Physics 443 HW #6
Due March 5, 2008

1. **Griffiths 4.13**.
   (a) Find $\langle r \rangle$ and $\langle r^2 \rangle$ for an electron in the ground state of hydrogen. Express your answers in terms of the Bohr radius.
   (b) Find $\langle x \rangle$ and $\langle x^2 \rangle$ for an electron in the ground state of hydrogen. *Hint* : This requires no new integration - note that $r^2 = x^2 + y^2 + z^2$, and exploit the symmetry of the ground state.
   (c) Find $\langle x^2 \rangle$ in the state $n = 2, l = 1, m = 1$. *Warning* : This is *not* symmetrical in x, y, z. Use $x = r \sin\theta \cos\theta$.

2. **Griffiths 4.16**. A hydrogenic atom consists of a single electron orbiting a nucleus with Z protons ($Z = 1$ would be hydrogen itself, $Z = 2$ is ionized helium, $Z = 3$ is doubly ionized lithium, and so on). Determine the Bohr energies $E_n(Z)$, the binding energy $E_1(Z)$, the bohr radius $a(Z)$ and the Rydberg constant $R(Z)$ for a hydrogenic atom. (Express your answers as appropriate multiples of the hydrogen values.) Where in the electromagnetic spectrum would the Lyman series fall, for $Z = 2$ and $Z = 3$? *Hint* : There's nothing much to *calculate* here- in the potential
$$V(r) = -\frac{e^2}{4\pi\epsilon_0}\frac{1}{r}, \quad e^2 \to Ze^2,$$
so all you have to do is make the same substitutiion in all the final results.

3. **Griffiths 4.29**.
   (a) Find the eigenvalues and eigenspinors of $\mathbf{S}_y$.
   (b) If you measured S_y on a particle in the general state χ
$$\chi = \begin{pmatrix} a \\ b \end{pmatrix} = a\chi_+ + b\chi_-,$$
   what values might you get, and what is the probability of each? Check that the probabilities add up to 1. *Note* : a and b need not be real.
   (c) If you measured S_y^2, what values might you get, and with what probabilities?

4. **Griffiths 4.30**. Construct the matrix $\mathbf{S}_r$ representing the component of spin angular momentum along an arbitrary direction $\hat{r}$. Use spherical coordinates, for which

$$\hat{r} = \sin\theta\cos\phi\hat{i} + \sin\theta\sin\phi\hat{j} + \cos\theta\hat{k}.$$

Find the eigenvalues and (normalized) eigenspinors of $\mathbf{S}_r$. *Answer* :

$$\chi_+^{(r)} = \begin{pmatrix} \cos(\theta/2) \\ e^{i\phi}\sin(\theta/2) \end{pmatrix}; \quad \chi_-^{(r)} = \begin{pmatrix} e^{-i\phi}\sin(\theta/2) \\ -\cos(\theta/2) \end{pmatrix}.$$

Note : You're always free to multiply by an arbitrary phase factor-say, $e^{i\phi}$-so your answer may not *look* exactly the same as mine.

5. **Griffiths 4.33**. An electron is at rest in an oscillating magnetic field

$$\mathbf{B} = B_0\cos(\omega t)\hat{k},$$

where B_0 and ω are constants.

(a) Construct the Hamiltonian matrix for this system.

(b) The electron starts out (at $t = 0$) in the spin-up state with respect to the x-axis (that is: $\chi(0) = \chi_+^{(x)}$). Determine $\chi(t)$ at any subsequent time. *Beware* : This is a time-*dependent* Hamiltonian, so you cannot get $\chi(t)$ in the usual way from stationary states. Fortunately, in this case you can solve the time-dependent Schrodinger equation

$$i\hbar\frac{\partial\chi}{\partial t} = \mathbf{H}\chi,$$

directly.

(c) Find the probability of getting $-\hbar/2$, if you measure S_x. *Answer* :

$$\sin^2\left(\frac{\gamma B_0}{2\omega}\sin(\omega t)\right).$$

(d) What is the minimum field (B_0) required to force a complete flip in S_x?

6. **Griffiths 4.36**.

(a) A particle of spin 1 and a particle of spin 2 are at rest in a configuration such that the total spin is 3, and its z component is $\hbar$. If you measured the z component of the angular momentum of the spin-2 particle, what values might you get, and what is the probability of each one?

(b) An electron with spin down is in the state ψ_{510} of the hydrogen atom. If you could measure the total angular momentum squared of the electron alone, (*not* including the proton spin), what values might you get, and what is the probability of each?

7. **Show that**

$$e^{i(\sigma\cdot\hat{n})\alpha/2} = \cos(\alpha/2) + i(\hat{n}\cdot\sigma)\sin(\alpha/2)$$

where the unit vector

$$\hat{n} = \sin\theta\cos\phi\hat{i} + \sin\theta\sin\phi\hat{j} + \cos\theta\hat{k}$$

The operator $\exp(i(\sigma\cdot\hat{n})\alpha/2)$ effects a rotation of the spinor χ through the angle α about the axis $\hat{n}$.

Prelim. March 7, 2008, 3:35 PM, 104 Rock. Review at 9:00AM.