Budapest, 26 June ‘07

Eotvos-Cornell ’07

Models of Neutrino Masses and Mixings: a Progress Report

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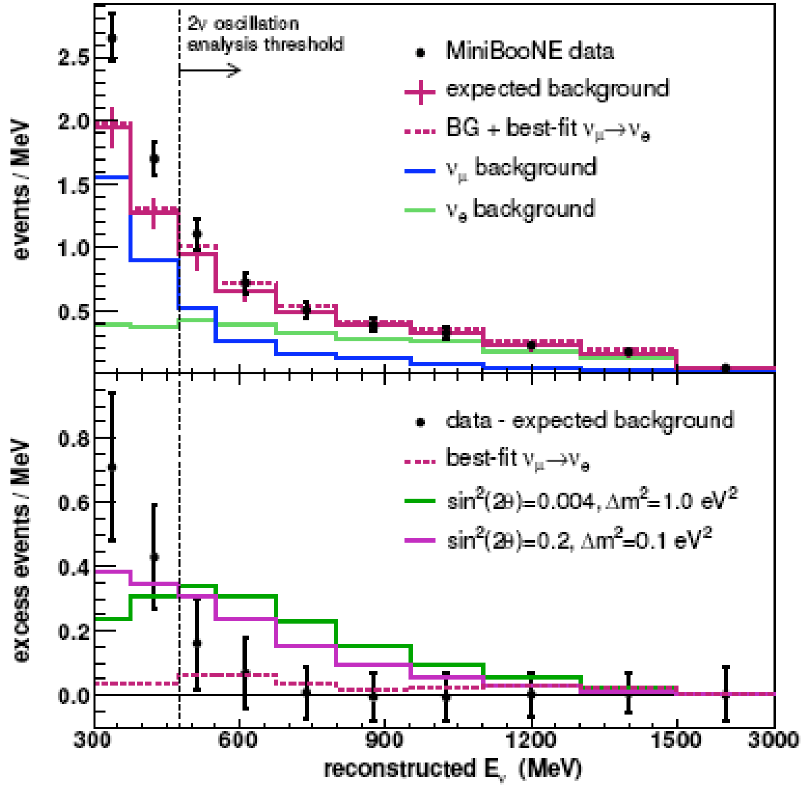
Progress in experiment

MiniBoone'07

MiniBoone has not reproduced the LSND signal in neutrinos (LSND was antineutrinos)
Will run also with antineutrinos



For the time being we can forget LSND and work with 3 light neutrinos

Still many uncertainties

- Absolute scale of ν masses?
- Degeneracy, normal or inverse hierarchy?
- no detection of $0\nu\beta\beta$ (proof that ν's are Majorana) ••••••

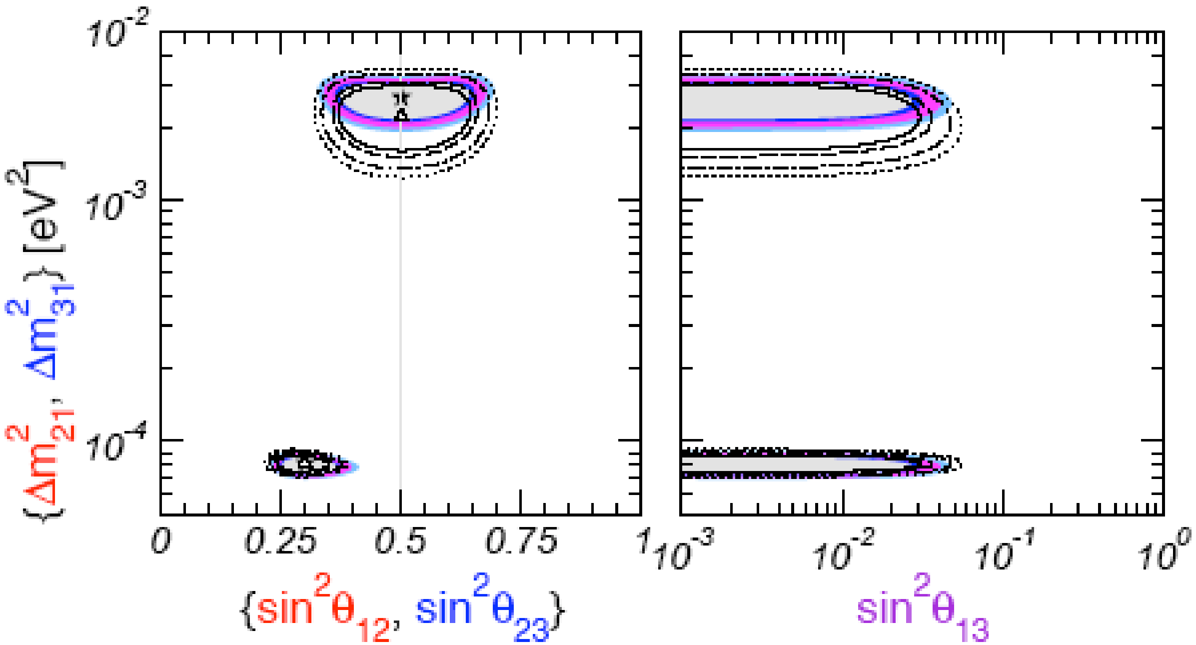
Different classes of models are still possible

Neutrino oscillation parameters

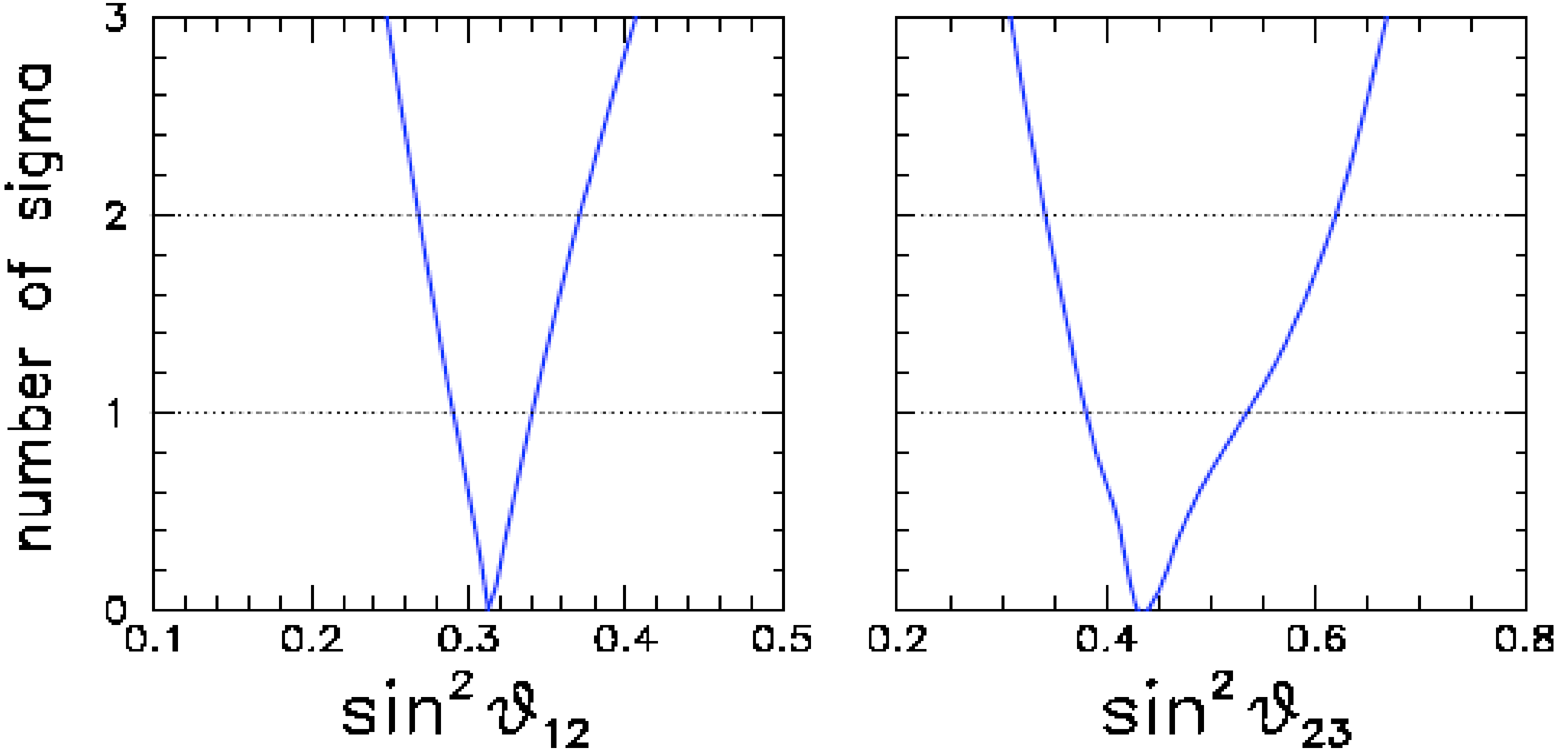
- 2 distinct frequencies
- 2 large angles, 1 small

Maltoni et al '06

parameter	best fit	2σ	3σ	4σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$	7.9	7.3–8.5	7.1–8.9	6.8–9.3
Δm^2_{31} $[10^{-3}\text{eV}^2]$	2.6	2.2–3.0	2.0–3.2	1.8–3.5
$\sin^2\theta_{12}$	0.30	0.26–0.36	0.24–0.40	0.22–0.44
$\sin^2\theta_{23}$	0.50	0.38–0.63	0.34–0.68	0.31–0.71
$\sin^2\theta_{13}$	0.000	≤ 0.025	≤ 0.040	≤ 0.058



Fogli et al '05



$$\delta m^2 = 7.92\,(1^{+0.09}_{-0.09}) \times 10^{-5}\ \mathrm{eV}^2$$

$$\Delta m^2 = 2.4\,(1^{+0.21}_{-0.26}) \times 10^{-3}\ \mathrm{eV}^2$$

$$\sin^2\theta_{12} = 0.314\,(1^{+0.18}_{-0.15})$$

$$\sin^2\theta_{23} = 0.44\,(1^{+0.41}_{-0.22})$$

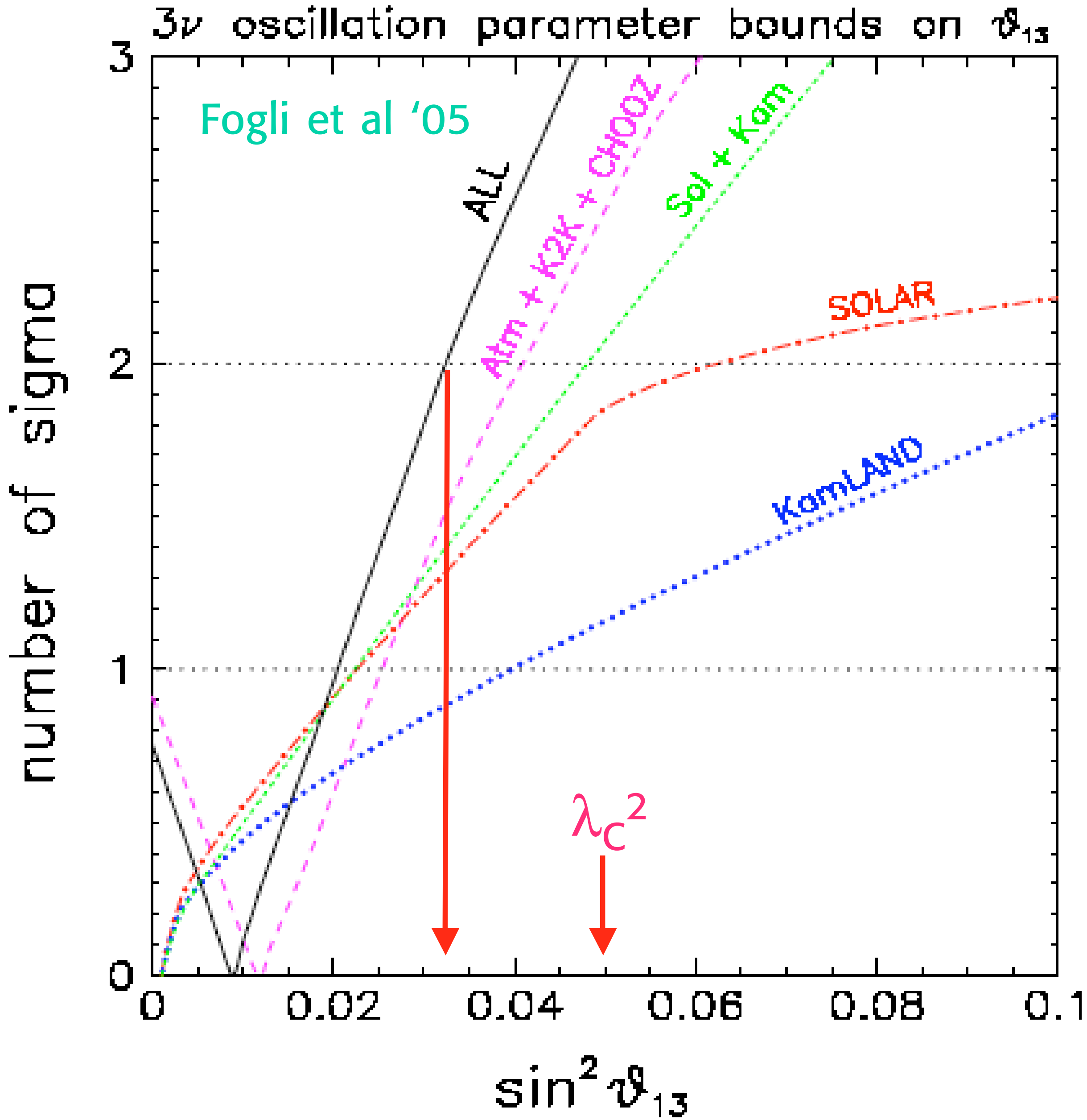
$$\sin^2\theta_{13} < 3.2 \times 10^{-2}$$

2σ ranges 95%

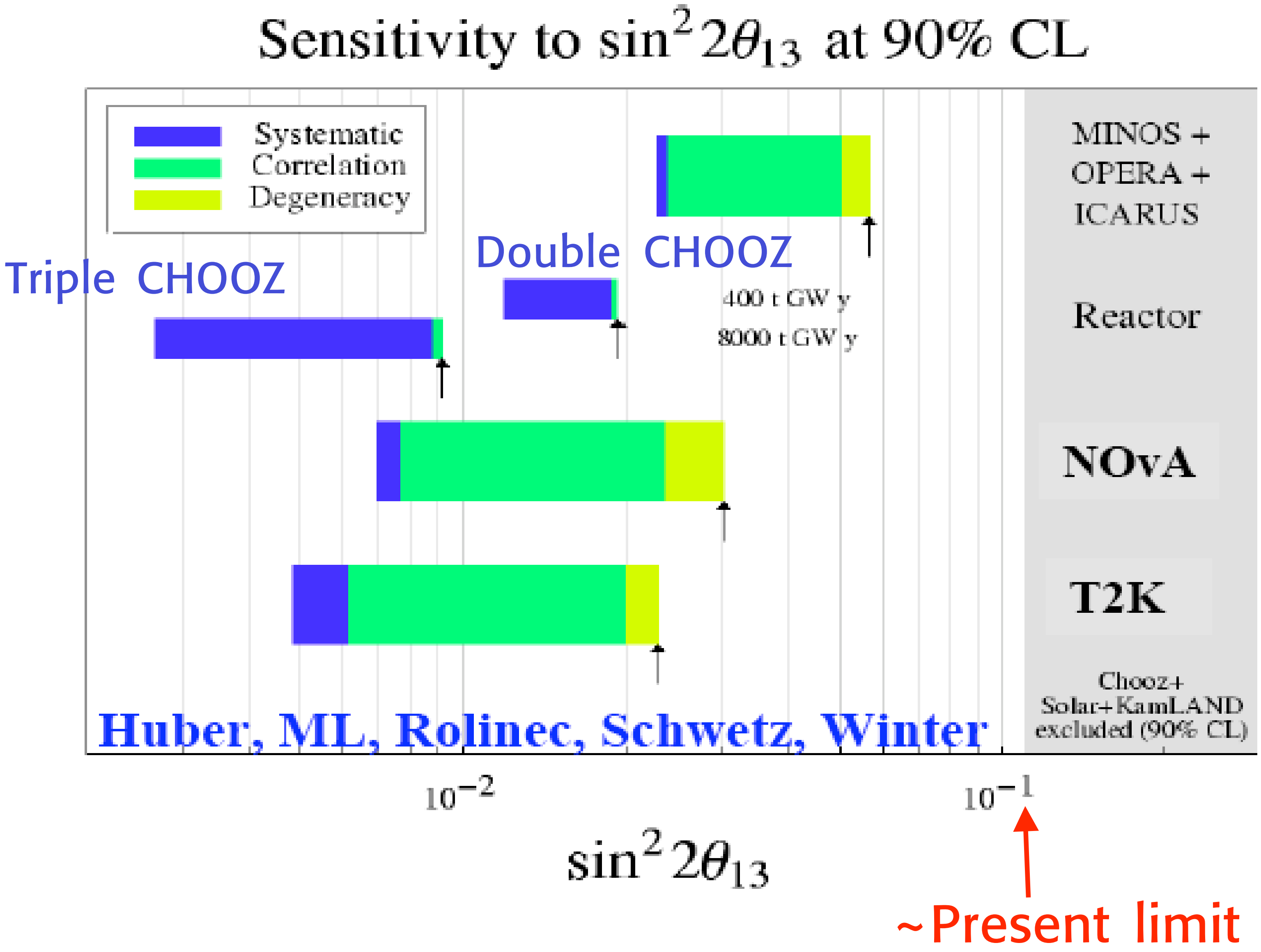
very precise (KamLAND)
very close to 1/3

θ_{13} bounds

The upper bound on $\sin\theta_{13}$ is close to $\lambda_C = \sin\theta_C$



Measuring θ_{13} is crucial for future ν-oscill's experiments (eg CP violation)



General remarks

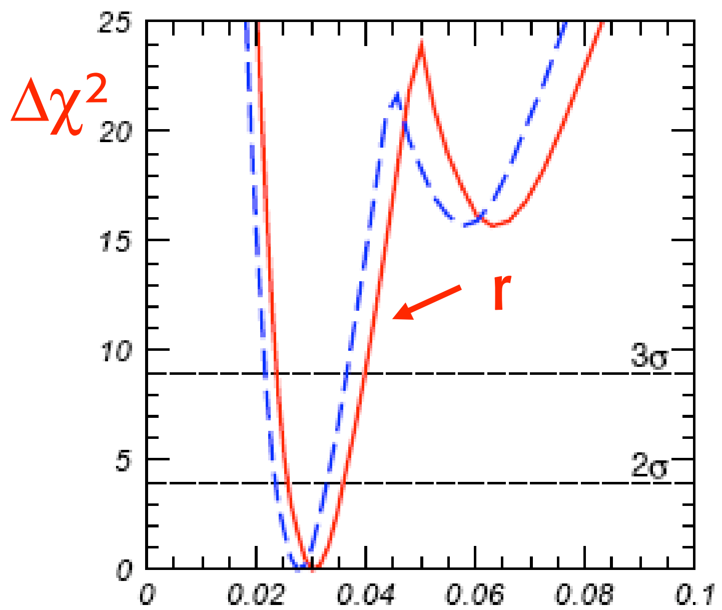
• After KamLAND, SNO and WMAP.... not too much hierarchy is needed for ν masses:

$$r \sim \Delta m^2_{sol}/\Delta m^2_{atm} \sim 1/30$$

Only a few years ago could be as small as 10^{-8}!

Precisely at 3σ: $0.024 < r < 0.040$

Maltoni et al '06



or

$m_{heaviest} < 0.2 - 0.7$ eV
$m_{next} > \sim 8\ 10^{-3}$ eV

For a hierarchical spectrum: $\frac{m_2}{m_3} \approx \sqrt{r} \approx 0.2$

Comparable to $\lambda_C = \sin\theta_C$: $\lambda_C \approx 0.22$ or $\sqrt{\frac{m_\mu}{m_\tau}} \approx 0.24$

Suggests the same "hierarchy" parameters for q, l, ν (small powers of λ_C) ⟶ e.g. θ_{13} not too small!

- Still large space for non maximal 23 mixing

2-σ interval $0.32 < \sin^2\theta_{23} < 0.62$

Maximal θ_{23} theoretically hard

- θ_{13} not necessarily too small
  probably accessible to exp.

Very small θ_{13} theoretically hard

So, in the past, most models were “normal”

"Normal" models: θ_{23} large but not maximal,
θ_{13} not too small (θ_{13} of order λ_C or λ_C^2)

Only a few “exceptional” models were considered:
"Exceptional" models: θ_{23} very close to maximal and/or
θ_{13} very small

Natural models of the “normal” type are not too difficult to build up (with normal or inverse or degenerate hierarchy)

Review: G.A., F. Feruglio, New J.Phys.6:106,2004 [hep-ph/0405048],

It is reasonable to attribute hierarchies in masses and mixings to differences in some flavour quantum number(s).

A simplest flavour (or horizontal) symmetry is $U(1)_F$

For example, simple models based on see-saw and $U(1)_F$ work for all quarks and leptons, explain all small numbers, are natural and compatible with (SUSY) GUT’s, e.g $SU(5)xU(1)_F$ (accomodation rather than prediction).

Larger flavour symmetry groups have also been studied.
They are more predictive but less flexible.
The problem of the "best" flavour group is still open.

The most ambitious models try to combine (SUSY) SO(10) GUT's with a suitable flavour group

"Exceptional" models with θ_{13}= 0 and θ_{23} maximal

The most general mass matrix for θ_{13}= 0 and θ_{23} maximal is given by
(after ch. lepton diagonalization!!!):

$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix}$$

Neglecting Majorana phases it depends on 4 real parameters
(3 mass eigenvalues and 1 mixing angle: θ_{12})

Inspired models based on μ–τ symmetry

Grimus, Lavoura..., Ma,.... Mohapatra, Nasri, Hai-Bo Yu ....

Actually, at present, since KamLAND, the most accurately known angle is θ_{12}

$$\sin^2\theta_{12} = 0.316 \pm 0.026 \sim 1/3$$

By adding $\sin^2\theta_{12} \sim 1/3$ to $\theta_{13} \sim 0$, $\theta_{23} \sim \pi/4$:

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Harrison, Perkins, Scott '02

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Comparison with experiment:

At 1σ:

Maltoni et al'06

$\sin^2\theta_{12} = 1/3$: 0.28-0.33

$\sin^2\theta_{23} = 1/2$: 0.44-0.56

$\sin^2\theta_{13} = 0$: < 0.014

The HPS mixing is clearly a very good approx. to the data!

Also called:
Tri-Bimaximal mixing

$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$

Actually, at present, since KamLAND, the most accurately known angle is θ_{12}

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$$m_\nu = \begin{bmatrix} x & y & y \\ y & z & w \\ y & w & z \end{bmatrix} \longrightarrow m = \begin{pmatrix} x & y & y \\ y & x+v & y-v \\ y & y-v & x+v \end{pmatrix}$$

$$\downarrow$$

$$\sin^2 2\theta_{12} = \frac{8y^2}{(x-w-z)^2 + 8y^2}$$

Tribimaximal Mixing

Tribimaximal Mixing

A simple mixing matrix compatible with all present data

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

In the basis of diagonal ch. leptons:

$m_\nu = U diag(m_1, m_2, m_3) U^T$

$$m_\nu = \frac{m_3}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \frac{m_2}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} + \frac{m_1}{6}\begin{bmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

Eigenvectors: $m_3 \to \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ $\quad m_2 \to \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad m_1 \to \frac{1}{\sqrt{6}}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

Note: mixing angles independent of mass eigenvalues

- For the HPS mixing matrix all mixing angles are fixed to particularly symmetric values

Sparked interest in constructing models that can naturally produce this highly ordered structure

Models based on the A4 discrete symmetry (even permutations of 1234) offer a minimal solution

Ma...;
GA, Feruglio hep-ph/0504165, hep-ph/0512103
GA, Feruglio, Lin hep-ph/0610165.......

Larger finite groups: T′, $\Delta(27)$....

Feruglio et al
Chen, Mahanthappa ....

Alternative models based on $SU(3)_F$ or $SO(3)_F$

Verzielas, G. Ross
....

King ....

Model building

Quality factors for models:
(higher standards by now!)

- Based on the most general lagrangian compatible with some simple symmetry or dynamical principle
- Should be complete: address at least charged leptons and neutrinos ($U_{P\text{-}NMS} = U^{+}_{e}U_{\nu}$, and the gauge symmetry connects ch. leptons and LH neutrinos)
- As many as possible small parameters (masses and mixings) should be naturally explained as a consequence.
- The necessary vev configuration should be a minimum of the most general potential for a region of parameter space
- The stability under radiative corrections and higher dim operators must be checked
- Simplicity, economy of fields and parameters, predictivity…

A4 is the discrete group of even perm's of 4 objects.
(the inv. group of a tetrahedron). It has 4!/2 = 12 elements.

An element is abcd which means 1234 --> abcd

C_1: 1 = 1234
C_2: T = 2314 ST = 4132 TS = 3241 STS = 1423
C_3: T^2 = 3124 ST^2 = 4213 T^2S = 2431 TST = 1342
C_4: S = 4321 T^2ST = 3412 TST^2 = 2143

Thus A4 transf.s can be written as:

1, T, S, ST, TS, T^2, TST, STS, ST^2, T^2S, T^2ST, TST^2

with: $S^2 = T^3 = (ST)^3 = 1$ [$(TS)^3 = 1$ also follows]

x, x' in same class if

C_1, C_2, C_3, C_4 are equivalence classes [$x' \sim gxg^{-1}$] g: group element

A4 has 4 inequivalent irreducible representations:
a triplet and 3 different singlets

$$3, 1, 1', 1''$$

(promising for 3 generations!)

Note:

as many representations as equivalence classes

$\Sigma d_i^2 = 12$ $\quad$ $9+1+1+1=12$

Three singlet inequivalent represent'ns:

Recall:

$S^2 = T^3 = (ST)^3 = 1$

$$\begin{cases} 1: & S=1,\ T=1 \\ 1': & S=1,\ T=\omega \\ 1'': & S=1,\ T=\omega^2 \end{cases}$$

$$\omega = \exp i\frac{2\pi}{3} = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$
$$\omega^3 = 1$$
$$1 + \omega + \omega^2 = 0$$
$$\omega^2 = \omega^*$$

The only indep. 3-dim represent'n is obtained by:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(S-diag basis)

An equivalent form:

$$VV^\dagger = V^\dagger V = 1$$

$$S' = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = VSV^\dagger \qquad T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{bmatrix} = VTV^\dagger \qquad V = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

(T-diag basis)

Cabibbo '78

A4 has only 4 irreducible inequivalent represt'ns: 1,1′,1″,3

Table of Multiplication:
1′x1′=1″; 1″x1″=1′;1′x1″=1
3x3=1+1′+1″+3+3

A4 is well fit for 3 families!

Ch. leptons l ~ 3

e^c, μ^c, τ^c ~ 1, 1′, 1″

In the S-diag basis consider 3: (a_1,a_2,a_3) $\xrightarrow{S}$ $(a_1,-a_2,-a_3)$

$\xrightarrow{T}$ (a_2,a_3,a_1)

For $3_1=(a_1,a_2,a_3)$, $3_2=(b_1,b_2,b_3)$ we have in $3_1 x 3_2$:

$$1 = a_1b_1 + a_2b_2 + a_3b_3$$

$$1' = a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3$$

$$1" = a_1b_1 + \omega a_2b_2 + \omega^2 a_3b_3$$

$$3 \sim (a_2b_3, a_3b_1, a_1b_2)$$

$$3 \sim (a_3b_2, a_1b_3, a_2b_1)$$

e.g. $1" = a_1b_1+\omega a_2b_2+\omega^2 a_3b_3 \xrightarrow{T} a_2b_2+\omega a_3b_3+\omega^2 a_1b_1 =$
$= \omega^2 [a_1b_1+\omega a_2b_2+\omega^2 a_3b_3]$

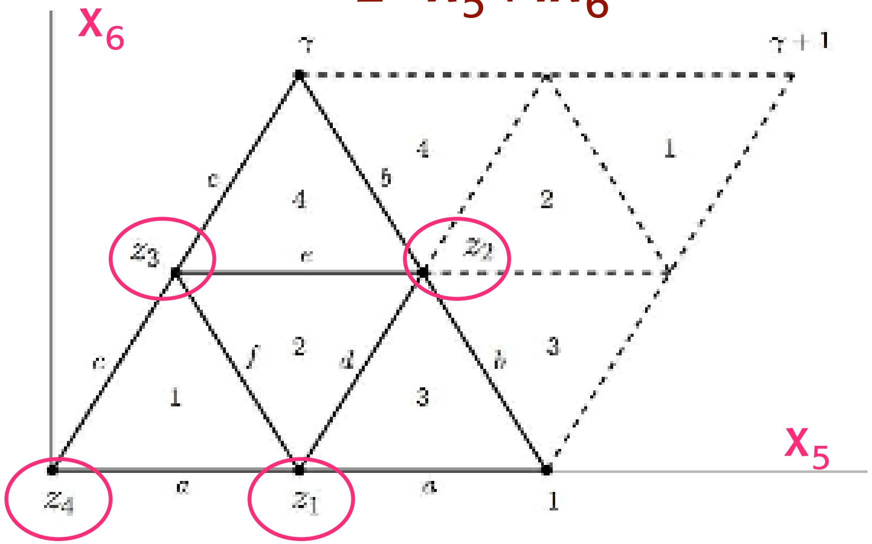
while, under S, 1" is inv.

What can be the origin of A4?

A4 (or some other discrete group) could arise from extra dimensions (by orbifolding with fixed points) as a remnant of 6-dim spacetime symmetry:

G.A.,F. Feruglio&Y. Lin, NP B775(2007)31

$z = x_5 + i x_6$



A torus with identified points:

$z \to z + 1$

$z \to z + \gamma \quad \gamma = \exp(i\pi/3)$

and a parity $z \to -z$

leads to 4 fixed points (equivalent to a tethraedron).

There are 4D branes at the fixed points where the SM fields live (additional gauge singlets are in the bulk)

⊕ A4 interchanges the fixed points

Under A4 the most common classification is:

lepton doublets $l \sim 3$

$e^c, \mu^c, \tau^c \sim 1, 1', 1''$ respectively

gauge singlet flavons $\phi, \phi', \xi, (\xi') \sim$ 3, 3, 1,(1) respectively

driving fields (for SUSY version) $\phi_0, \phi'_0, \xi_0 \sim$ 3, 3, 1

Additional symmetries:

- a broken $U(1)_F$ symmetry (ch. lepton masses) with e^c, μ^c, τ^c charges (3 or 4,2,0)
- one or more discrete parities (dep. on versions) : for example

Z: $(e^c, \mu^c, \tau^c) \to -i\,(e^c, \mu^c, \tau^c)$, $l \to i l$, $\phi \to \phi$, $(\xi,\phi') \to -(\xi,\phi')$

Structure of the model

$$\mathcal{L}_Y = y_e e^c(\varphi l) + y_\mu \mu^c(\varphi l)'' + y_\tau \tau^c(\varphi l)' + x_a \xi(ll) + x_d(\varphi' ll) + h.c. + \ldots$$

shorthand: Higgs and cut-off scale Λ omitted, e.g.:

$y_e e^c(\varphi l) \sim y_e e^c(\varphi l) h_d/\Lambda,$ $\quad x_a \xi(ll) \sim x_a \xi(l h_u l h_u)/\Lambda^2$

!!!

$$\langle\varphi'\rangle = (v', 0, 0)$$
$$\langle\varphi\rangle = (v, v, v)$$
$$\langle\xi\rangle = u$$

$$m_l = v_d \frac{v}{\Lambda} \begin{pmatrix} y_e & y_e & y_e \\ y_\mu & y_\mu\omega^2 & y_\mu\omega \\ y_\tau & y_\tau\omega & y_\tau\omega^2 \end{pmatrix}$$

the big plus of A4

Spectrum free.
Diagonalized by U_e:

$$m_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{pmatrix}$$

$$l \to \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} l = V l$$

⊕ From here it follows that U_{HPS} is the mixing matrix

m_ν in the basis of diagonal charged leptons is:

$$m_\nu\big|_{ldiag} \sim V^* \begin{bmatrix} a & 0 & 0 \\ 0 & a & d \\ 0 & d & a \end{bmatrix} V^* = \begin{pmatrix} a+2d/3 & -d/3 & -d/3 \\ -d/3 & 2d/3 & a-d/3 \\ -d/3 & a-d/3 & 2d/3 \end{pmatrix}$$

$$V = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$$

which in turn can be written as:

$$m_\nu\big|_{ldiag} \sim U^T \begin{bmatrix} a+d & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & -a+d \end{bmatrix} U$$

with:

$$U = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

The crucial issue is to guarantee the strict alignment

$$\begin{aligned} \langle \varphi' \rangle &= (v', 0, 0) \\ \langle \varphi \rangle &= (v, v, v) \\ \langle \xi \rangle &= u \end{aligned}$$

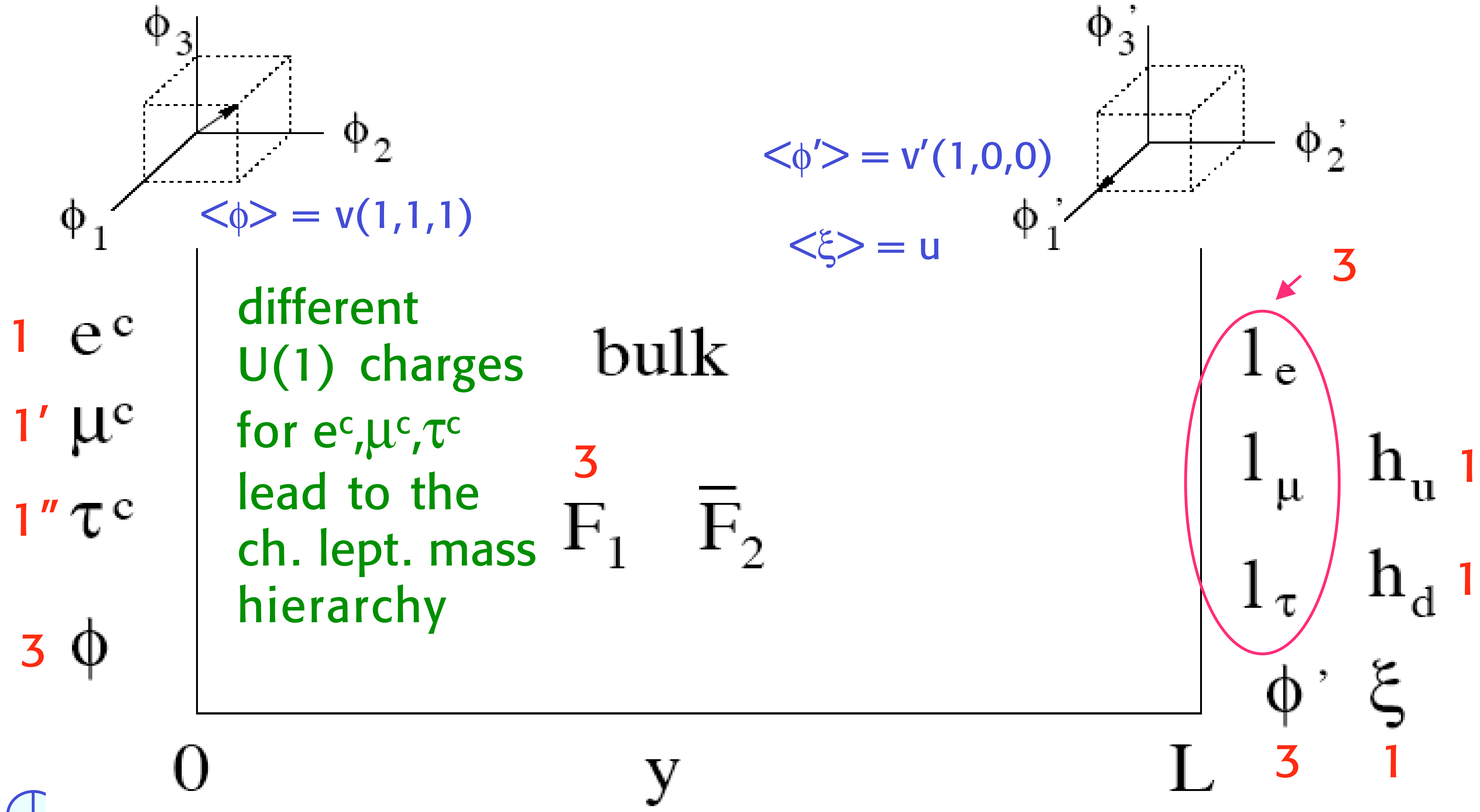
We have constructed a number of completely natural versions of the model, e.g.:

- a version in 5 dimensions (economic in flavon fields)
- a SUSY version in 4-dim (with a few more fields)

GA, Feruglio hep-ph/0504165

The model has 1 compactified extra dim. and 2 branes

(crucial issue: guarantee and protect the vev alignment)



In lowest approximation the action is:

$$\begin{aligned}
S &= \int d^4x dy \left\{ \left[iF_1\sigma^\mu\partial_\mu \overline{F}_1 + iF_2\sigma^\mu\partial_\mu \overline{F}_2 + \frac{1}{2}(F_2\partial_y F_1 - \partial_y F_2 F_1 + h.c.) \right] \right. \\
&- M(F_1F_2 + \overline{F}_1\overline{F}_2) \\
&+ V_0(\varphi)\delta(y) + V_L(\varphi', \xi)\delta(y-L) \\
&+ [Y_e e^c(\varphi F_1) + Y_\mu \mu^c(\varphi F_1)'' + Y_\tau \tau^c(\varphi F_1)' + h.c.]\,\delta(y) \\
&+ \left. \left[\frac{x_a}{\Lambda^2}\xi(ll)h_u h_u + \frac{x_d}{\Lambda^2}(\varphi' ll)h_u h_u + Y_L(F_2 l)h_d + h.c. \right] \delta(y-L) \right\} + \ldots
\end{aligned}$$

a Z-parity has also been imposed

$$(f^c, l, F, \varphi, \varphi', \xi) \xrightarrow{Z} (-if^c, il, iF, \varphi, -\varphi', -\xi)$$

After integrating out of the F fields one obtains the required effective 4-dim action

$$\mathcal{L}_Y = y_e e^c(\varphi l) + y_\mu \mu^c(\varphi l)'' + y_\tau \tau^c(\varphi l)' + x_a \xi(ll) + x_d(\varphi' ll) + h.c. + \ldots$$

In the flavour basis:

$$m_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a + 2d/3 & -d/3 & -d/3 \\ -d/3 & 2d/3 & a - d/3 \\ -d/3 & a - d/3 & 2d/3 \end{pmatrix}$$

m_ν = U diag(a+d,a,-a+d)U^T (in units of v_u^2/Λ) and U=U_{HPS}

In terms of physical param.s (moderate normal hierarchy):

$$|m_1|^2 = \left[-r + \frac{1}{8\cos^2\Delta(1-2r)}\right]\Delta m^2_{atm} \sim (0.017\ \text{eV})^2$$

$$|m_2|^2 = \frac{1}{8\cos^2\Delta(1-2r)}\Delta m^2_{atm} \sim (0.017\ \text{eV})^2$$

$$|m_3|^2 = \left[1 - r + \frac{1}{8\cos^2\Delta(1-2r)}\right]\Delta m^2_{atm} \sim (0.053\ \text{eV})^2$$

- A moderate fine tuning is needed for r

The model crucially depends on the precise vev alignment →

$$\begin{aligned} \langle\varphi'\rangle &= (v', 0, 0) \\ \langle\varphi\rangle &= (v, v, v) \\ \langle\xi\rangle &= u \end{aligned}$$

The extra dimension with 2 branes allows the decoupling of the ϕ and ξ,ϕ' potentials.
A discrete symmetry is also essential:
a separate continuous rotation symmetry on the 2 branes would make any disalignment illusory.

An alternative in 4 dimensions is a SUSY model with driving fields and a superpotential where all terms allowed by symmetry are present (with added fields ξ', ϕ_0, ϕ'_0, ξ_0).

In our models

- all terms allowed by symmetry are present
- all correct'ns are under control and can be made negligible

Extension to quarks

If we take all fermion doublets as 3 and all singlets as 1, 1′, 1″ (as for leptons): $Q_i \sim 3$, $u^c, d^c \sim 1$, $c^c, s^c \sim 1'$, $t^c, b^c \sim 1''$

Then u and d quark mass matrices are BOTH diagonalised by

$$U_u, U_d \sim \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}$$

As a result VCKM is unity: $V_{CKM} = U_u^+ U_d \sim 1$

So, in first approx. (broken by loops and higher dim operators), ν mixings are HPS and quark mixings ~identity

Corrections are far too small to reproduce quark mixings eg λ_C (for leptons, corrections cannot exceed $o(\lambda_C^2)$). But even those are essentially the same for u and d quarks)

Note: NOT straightforward to embed these models in a GUT: with these assignments A4 does not commute with SU(5)

If l ~ 3 then all 5* ~3, so that d^c_i ~ 3
if e^c, μ^c, τ^c ~ 1, 1', 1" then all 10_i ~ 1, 1', 1"

Realistic quark mass matrices are not easy to obtain from these assigments

For example, for u quarks at leading order:

$$m_u \sim 1.1 + 1'.1'' + 1''.1' \sim a\, u_1 u_1 + b\,(u_2 u_3 + u_3 u_2)$$

or

$$m_u \sim \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix}$$

Which implies $|m_c|=|m_t|$ and maximal U_{23}

Recent directions of research:

- Different (larger) finite groups

  Ma;
  Kobayashi et al;
  Luhn, Nasri, Ramond [$\Delta(3n^2)$];
  .....

- Trying to improve the quark mixings

  Carr, Frampton
  Feruglio et al
  .....

- Construct GUT models with approximate tribimaximal mixing

  Ma, Sawanaka, Tanimoto; Ma;
  Morisi, Picarello, Torrente Lujan;
  de Madeiros Verzielas, King, Ross [$\Delta(27)$];
  King, Malinsky [$SU(4)_C x SU(2)_L x SU(2)_R$];
  Chen, Mahanthappa .....

Better quarks: use T′ (also called $SL_2(F_3)$) the double covering group of A4 (A4 is not a subgroup of T′)

Aranda, Carone, Lebed
Carr, Frampton
Feruglio et al
Chen, Mahanthappa

24 transformations.
Irreducible representations: 1, 1′, 1″, 2, 2′, 2″, 3

General idea:

for quarks use 1(3rd family) + 2^a (1st&2nd families)

- t,(b) masses at renormalizable level (unsuppressed)
- V_{cb}, V_{ts} from doublet flavons (do not couple to leptons)
- 1st generation masses and mixings from subleading effects

Similar to old U(2) models

Barbieri, Dvali, Hall '96
Barbieri, Hall, Raby, Romanino '97
Barbieri, Hall, Romanino '97

Feruglio et al

Provides a solution to the problem of quarks

The lepton sector is identical to A4 models

Field	l	e^c	μ^c	τ^c	D_q	D_u^c	D_d^c	q_3	t^c	b^c	$h_{u,d}$	φ_T	φ_S	$\xi, \tilde{\xi}$	η	ξ''
T'	3	1	$1''$	$1'$	$2''$	$2''$	$2''$	1	1	1	1	3	3	1	$2'$	$1''$
Z_3	ω	ω^2	ω^2	ω^2	ω	ω^2	ω^2	ω	ω^2	ω^2	1	1	ω	ω	1	1
$U(1)_{FN}$	0	$2n$	n	0	0	n	0	0	0	0	0	0	0	0	0	0

The quark sector is decoupled from leptons and reproduces masses and mixings

$$\begin{aligned} w_q = \quad & y_t\,(t^c q_3)\,h_u + y_b\,(b^c q_3)\,h_d + \\ & y_1(\varphi_T D_u^c D_q)h_u/\Lambda + y_5(\varphi_T D_d^c D_q)h_d/\Lambda + \\ & y_2\,\xi''(D_u^c D_q)h_u/\Lambda + y_6\,\xi''(D_d^c D_q)h_d/\Lambda + \\ & \{y_3\ t^c(\eta D_q) + y_4\ (D_u^c \eta)q_3\}h_u/\Lambda + \{y_7\ b^c(\eta D_q) + y_8\ (D_d^c \eta)q_3\}h_d/\Lambda \end{aligned}$$

The adopted classification not suitable for a GUT version

Chen, Mahanthappa

It is a GUT model: SU(5)xT'

T~10; F~5

	T_3	T_a	$\overline{F}$	H_5	$H'_{\overline{5}}$	Δ_{45}	ϕ	ϕ'	ψ	ψ'	ζ	N	ξ	η
SU(5)	10	10	$\overline{5}$	5	$\overline{5}$	45	1	1	1	1	1	1	1	1
$^{(d)}T$	1	2	3	1	1	$1'$	3	3	$2'$	2	$1''$	$1'$	3	1
Z_{12}	ω^5	ω^2	ω^5	ω^2	ω^2	ω^5	ω^3	ω^2	ω^6	ω^9	ω^9	ω^3	ω^{10}	ω^{10}
Z'_{12}	ω	ω^4	ω^8	ω^{10}	ω^{10}	ω^3	ω^3	ω^6	ω^7	ω^8	ω^2	ω^{11}	1	1

For this the charged lepton sector is modified, while the neutrino mass terms are kept identical

For charged leptons, at leading level, only the τ gets mass and ν mixing is tribimaximal (for quarks: m_t is unsuppressed, b-τ unification)

$$\begin{aligned}
\mathcal{L}_{\rm TT} &= y_t H_5 T_3 T_3 + \frac{1}{\Lambda^2} y_{ts} H_5 T_3 T_a \psi\zeta + \frac{1}{\Lambda^2} y_c H_5 T_a T_a \phi^2 + \frac{1}{\Lambda^3} y_u H_5 T_a T_a \phi'^3 \\
\mathcal{L}_{\rm TF} &= \frac{1}{\Lambda^2} y_b H'_{\overline{5}} \overline{F} T_3 \phi\zeta + \frac{1}{\Lambda^3}\left[y_s \Delta_{45} \overline{F} T_a \phi\psi N + y_d H'_{\overline{5}} \overline{F} T_a \phi^2 \psi' \right] \\
\mathcal{L}_{\rm FF} &= \frac{1}{M_x \Lambda}\left[\lambda_1 H_5 H_5 F F \xi + \lambda_2 H_5 H_5 F F \eta \right] ,
\end{aligned}$$

From higher dimension operators masses for light quarks and leptons, mixing angles

Tribimaximal mixing modified by ch. lepton diagonalisation

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,\mathrm{TBM}} - \frac{1}{2}\theta_c\cos\beta$$

$$\theta_{13} \simeq \theta_c/3\sqrt{2}$$

θ_c = Cabibbo angle

β = undet. phase

a new form of quark-lepton complementarity

Raidal
Minakata, Smirnov
Antusch, King
GA, Feruglio, Masina
....

An interesting model. But:

- a complicated flavon structure
- alignment problem not discussed

Conclusion

From experiment: a good first approximation for quarks:

$$V_{CKM} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and for neutrinos

$$U = \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Models based on A4 indeed lead to this pattern

All this is highly non trivial but no real illumination has followed!!