P6574 HW #1
Due January 30, 2015

1. **Spin 1/2 (S&N 3.10)**

   (a) Consider a pure ensemble of identically prepared spin $\frac{1}{2}$ systems. Suppose the expectation values $\langle S_x \rangle$,$\langle S_z \rangle$ and the sign of $\langle S_y \rangle$ are know. Show how we may determine the state vector. Why is it unnecessary to know the magnitude of $\langle S_y \rangle$?

   (b) Connsider a mixed ensemble of spin $\frac{1}{2}$ systems. Suppose the ensemble averages $[S_x]$, $[S_y]$, and $[S_z]$ are known. Show how we may construct the 2×2 density matrix that characterizes the ensemble.

2. **Time Evolution (S&N 3.11)**

   (a) Prove that the time evolution of the density operator ρ (in the Schrodinger picture) is given by
   $$\rho(t) = \mathcal{U}(t, t_0)\rho(t_0)\mathcal{U}^\dagger(t, t_0).$$

   (b) Suppose we have a pure ensemble at $t = 0$. Prove that it cannot evolve into a mixed ensemble as long as the time evolution is governed by the Schrodinger equation.

3. **Spin 1 (S&N 3.12)**
   Consider an ensemble of spin 1 systems. The density matrix is now a 3×3 matrix. How may independent (real) parameters are needed to characterize the density matrix? What must we know in addition to $[S_x], [S_y]$, and $[S_z]$ to characterize the ensemble completely?