P443 HW #13
Due May 5, 2008

1. **Griffiths 11.12**. Calculate the total cross-section for scattering from a Yukawa potential, in the Born Approximation. Express your answer as a function of E.

2. **Scattering from a square well**. Evaluate the Born approximation to scattering of particles by the spherical square-well potential.

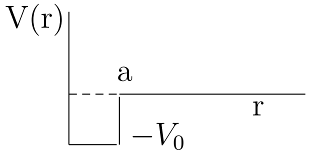

Figure 1: Square well potential

Show that

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{2mV_0a^3}{\hbar^2}\right)^2 \left(\frac{\sin Ka - Ka\cos Ka}{K^3a^3}\right)^2$$

and that the low energy limit of this is

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{2mV_0a^3}{3\hbar^2}\right)^2 (1 - \frac{1}{5}K^2a^2 + ...)$$

where

$$K = 2k\sin\frac{\theta}{2}$$

Evaluate the total cross section in the limits of low and high energies; the answers are

$$\sigma = \pi\left(\frac{4m}{3\hbar^2}\right)^2 (V_0a^3)^2(1 - \frac{2}{5}k^2a^2 + ...) \text{ low energy}$$

and

$$\sigma = 2\pi\left(\frac{m}{\hbar^2}\right)^2 \left(\frac{V_0a^3}{ka}\right)^2 \text{ high energy}$$

By the way:

$$\int \frac{(\sin x - x\cos x)^2}{x^5}dx = -\frac{1}{4}\left[\frac{(\sin x - x\cos x)^2}{x^4} + \frac{\sin^2 x}{x^2}\right]$$

3. **Dirac Equation**

The Dirac equation is

$$i\hbar\frac{\partial\psi}{\partial t} = [c\alpha\cdot\mathbf{p} + \beta mc^2]\psi$$

where

$$\alpha = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

and

$$\psi(\mathbf{r},t) = \begin{pmatrix} \psi_1(\mathbf{r},t) \\ \psi_2(\mathbf{r},t) \end{pmatrix}$$

and ψ_1 and ψ_2 are each two component spinors. If we assume a time dependence $\psi(\mathbf{r},t) = \psi(\mathbf{r})e^{-iEt/\hbar}$, then the Dirac equation becomes

$$E\psi = [c\alpha\cdot\mathbf{p} + \beta mc^2]\psi \tag{1}$$

In order to include a magnetic field we replace $\mathbf{p} \to \mathbf{p} - q\mathbf{A}$.

(a) Show that in the nonrelativistic limit

$$E_S\psi_1 = \frac{[\sigma\cdot(\mathbf{p} - q\mathbf{A})][\sigma\cdot(\mathbf{p} - q\mathbf{A})]}{2m}\psi_1 \tag{2}$$

where $E_S = E - mc^2$.

(b) Prove the identity

$$\sigma\cdot\mathbf{A}\sigma\cdot\mathbf{B} = \mathbf{A}\cdot\mathbf{B} + i\sigma\cdot\mathbf{A}\times\mathbf{B}$$

for any vectors $\mathbf{A}$ and $\mathbf{B}$.

(c) Show that

$$(\mathbf{p} - q\mathbf{A})\times(\mathbf{p} - q\mathbf{A}) = iq\hbar\mathbf{B}$$

(d) And that Equation 2 becomes

$$\left[\frac{(\mathbf{p} - q\mathbf{A})^2}{2m} - \mu\cdot\mathbf{B}\right]\psi_1 = E_S\psi_1$$

where $\mu = \frac{q}{m}\mathbf{s}$ and $\mathbf{s} = \frac{\hbar}{2}\sigma$.