CON 90-4
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# Damping Partition Numbers for Distorted Orbits- Revisited

Damping partition numbers depend on the displacement of the closed orbit in the lattice quadrupoles. If the orbits of electrons and positrons are separated the partition numbers are generally not equal. The partition numbers for electrostatically separated beams and unequal energy beams are computed.

In CON 89-10 I reported on the assymetry in damping partition numbers for electrostatically separated electrons and positrons. An error has since been identified in the relevant code. The corrected numbers bear little resemblance to those of the earlier note and all are advised to discard page 2 of said document. The numbers in the revised table (1) indicate that there is a significant assymetry of partition numbers for $e^+$ and $e^-$ for the case of east-west symmetric pretzels in the present HEP lattice (D9920A159.917_3S). Note furthermore that while for colliding beams positrons have a shorter transverse damping time than electrons, for injection (south separators off) the situation is reversed.

Table 1. G9920a149.9a7_3s 5.17GeV

| | No Pretzel | Injection(+/-) | Luminosity |
|---|---|---|---|
| $J_x$ | 0.987 | 0.852/1.122 | 1.129/0.845 |
| $J_\epsilon$ | 2.013 | 2.148/1.878 | 1.871/2.155 |
| $\tau_x$ (ms) | 28.6 | 33.1/25.2 | 25.0/33.4 |
| $\tau_z$ (ms) | 28.2 | 28.2 | 28.2 |
| $\tau_\epsilon$(ms) | 14.0 | 13.1/15.0 | 15.1/13.1 |
| $\sigma_E/E \times 10^{-3}$ | 0.598 | 0.579/0.619 | 0.620/0.578 |
| $\epsilon_x(mm-mrad)$ | 0.142 | 0.164/0.125 | 0.124/0.166 |

Of course in a perfectly uniform lattice, that is one with all quadrupoles of equal focal length and uniform dispersion, the partition numbers are independent of the amplitude of a full wavelength bump. The observed assymetry is a residual effect and indeed can be significantly reduced by a suitable tailoring of the lattice. The damping characteristics for a modified version of the HEP lattice are summarized in Table 2. The partition numbers are nearly identical for electrons and positrons in colliding beam conditions. The assymetry in the case of the injection orbits is however somewhat greater than in the original lattice. That the optical characteristics of the modified and original HEP lattices are essentially the same can be seen by a comparison of Figures 1 and 2. The modified lattice was generated with the help of a constraint to the quadrupole distribution added via the design program.

Table 2. D9920a052.9a7_3s_part 5.17GeV

| | No Pretzel | Injection(+/-) | Luminosity |
|---|---|---|---|
| $J_x$ | 0.987 | 0.708/1.266 | 0.975/0.999 |
| $J_\epsilon$ | 2.013 | 2.292/1.734 | 2.025/2.002 |
| $\tau_x$ (ms) | 28.6 | 39.9/22.3 | 28.9/28.3 |
| $\tau_z$ (ms) | 28.2 | 28.2 | 28.2 |
| $\tau_\epsilon$(ms) | 14.0 | 12.3/16.2 | 13.9/14.1 |
| $\sigma_E/E \times 10^{-3}$ | 0.598 | 0.560/0.644 | 0.596/0.600 |
| $\epsilon_x(mm - mrad)$ | 0.140 | 0.195/0.109 | 0.142/0.138 |

In the scheme where beams are separated by virtue of a small energy difference (See CON 90-2) the assymetry in the partition numbers is no longer a nearly cancelling effect. A rough estimate of the effect proceeds as follows. As noted in CON 89-10 the partition numbers $J_x = 1 - D$ and $J_\epsilon = 2 + D$ where

$$D = \frac{\sum_i \eta_i \left(G_i^3 + 2x_i k_i^2\right) l_i}{\sum_i \left(G_i^2 l_i + (x_i k_i)^2 l_i\right)}.$$

The sum is over dipoles and quadrupoles. $x_i$ is the displacement of the orbit in a quad of strength $k_i$. $G = 1/\rho$ and $\rho$ is the bending radius in the dipoles. The term proportional to $k_i^2$ in the denominator is typically very small and will be neglected. In an isomagnetic approximation of CESR with undisplaced trajectories $D \sim \eta G \sim 1.5/90 \sim 0.016$. An off energy beam has displacement $x = \eta\delta$. If there are equal numbers of bends and quadrupoles the ratio of the off energy to on energy terms is for typical CESR values

$$\frac{2\delta\eta_i k_i^2 l_i}{G_j^3 l_j} \sim \frac{2(0.01)(1.5m)(.22m^{-2})^2(.6m)}{(1/90m)^3 7m} \sim 100.$$

Then $D(\delta = 0.01) \sim 100D(\delta = 0) \sim 1.6$, $J_x$ is less than zero and the high energy beam is antidamped. For the case of the full wavelength pretzel bumps there is approximate cancellation, but for an off energy beam there is none.

A summary of the damping characteristics is tabulated for a lattice designed to separate off energy beams. It is clear that some kind of quadrupole wiggler or combined function magnet in both the high and low energy path length bypass is required to stabilize the high energy beams and to guarantee equal emittances.

Table 3. twoeng28_11. 5.17GeV

| | On Energy | $\delta E/E = (+/-)0.009$ |
|---|---|---|
| $J_x$ | 0.991 | -0.653/2.635 |
| $J_\epsilon$ | 2.009 | 3.653/0.365 |
| $\tau_x$ (ms) | 27.4 | -41.5/10.3 |
| $\tau_z$ (ms) | 27.1 | 27.1 |
| $\tau_\epsilon$(ms) | 13.5 | 7.4/73.6 |
| $\sigma_E/E \times 10^{-3}$ | 0.612 | 0.452/1.428 |
| $\epsilon_x(mm - mrad)$ | 0.114 | -/0.042 |

The partition numbers can be compensated with a combined function magnet or quadrupole wiggler placed in the region where the beams are separated into distinct guide fields. For a magnet to be useful in the part of the machine where the beam displacement is simply proportinal to $\eta\delta$, it must have a bend field with magnitude that decreases linearly with displacement from the design orbit. The disparity in partition numbers is due to the sum of terms proportional to $k_i^2$ in the expression for $D$. For a configuration in which the beams share a common guide field through most of the ring but are then separated into a path length bypass the sum is over the common quadrupoles. A combined function bend located in the bypass contributes $2\eta\frac{1}{\rho}kl$ to the sum. Then if

$$2\eta\frac{1}{\rho}kl = -\Sigma_{i=1}^{Nquads} 2\eta_i^2 k_i^2 l_i \delta$$

the on energy partition numbers are restored. If $\eta_i \sim 1.5m$,$k_i \sim 0.2m^{-2}$,$l_i = 0.6m$,$\delta = 0.01$, and $Nquads = 40$ then the sum over the shared quadrupoles is $\Sigma \sim 0.045m^{-1}$. The sum can be compensated by a combined function magnet if within the magnet $\eta = 1.5m$,$\rho = 35m$,$l = 9m$, and $k = 0.06m^{-2}$. Such a bend (or bends) might replace one of the standard bends in the bypass. Alternatively, a quadrupole wiggler can be used to restore symmetry of damping if $2k^2x\eta l \sim -\Sigma$. Parameters of such a magnet if $\eta = 1.5m$, and the displacement in the magnet is $1.6cm$ are $k = 0.8m^{-2}$,$l = 1m$.

To summarize: In the 3S HEP lattice with symmetric pretzels, the assymetry in the transverse damping times is about 30%. The equality of partition numbers can be restored without difficulty by appropriately constraining the quadrupole distribution, and a lattice with partition numbers independent of pretzel amplitude has been generated. Partition numbers of beams that are off energy by $\pm 1\%$ can be restored to the on energy values with a combined function magnet or quadrupole wiggler.