m_Q \hat{\Lambda}$
- one-loop masses for pseudo-moduli $m^2_{1-\text{loop}} \sim \frac{m_Q \hat{\Lambda}}{16\pi^2}$

Why are higher order Kähler corrections unimportant?

We used quadratic approximation for Kähler potential:

$$K = M^\dagger M$$

Higher order terms: $K \sim \frac{(\mathrm{Tr} M^\dagger M)^2}{\hat{\Lambda}^2} + \dots$

Potential: $V = K^{-1}_{ij^*} F_i F^*_j ,\ F_M = m_Q \hat{\Lambda}$

Non-claculable contribution to the mass

$$\Delta m^2 \sim m_Q^2 \ll m^2_{\mathrm{calculable}} \sim m_Q \hat{\Lambda}$$

$\frac{m_Q}{\hat{\Lambda}} \ll 1$ allows control on non-calculable terms

The situation may change in $N_f = N_c$ case

$N_f = N_c$ *ISS conjecture*

Is there a meta-stable SUSY breaking vacuum here?

ISS approach: decouple one flavor

Limit: $(m_Q)_{N_f} \rightarrow \infty$, but $(m_Q)_1 \ldots (m_Q)_{N_c}$ finite

$\hat{M},\ \hat{B},\ \hat{\bar{B}}$: $N_f = N_c + 1$ mesons and baryons

$$\hat{M} = \begin{pmatrix} M_{ij} & \hat{M}_{i,N_c+1} \\ \hat{M}_{N_c+1,j} & M_{N_c+1,N_c+1} \end{pmatrix}$$

$$\hat{B} = (B_i, B) \quad \hat{\bar{B}} = (\bar{B}_i, \bar{B})$$

Take $(m_Q)_{N_f} \to \infty$

- $\hat{M}_{i,N_c+1},\ \hat{M}_{N_c+1,i},\ B_i, \bar{B}_i$ decouple
- $\mathcal{A} \sim M_{N_c+1,N_c+1}$ no dynamics $\Rightarrow$ Lagrange multiplier
- deformed moduli space $\det M - B\bar{B} = \Lambda^{2N_c}$
- $\Lambda^{2N_c} = (m_Q)_{N_c+1} \hat{\Lambda}^{2N_c-1}$

$(m_Q)_{N_c+1} \to \infty$

$\hat{\Lambda}^{2N_c-1} \to 0$

Take $(m_Q)_{N_f} \to \infty$

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  $(m_Q)_{N_c+1} \to \infty$ $\qquad$ $\hat{\Lambda}^{2N_c-1} \to 0$

  SUSY-breaking meta-stable minimum?

- $M_{ij} \to 0$, $B = -\bar{B} \to \Lambda^N$
- conjecture: the minimum is there... **Kähler??**

Kähler potential - beyond the quadratic approximation

Kähler metric

$$g^{-1}_{MM^\dagger} \sim \frac{\mathrm{Tr} M^\dagger M}{\Lambda^2} + \frac{\mathrm{Tr} M \mathrm{Tr} M^\dagger}{\Lambda^2} + \frac{(B_+ + B_+^\dagger)^2}{\Lambda^2} + \dots$$

Potential:

$$V = g^{-1}_{MM^\dagger} |F_M|^2$$

$$F_M = m_Q \Lambda.$$

$$V \sim m_Q^2 (\mathrm{Tr} M M^\dagger + \mathrm{Tr} M \mathrm{Tr} M^\dagger + (B_+ + B_+^\dagger)^2)$$

Contributions stays finite at $N_f = N_c$

Limit $N_f = N_c$ ***- masses***

Compare the contribution to the scalar masses

- calculable, tree $m^2 \sim \frac{m_Q^2 \hat{\Lambda}}{m_{N_c+1}} \longrightarrow 0$
- pseudo-moduli $m^2 \sim \frac{1}{16\pi^2} \frac{m_Q \hat{\Lambda}}{m_{N_c+1}} \longrightarrow 0$
- uncalculable $m^2_{\mathrm{uncalc}} \sim m_Q^2$ **finite**
- we even do not know the sign of m^2

Another deformation - ITIY

Itriligator, Thomas, 1996; Izawa, Yanagida, 1996

Try another deformation

- does the extremum survive?
- is it still a minimum?
- where is calculability lost?

Add singlets. Under $SU(N_f)_L \times SU(N_f)_R$:

- $S_{ij} \quad (\bar{\mathbf{N}}, \mathbf{N})$
- $T \quad (\mathbf{1}, \mathbf{1})$, $\quad \bar{T} \quad (\mathbf{1}, \mathbf{1})$

Low-energy Superpotential

$$W = \mathcal{A}(\det M - B\bar{B} - \Lambda^{2N}) + \lambda \mathrm{Tr}(SM) + \kappa(TB + \bar{T}\bar{B}) + m_Q \mathrm{Tr} M + \frac{m_S}{2} S^2 + \frac{m_T}{2}(T^2 + \bar{T}^2)$$

moduli space deformation

ISS mass-term

ITIY coupling

singlet masses - m_Q has no effect w/o them

ISS $N_f = N_c$ ***limit***

Decoupling limit:

$$\frac{\lambda^2}{m_S} \to 0; \quad \frac{\kappa^2}{m_T} \to 0$$

SUSY-breaking solution should:

- define $B_\pm \sim B \pm \bar{B}$
- $M \to 0$, $B_+ \to 0$, $B_- \to \Lambda$ at the decoupling limit
- $F_M \propto m_Q$

SUSY solution:

- decoupling limit - finite distance from the origin
- sufficiently far from SUSY-breaking solution

SUSY solution

1.

$$|\mathrm{Tr}M| = \Lambda^2; \quad B_\pm = T_\pm = 0; \quad |S| = -\frac{\lambda}{m_S}\Lambda^2$$

2. two more solutions with

$$M \propto \left(\frac{m_Q m_T}{\kappa^2}\right)^{\frac{1}{N-1}}$$

Decoupling limit: the first solution - relevant

Two other solutions: $M \to \infty$

Non-SUSY extremum

- quadratic approximation for Kähler
- look for solution along *baryonic* branch
- we take $(m_Q)_{ij} \propto \delta_{ij}$;

Non-SUSY solution near the origin

...and decoupling limit

$$\mathcal{A} = \frac{\kappa^2}{m_T}; \qquad B_-^2 \sim \Lambda^{2N}; \qquad M \sim \left(\frac{\lambda^2}{m_S}\frac{m_T}{\kappa^2}\right)^{\frac{1}{N-2}}$$

$$B_+ = T_+ = 0$$

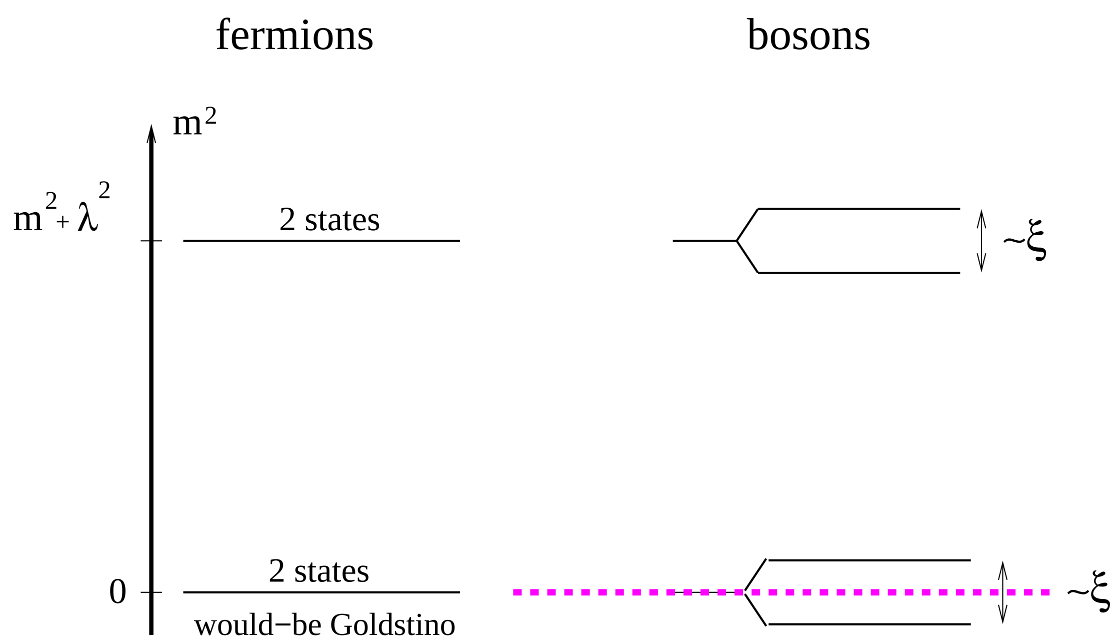
- $$M \to 0 \qquad \Longrightarrow \qquad \frac{\lambda^2/m_S}{\kappa^2/m_T} \to 0$$
- S_{ij} decouple faster than T and $\bar{T}$

Spectrum at SUSY-breaking point

- spectrum for B and T - supersymmetric
- ISS: mesino is Goldstino
- expect: Goldstino reduces to mesino in decoupling limit
- before: admixture of M and S
- one SUSY-breaking parameter: ξ

$$\xi \propto \frac{\kappa^2}{m_T}\left(\frac{\lambda^2}{m_S}\frac{m_T}{\kappa^2}\right)^{\frac{N-3}{N-2}} m_Q^* \quad \text{large } N \quad \xi \propto \frac{\lambda^2}{m_S}$$

Saddle point



Always one state below zero - instability.

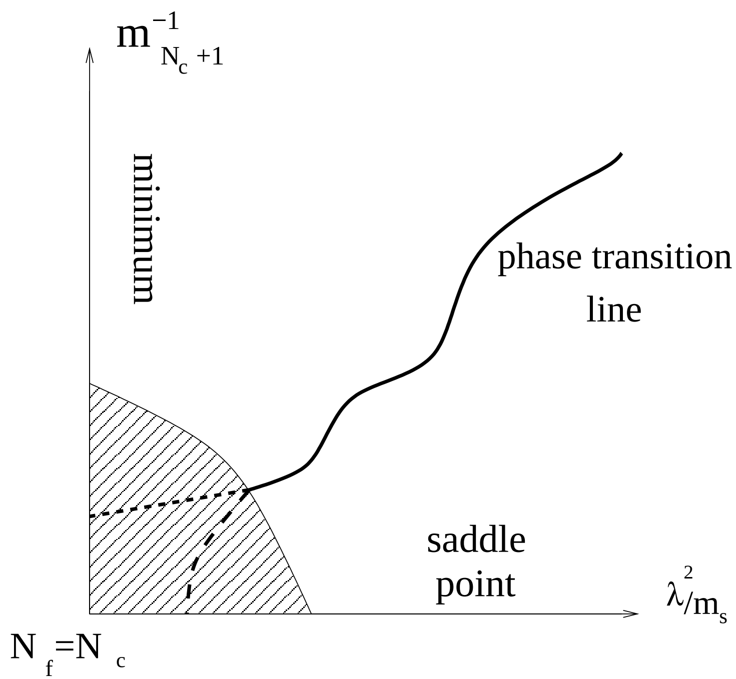
Range of validity

- wanted: calculable contributions $\gg$ uncalculable
- the lowest state $m^2 \sim \pm|\xi|$
- demand $\frac{(\lambda\Lambda)^2 m_Q}{m_S} \gg m_Q^2$

  calculable — uncalculable

- $\frac{(\lambda\Lambda)^2}{m_S} \gg m_Q$
- one can choose e.g. $m_Q \ll (\lambda\Lambda) \ll m_S \lesssim \Lambda$
- similar bound on $\frac{\kappa^2}{m_T}$

$N_f = N_c$ ***conjecture overview***



- ISS points undergo phase transition
- shaded region: ISS point is governed by non-calculable contributions from Kähler

Phenomenological consequences - Pentagon

Banks, 2006, "Pentagon" model

- $N_f = N_c = 5$ with diagonal ISS mass $\Delta W = m_Q \mathrm{Tr} M$
- use ISS-conjectered minimum
- flavor symmetry $SU(5)_{\mathrm{diag}}$
- embed the SM into the flavor symmetry
- μ-problem: need the singlet S: $\Delta W = S H_u H_d$
- $SU(3) \times SU(2) \times U(1)$ unbroken - S couples to the quarks through Y_{ij}
- $\Delta W_{SM} = \lambda S \mathrm{Tr}(Y M)$
- messengers - off diagonal components of M

Spectrum of Pentagon

Metastability $m_Q \ll \Lambda_5$

Consider first $\lambda \ll 1$ to avoid destabilization

ISS minimum? Answer in ΔK

let's believe ISS conjecture

Statement: weakly coupled messengers - $\mathrm{STr}[\mathrm{mess}] > 0$

Poppitz and Trivedi, 1997:

large **negative** contributions to squarks m^2

Wrong-sign contributions to squark masses

- small λ - tachyonic squarks, $SU(3)_C$ is broken
- $m^2[\text{squark}] \propto \log(\Lambda_5/m_F)$

$$m_F \sim \lambda^{\#}\sqrt{m_Q \Lambda_5}$$

- λ large - back to ITIY-like, no stable minimum

Where is Pentagon viable?

- the theory is strongly coupled, corrections - uncalculable
- even if the minimum exists it is **not** near the origin, it's not the ISS-conjectured minimum

λ is large or small - Pentagon is ruled out. Intermediate λ - we do not know. Unlikely to have viable minimum.

Conclusions

- there is no clear indication that the meta-stable SUSY-breaking vacuum exists in $N_f = N_c$ SQCD
- no information can be gained by deforming the theory
- minimum of one deformation - saddle point in another
- coupled singlets - the instability may be generic
- Pentagon - the coupling to singlet can not be too large or too small
- if the conjectured minimum of Pentagon exists:
  - it's uncalculable
  - it's not ISS minimum