P443 HW#10
Due April 9, 2008

1. **Griffiths 7.13**. Find the lowest bound on the ground state of hydrogen you can get using a gaussian trial wave function

$$\psi(\mathbf{r}) = Ae^{-br^2},$$

where A is determined by normalization and b is an adjustable parameter. *Answer* : -11.5eV.

2. **Griffiths 7.14**. If the photon had a nonzero mas ($m_\gamma \neq 0$), the Coulomb potential would be replaced by the **Yukawa potential**,

$$V(\mathbf{r}) = -\frac{e^2}{4\pi\epsilon_0}\frac{e^{-\mu r}}{r},$$

where $\mu = m_\gamma c/\hbar$. With a trial wave function of your own devising, estimate the binding energy of a "hydrogen" atom with this potential. Assume $\mu a \ll 1$, and give your answer correct to order $(\mu a)^2$.

3. **Griffiths 7.15**. Suppose you are given a quantum system whose Hamiltonian H_0 admits just two eigenstates, ψ_a (with energy E_a), and ψ_b (with energy E_b). They are orthogonal, normalized, and nondegenerate (assume E_a is the smaller of the two energies). Now we turn on a perturbation H', with the following matrix elements:

$$\langle \psi_a \mid H' \mid \psi_a\rangle = \langle \psi_b \mid H' \mid \psi_b\rangle\,; \qquad \langle \psi_a \mid H' \mid \psi_b\rangle = \langle \psi_b \mid H' \mid \psi_a\rangle = h,$$

where h is some specified constant.

   (a) Find the exact eigenvalues of the perturbed Hamiltonian.

   (b) Estimate the energies of the perturbed system using second-order perturbation theory.

   (c) Estimate the ground state energy of the perturbed system using the variational principle, with a trial function of the form

$$\psi = (\cos\phi)\psi_a + (\sin\phi)\psi_b,$$

   where ϕ is an adjustable parameter. *Note* : Writing the linear combination in this way is just a neat way to guarantee that ψ is normalized.

(d) Compare your answers to (a), (b), and (c). Why is the variational principle so accurate, in this case?

Prelim II - April 11, 2008, 9:00AM
Includes all material through assignment 9.