P443 HW #12
Due April 28, 2008

1. **Griffiths 10.1**. The case of an infinite square well whose right wall expands at a *constant* velocity (v) can be solved *exactly*. A complete set of solutions is

$$\Phi_n(x,t) \equiv \sqrt{\frac{2}{w}} \sin\left(\frac{n\pi}{w}x\right) e^{i(mvx^2 - 2E_n^i at)/2\hbar w}, \tag{1}$$

where $w(t) \equiv a + vt$ is the (instantaneous) width of the well and $E_n^i \equiv n^2\pi^2\hbar^2/2ma^2$ is the nth allowed energy of the *original* well (width a). The *general* solution is a linear combination of the Φ's:

$$\Psi(x,t) = \sum_{n=1}^{\infty} c_n \Phi_n(x,t);$$

the coefficients c_n are *independent of t*.

   (a) Check that Equation 1 satisfies the time-dependent Schrodinger equation, with the appropriate boundary conditions.

   (b) Suppost a particle starts out ($t = 0$) in the ground state of the initial:

$$\Psi(x,0) = \sqrt{\frac{2}{a}} \sin\left(\frac{\pi}{a}x\right).$$

   Show that the expansion coefficients can be written in the form

$$c_n = \frac{2}{\pi}\int_0^{\pi} e^{-i\alpha z^2} \sin(nz)\sin(z)dz,$$

   where $\alpha \equiv mva/2\pi^2\hbar$ is a dimensionless measure of the speed with which the well expands. (Unfortunately, this integral cannot be evaluated in terms of elementary functions.)

   (c) Suppose we allow the well to expand to twice its orignal width, so the "external" time is given by $w(T_e) = 2a$. The "internal" time is the *period* of the time-dependent exponential factor in the (initial) ground state. Dtermine T_e and T_i, and show that the adiabatic regime corresponds to $\alpha \ll 1$, so that $\exp(-i\alpha z^2) \approx 1$ over the domain of integration. Use this to determine the expansion coefficients, c_n. Construct $\Psi(x,t)$, and confirm that it is consistent with the adiabatic theorem.

(d) Show that the phase factor $\Psi(x,t)$ can be written in the form

$$\theta(t) = -\frac{1}{\hbar}\int_0^t E_1(t')dt',$$

where $E_n(t) = n^2\pi^2\hbar^2/2mw^2$ is the *instantaneous* eigenvalue, at time t. Comment on this result.

2. **Griffiths 10.3**

(a) Use the expression for geometric phase

$$\gamma_n(t) = i\int_{R_i}^{R_f} \langle \psi_n \mid \frac{\partial \psi_n}{\partial R}\rangle dR,$$

to calculate the geometric phase change when the infinite square well expands adiabatically from width w_1 to width w_2. Comment on this result.

(b) If the expansion occurs at a constant rate ($dw/dt = v$), what is the dynamic phase change for this process?

(c) If the well now contracts back to its original size, what is Berry's phase for the cycle?

3. **Griffiths, 10.6**. Work out the analog of Griffiths Equation 10.62 for a particle of spin 1. *Answer* : $-\Omega$.