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Perturbation of Dispersion and Damping Partition Numbers Due to Pretzel Orbit and Wigglers

W. Lou and D. Rubin

Abstract

The horizontal displacement of the closed orbit due to the crossing angle pretzel effects a distortion of the dispersion function [**?**]. The antisymmetry of both the closed orbit and the error in the dispersion increases the damping rate for horizontal oscillations by about 10%. Another pretzel effect is a result of the displacement of the orbit in the wigglers. The decrease in the magnitude of the wiggler bending field with horizontal position increases horizontal damping rate for displacements with the same sign as the dispersion, and decreases horizontal damping rate for displacements of opposite sign.

Pretzel Generated Dispersion

The guide field generates dispersion as follows:

$$\eta(s) = \frac{\sqrt{\beta(s)}}{\sin \pi\nu} \int \sqrt{\beta(s')} G(s') \cos(\phi(s) - \phi(s') - \pi\nu) ds', \tag{1}$$

where $G = 1/\rho$, and ρ is the effective bending radius of the element at s'. If there is no closed orbit distortion then $G(s)$ is non zero only within the dipoles. But for a pretzelled orbit, there are contributions from the electrostatic separators, quadrupole and sextupole fields. In particular $G(s') = \frac{eE_{sep}(s')}{E_{beam}}$ for the separators, $G(s') = k(s')x(s')$ for quadrupoles and $G(s') = S(s')x(s')^2$ for sextupoles. $x(s')$ is the horizontal displacement of the orbit at s'. Also, the phase difference $\phi(s) - \phi(s')$ depends on the pretzel amplitude through the sextupole distribution. Sextupole and quadrupole distributions are east-west symmetric. The pretzel $x(s')$ and phase differences are antisymmetric. The contribution to the dispersion generated by a pretzel corresponding to a $2.3mrad$ crossing angle is shown in Figure 1. The result is very nearly pure antisymmetric.

The pretzel off dispersion is shown in Figure 2 for reference.

Damping Partition Numbers

The combination of pretzel dependent horizontal dispersion and pretzel orbit contributes to a change in the damping partition numbers [?]: $J_x = 1 - \mathcal{D}$ and $J_z = 2 + \mathcal{D}$, where

$$\mathcal{D} = \frac{\oint \eta(G^3 + 2Gk)ds}{I_2}. \tag{2}$$

and $I_2 = \oint G^2 ds$. In the case of the on axis orbit (pretzel off), there is no contribution from the second term in the numerator since the focusing strength is zero in bending magnets, and the curvature is zero in the quadrupoles. For the displaced orbit the effective bending radius within the quadrupoles, sextupoles and separators is as noted above,

$$G_{pretz} = k(s)x(s) + S(s)x(s)^2 + \frac{eE_{sep}(s)}{E_{beam}}. \tag{3}$$

The effective focusing strength includes contributions from the sextupoles, $k_{pretz}(s) = k(s) + 2S(s)x(s)$. Including the pretzel dependent change in the dispersion we have that

$$\mathcal{D} = \frac{\oint(\eta + \Delta\eta)((G + G_{pretz})^3 + 2G_{pretz}k_{pretz})ds}{I_2 + \Delta I_2}. \tag{4}$$

Again, since the guide field has separated focusing, sextupole and bending fields, equation (4) can be rewritten as

$$\mathcal{D} = \frac{\oint(\eta + \Delta\eta)(G^3 + (G_{pretz})^3 + 2G_{pretz}k_{pretz})ds}{I_2 + \Delta I_2}. \tag{5}$$

The pretzel off dispersion, $\eta(s)$, as well as the quadrupole focusing strength $k(s)$, and the dipole curvature $G(s)$, are east west symmetric. The pretzel orbit $x(s)$, and the pretzel dependent dispersion $\Delta\eta$ are antisymmetric. Since $x^2k \ll 1$, and $Sx \ll k$,

$$\Delta\mathcal{D} \sim \frac{2\oint \Delta\eta(s)x(s)k(s)^2 ds}{I_2 + \Delta I_2} \tag{6}$$

where $\Delta I_2 = \oint (G_{pretz})^2 ds$.

For the crossing angle optics m9a17b301.ge92s_5265, with $\theta^* = 2.3mrad$, $\Delta\mathcal{D} = -0.119$. Evidently the horizontal damping rate increases with crossing angle. The increase in horizontal damping rate and corresponding decrease in longitudinal damping rate can be understood qualitatively by noting that the largest contribution to the integral in (6) comes in the horizontally focusing quads in the IR and nearest L3. We see in Figure 1 that dispersion and displacement have opposite sign in those quads. Therefore radiation decreases with the energy of the particle in those elements, decreasing the damping rate of the energy oscillations and increasing the horizontal damping rate. Because the pretzel is antisymmetric, the change in damping rate is the same for both beams. The horizontal emittance scales inversely with $J_x = 1 - \mathcal{D}$, and the emittance decreases with the full crossing angle pretzel. Since both the dispersion error and the displacement increase approximately linearly with crossing angle, we expect that $\Delta\mathcal{D} \sim (\theta^*)^2$, as indicated in Table 1.

In the seven bunch, head-on optics, the pretzel dependent dispersion error is east-west symmetric since the pretzel is east-west symmetric. The pretzel dependent contribution to the dispersion in the head-on optics is shown in Figure 3.

For the symmetric pretzel $\mathcal{D}$ increases by 0.173 for the full separation pretzel for the positron beams and to 0.231 for the electron beam. (Electrons and positrons behave differently with the symmetric pretzel.) Dispersion and orbit are very nearly in phase throughout most of the ring (see Figure 3) so the effect is to decrease the horizontal damping time for the symmetric pretzel.

Wigglers

The wiggler field effects the partition numbers via the contribution to the radiaton integrals I_2 and I_4 and both depend on the horizontal displacement

Table 1: $\Delta\mathcal{D}$, partition numbers and horizontal damping time are shown for various crossing angle 9-bunch and head-on 7-bunch configurations.

Optics and pretzel amplitude	$\Delta\mathcal{D}$	J_x	J_z	$\tau_x(ms)$
Crossing angle $\theta^* = 0.0$	0.0	0.987	2.013	27
Crossing angle $\theta^* = 1.12mrad$	-0.028	1.015	1.985	26
Crossing angle $\theta^* = 2.3mrad$	-0.119	1.106	1.894	24
Crossing angle $\theta^* = 3.15mrad$	-0.269	1.256	1.744	21
Head-on 7-bunch (pretzel off)	0.0	0.984	2.016	27
Head-on 7-bunch (pretzel on e^-)	0.157	0.827	2.173	32
Head-on 7-bunch (pretzel on e^+)	0.215	0.769	2.231	34

of the beam. The vertical component of the magnetic field in the wiggler is given by[?]

$$B_y = B_0 \cos(k_x x) \cosh(k_y y) \cos(k_z z) \tag{7}$$

Then in the midplane ($y = 0$) and in the limit $k_x x \ll 1$,

$$\frac{\partial B_y}{\partial x} \sim B_0 k_x^2 x \cos(k_z z)$$

The gradient of the vertical field decreases linearly with horizontal displacement and the effective quadrupole strength

$$k = \frac{ec}{E}\frac{\partial B_y}{\partial x}\cos(k_z z) = \frac{ec}{E} B_0 k_x^2 x \cos(k_z z).$$

For the CESR wigglers $B_0 = 1.2T$ and $k_x = 6m^{-1}$. The curvature on axis is $G_0 = \frac{ec}{E} B_0$ and the change in $\mathcal{D}$ due to the horizontal displacement of the beam in the wiggler is

$$\Delta\mathcal{D} \sim \frac{\int_0^l 2\eta G k dz}{I_2} = \frac{\int 2\eta G_0 \cos(k_z z) G_0 k_x^2 x \cos(k_z z) dz}{I_2} \tag{8}$$

$$\Delta\mathcal{D} \sim \eta \frac{G_0^2 k_x^2 x l}{I_2}, \tag{9}$$

where $l = 2.45m$ is the length of the wiggler. A crossing angle of $2.3mrad$ corresponds to an $11mm$ displacement in the wiggler and the dispersion

$\eta = 2.1m$. With both wigglers closed $I_2 = 0.105$. At a beam energy of $E = 5.265GeV$, the change in $\mathcal{D}$ due to the pretzel displacement in one wiggler is $\Delta\mathcal{D} \sim 0.091$. Since the pretzel is antisymmetric, the displacement in the east and west wigglers is equal but opposite, and there is no net change in $\mathcal{D}$ with crossing angle. (For a description of the calculation with the MAD and RACETRACK codes see W.Lou, CON 94-18.)

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References

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