CORRECTION OF TRANSVERSE COUPLING IN A STORAGE RING

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Abstract

At the Cornell Electron Storage Ring (CESR) we have developed a technique for local measurement and correction of the transverse betatron coupling. We compare this to the standard "global" coupling correction that minimizes the closest approach of the two normal mode frequencies as observed on a spectrum analyzer. In our local coupling technique we measure [1] and correct the coupling of the transverse betatron modes at about 90 different points in the machine. Our measurements consist of experimental values for the the normalized aspect ratio of the beam, the square root of the vertical to the horizontal emittance. DIMAT [2], a single beam simulation program, has been modified to calculate coupling as a function of skew quadrupole strengths. A least square fitting routine finds values of skew quadrupole strengths which reproduce our measured coupling data. These results are used to change the strengths of about 12 of the skew quadrupoles in CESR and so reduce the coupling. After 2 iterations we can reduce the normalized aspect ratio to about 0.015.

Introduction

In an electron storage ring there is generally coupling between the horizontal and vertical planes. This may be caused by experimental solenoids, misalignments of normal focusing quads, or sextupoles. In the presence of coupling the normal modes of oscillation no longer correspond to purely horizontal or purely vertical motion. The $\overline{C}$'s are parameters based on the elements of the full turn transfer matrix and they characterize how the normal modes are linked to horizontal and vertical motion. We can calculate the normal mode emittances from the $\overline{C}$'s and the normal mode twiss parameters.

Typically the coupling is parametrized by the closest approach or splitting of the normal mode tunes. When the tunes can be brought together by an adjustment of the purely horizontal and vertical focussing quads the machine is "globally decoupled". Global decoupling is quick and useful but does not guarantee correction of coupling errors. We show its effects on the $\overline{C}$. A scheme for measuring and compensating local coupling errors has been developed and used at CESR. Results are presented and indicate a decrease in vertical beam size of roughly a factor of 3. Sources of error in the measurements are discussed.

Normal Mode Decomposition [3-5]

A full turn coupled transfer matrix T is decomposed into normal modes as follows:

$$T = \begin{pmatrix} M & n \\ m & N \end{pmatrix} = VUV^{-1} \tag{1}$$

where

$$U = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} \;;\; V = \begin{pmatrix} \gamma I & C \\ -C^{\dagger} & \gamma I \end{pmatrix} \quad \text{and} \quad \gamma^2 + |C| = 1 \tag{2}$$

A is the full turn transfer matrix for one of the normal modes and so is written in the form

$$A = \begin{pmatrix} \cos 2\pi\nu_A + \alpha_A \sin 2\pi\nu_A & \beta_A \sin 2\pi\nu_A \\ -\gamma_A \sin 2\pi\nu_A & \cos 2\pi\nu_A - \alpha_A \sin 2\pi\nu_A \end{pmatrix} \tag{3}$$

and similarly for B. T,U, and V are 4x4 matrices. A,B, and C are 2x2 matrices and I is the 2x2 identity matrix. The laboratory phase space coordinates X are related to the normal mode coordinates W by $X = VW$. The same relation, $X = VW$, holds for energy displacements so the normal mode dispersions may be calculated from $(\eta_u, \eta_u', \eta_v, \eta_v')^T = V^{-1}(\eta_x, \eta_x', \eta_y, \eta_y')^T$. Given the normal mode twiss parameters and dispersions the normal mode emittances may be calculated in the usual way [6].

These normal mode coordinates are normalized to remove the α and β dependence.

$$\overline{W} = GW = \begin{pmatrix} G_A & 0 \\ 0 & G_B \end{pmatrix} W \;;\; \text{where} \quad G_A = \begin{pmatrix} \frac{1}{\sqrt{\beta_A}} & 0 \\ \frac{\alpha_A}{\sqrt{\beta_A}} & \sqrt{\beta_A} \end{pmatrix}$$

and similarly for G_B. Applying the same normalization to V yields

$$\overline{V} \equiv GVG^{-1} = \begin{pmatrix} \gamma I & G_A C G_B^{-1} \\ -G_B C^{\dagger} G_A^{-1} & \gamma I \end{pmatrix} = \begin{pmatrix} \gamma I & \overline{C} \\ -\overline{C}^{\dagger} & \gamma I \end{pmatrix}$$

Relative Phase and Amplitude [7]

Consider the motion in the x-y coordinate system as a consequence of the excitation of only the A mode. Take as an initial vector $\overline{W}_0 = (\epsilon_A, 0, 0, 0)^T$ Then n turns later when the normal mode A has propagated through some phase $\phi_A = 2\pi n\nu_A$, the physical state of the system is

$$X = VW_n = VU^nW_0 = G^{-1}\overline{V}\;\overline{U}^n\;\overline{W}_0 \tag{4}$$

Define

$$\cos\Delta\phi_A \equiv \frac{-\overline{C}_{22}}{\sqrt{\overline{C}_{12}^2 + \overline{C}_{22}^2}} \;;\; \sin\Delta\phi_A \equiv \frac{\overline{C}_{12}}{\sqrt{\overline{C}_{12}^2 + \overline{C}_{22}^2}} \tag{5}$$

Then

$$X = \begin{pmatrix} \epsilon_A\gamma\sqrt{\beta_A}\cos\phi_A \\ (x') \\ \epsilon_A\sqrt{\beta_B}\sqrt{\overline{C}_{12}^2 + \overline{C}_{22}^2}\,[\cos(\phi_A + \Delta\phi_A)] \\ (y') \end{pmatrix} = \begin{pmatrix} x_{amp}\cos\phi_x \\ (x') \\ y_{amp}\cos\phi_y \\ (y') \end{pmatrix}$$

$$\left(\frac{y}{x}\right)_A \equiv \frac{y_{amp}}{x_{amp}} = \frac{1}{\gamma}\sqrt{\frac{\beta_B}{\beta_A}}\sqrt{\overline{C}_{12}^2 + \overline{C}_{22}^2} \;;\; (\phi_y - \phi_x)_A = \Delta\phi_A$$

where $(y/x)_A$ is the ratio of the y amplitude to the x amplitude for the A mode and $(\phi_y - \phi_x)_A$ is the phase difference between the two motions for the A mode.

Inverting these expressions gives

$$\overline{C}_{12} = \gamma\sqrt{\frac{\beta_A}{\beta_B}}\left(\frac{y}{x}\right)_A \sin\Delta\phi_A \;;\; \overline{C}_{22} = -\gamma\sqrt{\frac{\beta_A}{\beta_B}}\left(\frac{y}{x}\right)_A \cos\Delta\phi_A \tag{6}$$

Similarly if only the B mode is excited,

$$\overline{C}_{12} = \gamma\sqrt{\frac{\beta_B}{\beta_A}}\left(\frac{x}{y}\right)_B \sin\Delta\phi_B \;;\; \overline{C}_{11} = \gamma\sqrt{\frac{\beta_B}{\beta_A}}\left(\frac{x}{y}\right)_B \cos\Delta\phi_B$$

where $(x/y)_B$ is the ratio of the x amplitude to the y amplitude for the B mode and $\Delta\phi_B = (\phi_x - \phi_y)_B$ is the phase difference between the two motions for the B mode.

Thus three of the four elements of $\overline{C}$ are determined from the experimentally measurable quantities $(y/x)_A$, $(x/y)_B$, $\Delta\phi_A$, and $\Delta\phi_B$.

Physical Meaning of $\overline{C}$

The real space ellipse traced out on successive turns by a particle with only one normal mode excited is written as

$$x = x_{amp}\cos\phi \;;\; y = y_{amp}\cos(\phi + \Delta\phi) \tag{7}$$

This defines an ellipse whose axes have lengths of d and f and are rotated at an angle of θ with respect to the x and y axes. If only the A mode is excited then Fig.1 shows how these relate to the horizontal and vertical motion.

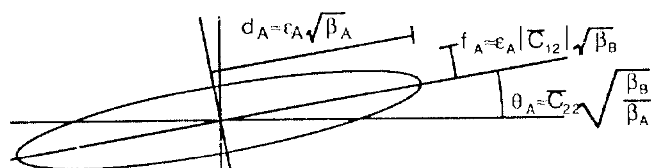

Figure 1 : Beam ellipse for weak coupling and the A mode excited

For weak coupling, where $|\overline{C}_{ij}|^2 \ll 1$ and $\gamma \approx 1$ we find $\theta_A \approx \overline{C}_{22}\sqrt{\frac{\beta_B}{\beta_A}}$; $d_A \approx \epsilon_A\sqrt{\beta_A}$; $f_A \approx \epsilon_A|\overline{C}_{12}|\sqrt{\beta_B}$

In this approximation, after scaling by the ratios of the root betas, $\overline{C}_{22}$ is interpreted as the angle through which the ellipse has been rotated and $\overline{C}_{12}$ is interpreted as the ratio of the lengths of the minor to major axes.

The results for the B mode excitation are easily obtained by switching the horizontal and vertical axes in Figure 1 and $\beta_A \longleftrightarrow \beta_B$; $\epsilon_A \longrightarrow \epsilon_B$; $\overline{C}_{22} \longrightarrow \overline{C}_{11}$.

This can also be seen in another manner which will later prove useful. Recall equations (6) which hold for only the A mode excited. Since $\Delta\phi_A$ is the difference in phase between the horizontal and vertical motions, the $\overline{C}_{22}$ is a measure of the component of the vertical motion that is in phase with the horizontal motion and the $\overline{C}_{12}$ is a measure of the component of the vertical motion that is 90° out of phase with the horizontal motion. So this immediately associates $\overline{C}_{22}$ with the tilt angle of the real space ellipse and $\overline{C}_{12}$ with the blow up of the real space ellipse.

Global Decoupling

The most common way to adjust the coupling is to globally decouple the machine, a process that is best described operationally. The strengths of normal quads are adjusted to bring the normal mode tunes together. Typically the tunes cannot be made equal. The minimum tune split is used as a measure of the coupling in the machine. Skew quads are then used to minimize the tune split. Finally the normal quads are returned to their original values to bring the machine back to its normal operating point. We examine the global decoupling in terms of the coupling parameters $\overline{C}$.

Set Up Notation

- Recall equation (1) and define $H = m^{\dagger} + n$ and $\overline{H} = G_A H G_B^{-1}$. Also define a normalized skew quad strength $q = \sqrt{\beta_A \beta_B}/f$ where f is the focal length of the skew quad. Then

$$(\cos 2\pi\nu_A - \cos 2\pi\nu_B)^2 = \frac{1}{4}\left[\,\mathrm{Tr}\,(M-N)\right]^2 + \det \overline{H}$$

The matrices M and N do not change to first order in the coupler strengths q. [4] So $\mathrm{Tr}M \approx 2\cos 2\pi\nu_x$ where ν_x is the horizontal tune with all couplers turned off and similarly for $\mathrm{Tr}N$. Near the difference coupling resonance $\det \overline{H}$ is always positive. $\overline{H}$ may be written as

$$\overline{H} = \overline{H}_+ \sin\pi\,(\nu_A + \nu_B) + \overline{H}_- J \sin\pi\,(\nu_A - \nu_B)$$

and to first order in coupler strengths q, $\overline{H}_+$ and $\overline{H}_-$ are

$$\overline{H}_{\pm} = R\left[-\pi\,(\nu_A \mp \nu_B)\right] \sum_{k=SQ} q_k R\left(\phi_{A,pk} \mp \phi_{B,pk}\right) \qquad (8)$$

The sum is over all skew quads. There is also a contribution to the $\overline{H}_{\pm}$ from solenoids, but since this does not change the character of the problem we will ignore it. $R\,(\phi)$ is a 2×2 rotation matrix so it has the form

$$R\,(\phi) = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}$$

The $\phi_{A,pk}$ is the phase advance of normal mode A between the k^{th} skew quad and the observation point p. Notice that for observation points between couplers the $\overline{H}_+$ propagates like the difference of the phase advances and the $\overline{H}_-$ propagates like the sum of the phase advances. CESR runs near the coupling resonance so generally the $\overline{H}_+$ varies slowly as a function of longitudinal position and the $\overline{H}_-$ varies quickly. Also the $\overline{H}_{\pm}$ have the form of a constant times a rotation matrix, so that $\det \overline{H}_{\pm} = 0$ iff $\overline{H}_{\pm} = 0$. Also

$$\det \overline{H} = \left(\det \overline{H}_+\right)\sin^2\pi\,(\nu_A + \nu_B) - \left(\det \overline{H}_-\right)\sin^2\pi\,(\nu_A - \nu_B) \qquad (9)$$

The $\overline{C}$ and γ that appeared earlier in the matrix V are related to these by

$$\overline{C} = \frac{-\overline{H}}{\gamma\,\mathrm{Tr}\,(A-B)} \quad ; \quad \gamma = \sqrt{\frac{1}{2} + \frac{1}{2}\left|\frac{\mathrm{Tr}\,(M-N)}{\mathrm{Tr}\,(A-B)}\right|}$$

Define $\overline{C}_{\pm}$ to be the part of $\overline{C}$ corresponding to the $\overline{H}_{\pm}$. If we are not close to the coupling resonance then $\gamma \approx 1$. If we are very close to the coupling resonance then $\gamma \approx 1/\sqrt{2}$.

Global Decoupling

- The first step in global decoupling is to adjust normal quads to bring the normal mode tunes together. This corresponds to setting $\mathrm{Tr}(M-N) = 0$. The closest approach of the tunes is given by $|\cos 2\pi\nu_A - \cos 2\pi\nu_B| = \sqrt{\det \overline{H}}$.

Next adjust skew quads to bring the tunes together, that is to get $0 = |\cos 2\pi\nu_A - \cos 2\pi\nu_B| = \sqrt{\det \overline{H}}$. From equation (9) this requires that $\overline{H}_+ = 0$ but places no restriction on $\overline{H}_-$. $\overline{C}$ may generally be written as

$$\overline{C} = \frac{-\overline{H}_+ \sin\pi\,(\nu_A + \nu_B)}{\gamma\,\mathrm{Tr}\,(A-B)} + \frac{-\overline{H}_- J \sin\pi\,(\nu_A - \nu_B)}{2\gamma\,(\cos 2\pi\nu_A - \cos 2\pi\nu_B)}$$

So near the coupling resonance,

$$\overline{C} \approx \frac{\mp 1}{2\gamma}\frac{\overline{H}_+}{\sqrt{\det \overline{H}_+}} + \frac{-1}{2\gamma \sin 2\pi\nu_A}\overline{H}_- J$$

Recall that $\overline{H}_+/\sqrt{\det \overline{H}_+}$ is just a rotation matrix, so that even as $\overline{H}_+$ goes to zero, $\overline{C}$ remains finite.

Now adjust the normal quads to return the tunes to the operating point. Recall eqn. (8). As the tunes are brought back to the operating point the phase advances between the skew quads and the observation point change. So the $\overline{H}_{\pm}$ also change and $\overline{H}_+ \neq 0$. If the operating point is not too far from the coupling resonance then the phase advances will only change slightly and $\overline{H}_{\pm}$ will remain near their values on the coupling resonance. So the $\overline{C}_+$ will remain small. The $\overline{H}_-$ was never constrained so $\overline{C}_-$ is also unconstrained. This can be seen in the DIMAT output shown in Figures 2 and 3. Figure 2 shows the coupling produced by powering two skew quads. It happens to contain more $\overline{C}_+$, which propagates between couplers as the difference of the normal mode phase advances, than $\overline{C}_-$, which propagates between couplers as the sum of the normal mode phase advances. Figure 3 shows the coupling after the machine has been globally decoupled. The global decoupling has left almost no $\overline{C}_+$ but has not changed the amount of $\overline{C}_-$.

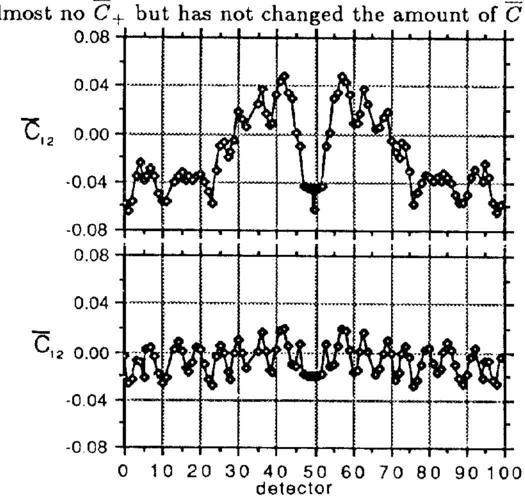

Figure 2 (top) : $\overline{C}_{12}$ at each detector due to two symmetrically placed, anti-symmetrically powered skew quads.
Figure 3 (bottom) : $\overline{C}_{12}$ at each detector due to two pairs of symmetrically placed, anti-symmetrically powered skew quads. The machine has been globally decoupled.

Global decoupling makes $\overline{C}_+$ very small if the operating point is not far from the coupling resonance but it does not affect the $\overline{C}_-$. There is nothing special about a globally decoupled machine unless it is at the coupling resonance. In fact if a machine is not at the coupling resonance, global decoupling is ill defined, because the change in the $\overline{H}_{\pm}$ as the tunes are brought together depend on the detailed changes in the phase advances between the skew quads and the observation point. This in turn depends on which quads are used to bring the tunes together. For example, using two pairs of symetrically located quads, bring the machine to the coupling resonance, then globally decouple, and return to the operating tunes using the same pairs of quads. Now if the machine is returned to the coupling resonance using two different pairs of quads the machine is generally not globally decoupled.

Reduction of Coupling in CESR

Measurement

- A normal mode of the beam is coherently excited by a shaker. The signals from each button of each beam detector are sent through a spectrum analyzer along with a reference signal from the shaker drive. The relative phase and amplitude of the vertical and horizontal motions is determined. From excitation of normal mode A, the $\Delta\phi_A$ and $(y/x)_A$ are measured and $\overline{C}_{12}$ and $\overline{C}_{22}$ are determined. Similarly excitation of normal mode B yields $\overline{C}_{11}$ and $\overline{C}_{12}$.

For weak coupling, normal modes A and B closely correspond to horizontal and vertical motion respectfully. So when normal mode A is shaken there is a large coherent horizontal signal and a much smaller coherent vertical signal. There is also noise from the incoherent motion in the beam. Since the incoherent horizontal motion is much greater than the incoherent vertical motion the data from excitation of normal mode A is much cleaner than the data from excitation of normal mode B.

Also recall that for normal mode A, the $\overline{C}_{22}$ measures the component of the vertical motion that is in phase with the horizontal motion. So, the $\overline{C}_{22}$ are susceptible to large errors if any of the large coherent horizontal signal "leaks" into the small coherent vertical signal. This leakage takes place in the detector and in the electronics that process the detector signal. The $\overline{C}_{12}$ are not susceptible to this effect because they measure horizontal and vertical signals that are 90° out of phase with each other. Any leakage produces horizontal and vertical motions that are either exactly in phase or 180° out of phase with each other. So the $\overline{C}_{12}$ data are much cleaner than the $\overline{C}_{22}$ data. Our analysis is based only on the $\overline{C}_{12}$ data.

Procedure - We began by globally decoupling the machineto remove most of the $\overline{C}_+$. The rms size of the $\overline{C}_{12}$ is about 0.033. Figure 4 shows this data. For each iteration we shook normal mode A and measured the $\overline{C}_{12}$ and the $\overline{C}_{22}$ at each of the beam detectors. Then DIMAT was used to find the strengths of skew quads that would best reproduce the $\overline{C}_{12}$ data. These changes were dialed into CESR. For the first iteration three pairs of skew quads were used. After the first iteration the rms size had been reduced by about 25% to about 0.024. If everything had worked perfectly all the $\overline{C}_{12}$ would have been zero. Since they were not all zero we did a second iteration. This time it was neccessary to use 4 pairs of skew quads to get a good fit from DIMAT. After the second iteration the rms had been reduced about 45% to about 0.013. These are shown in Figure 5.

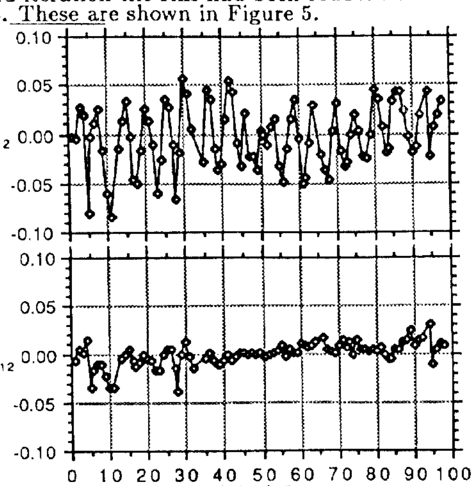

Figure 4 (top) : $\overline{C}_{12}$ at each detector. Machine has been globally decoupled, but still shows local coupling.

Figure 5 (bottom) : $\overline{C}_{12}$ at each detector after two iterations of local decoupling.

Sources of Error - A major source of error is β errors, or more precisely differences between the β's in the machine and in the simulation. These are caused by errors in the quad strengths. We assume that the simulation has been adjusted so that its normal mode tunes match those in the machine. This is especially important at CESR, where we operate near the coupling resonance so that a small change in one of the tunes may produce a large change in the distance to the coupling resonance.

The β errors affect the results in 3 ways. First, in going from the measured relative phases and amplitudes of the horizontal and vertical motion to the $\overline{C}$'s the dependence on the twiss parameters is taken out. Second, the effect that the skew quads have on the $\overline{C}$'s is scaled by $\sqrt{\beta_A\beta_B}$. Because the calibration of the skew quad, its focal length as a function of excitation, is usually determined through a coupling measurement this error appears twice, once for the β errors at the skew quad when it is calibrated and once for the β errors at the skew quad when it is used. The exception to this is when the calibration is done just before the skew quad is used, as then the β errors are included in the calibration. Finally, these β errors change the phase advances between the skew quads and the observation points. The coupling effect from this strongly depends on the configuration of couplers in the machine.

Fortunately even fairly large differences (as much as 20-25%) between the β's in the machine and in the simulation can be tolerated; the method will still reduce the coupling. However if the errors are large the reduction in coupling for each iteration will be small. For the data shown above the fractional differences between the β's in the machine and the simulation were large, about 25%. If the β's are measured and the values of the normal quads in the simulation are adjusted to produce β's that match those measured in the machine, the differences between the β's in the machine and in the simulation can be reduced to about 5%. If this had been done, only one iteration would have been required to go from the coupling in Figure 4 to the coupling in Figure 5.

By its nature, correction of local coupling requires many skew quads distributed around the ring. Although CESR has 9 pairs of skew quads, in the test above only 4 pairs were used. As a result there is coupling remaining in Figure 5 between detectors 5 and 30. About half of this cannot be removed by the 4 pairs of skew quads used.

In addition to all of these systematic errors there are also random errors of about 0.005 in the $\overline{C}_{12}$.

In spite of all these problems after two iterations we were able to reduce the coupling in CESR to the level of $\overline{C}_{12} \approx 0.013$. This requires about 2-3 hours of machine time, about the same amount of time required to correct the β's in the machine. With our current knowledge, after about 2-3 iterations we should be able to reduce the coupling to the level of the noise in the measurements, that is about $\overline{C}_{12} \approx 0.005$.

We have shown how the $\overline{C}$'s relate to the vertical beamsize and the normal mode emittances but have not discussed this in detail.This is because the vertical beamsizes produced by the coupling generally found in CESR is about the same as the resolution of our vertical beam size monitor. So we have no good experimental data.

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