OPTIMIZATION OF MUON CAPTURING IN g-2 RING

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Abstract

We consider the dynamics of muons injected into the g-2 ring now under construction at FERMILAB. We present the concept for the pulsed injection kicker magnet that optimizes capture efficiency and minimizes coherent betatron oscillations of stored muons. Some engineering details of the high voltage kicker and pulser are presented.

OVERVIEW

Experiment E-821 [1], was performed at Brookhaven National Laboratory for measurements of anomalous magnetic moment of muon $a_\mu = (g_\mu - 2)/2$ where g_μ is the gyromagnetic ratio $\vec{\mu}_\mu = g_\mu(e/2m)\vec{s}$, $\vec{s}$ stands for the vector of spin with accuracy 0.54 ppm. The idea of the experiment is straightforward. The angular frequency $\vec{\omega}_a$, which corresponds to the difference between the spin precession frequency and the cyclotron frequency $\vec{\omega}_c = eB/mc$

$$\vec{\omega}_a = -\frac{e}{m_\mu}\left[a_\mu \vec{B} - \left(a_\mu - \tfrac{1}{\gamma_\mu^2 - 1}\right)\vec{\beta}\times\vec{E}\right], \qquad (1)$$

identifies the anomalous magnetic moment. Given the magnetic field along the trajectory of the muons, then using (1) one can calculate a_μ. In the experiment, the positrons from the time of flight decay $\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu$ are detected with 24 Pb-scintillating fiber calorimeters distributed evenly around the inner circumference of the storage ring [2], [3]. As the direction of the emission of the decay positrons is correlated with the orientation of the muon spin vector, the time dependence (variation) of the counts in each calorimeter is associated with the precession of the muon spin in the guiding magnetic field of the ring. For the muon energy, which corresponds to the $\gamma_\mu = \sqrt{1 + 1/a_\mu} \cong 29.3$, the term that multiplies the cross product of velocity and electric field in (1) acquires a zero value, so the spin precession becomes insensitive to the electric field in the first order.

Now the entire (*g*-2) ring will be moved from BNL to FERMILAB, in preparation for a next-generation experiment to be performed with increased statistics and improved systematics allowing a measurement of the anomalous moment with an accuracy of better than 0.14 ppm [4]. As the efficiency of muon capture contributes to better statistics, and smaller coherent betatron oscillations to reduction of an important systematic effect, we are well advised to consider improvements to the injection kicker system.

One of the key components is a fast kicker pulser, which provides on axis injection into the muon storage ring [5]. The pulser in the E-821was a simple LCR contour triggered by a thyratron with sub-critical decrement arranged with the help of resistor *R*~12Ω installed in a discharge loop arranged with *C*=10nF and effective inductance *L*~1μH. During the E-821 run it was found that the muon accumulation efficiency had not yet plateaued when the pulser reached its maximum value, suggesting that at very least, the total number of muons captured would be increased with a more powerful kicker. We recommend a Blumlein-type pulser for these purposes.

OPTICAL PARAMETERS

Parameters of the ring, which are important for the kicker design, are represented in Table 1. Transverse focusing of muons is provided by electrostatic quadrupoles that occupy significant fraction of the orbit perimeter. In view of very nearly uniform focusing we assume that the field index *n* is constant around the ring. The average field index

$$n = -\frac{R}{\beta c B}\frac{\partial E_y}{\partial r}\ ,\ (CGS,\ Gs,\ cm), \qquad (2)$$

where β stands for the normalized velocity of muon, *R* is the radius of trajectory, *B* is the guiding magnetic field and *E* the electric field. Then the betatron functions, tune shifts and dispersion are

$$\beta_x = \frac{R}{\sqrt{1-n}},\ \beta_y = \frac{R}{\sqrt{n}},\ Q_x = \frac{R}{\beta_x},\ Q_y = \frac{R}{\beta_y}\ \eta = \frac{R}{1-n}. \qquad (3)$$

The physical aperture of (*g*-2) ring has a radius of 4.5 cm so the maximum and minimum radial offset is $x_{max/min}$=±4.5cm respectively and the maximum and minimum energy offset is $\delta_{max/min} = x_{max/min}/\eta$.

Table 1. Muon ring parameters

Muon momentum [GeV/c]	3.09
R[m]	7.112
Index *n*	0.139
Qx, *Qy*	0.927; 0.373
β_x, β_y, η_x [m]	7.67, 19.1, 8.26
x_{max}, x_{min} [cm]	+4.5, -4.5
Energy aperture [%]	0.545
Offset at inflector exit [cm]	7.6

The muons exiting the inflector have finite energy and angular spread, and are displaced x_0 = 7.6cm from the central orbit. For a muon with fractional energy offset δ, the radial offset (x_0) has a betatron contribution and an energy contribution.

$$x_0 = x_\beta + \eta\delta. \qquad (4)$$

Clearly the betatron contribution depends on energy and we write

$$x_\beta(\delta) = x_0 - \eta\delta \qquad (5)$$

At betatron phase φ_x (φ_x=0 at the inflector exit),

$$x(s) = x_\beta(\delta)\cos\varphi_x + \eta\delta \quad (6)$$

The muon with energy offset δ has zero betatron amplitude at $\varphi = \pi/2$ and displacement from the on energy central trajectory of $x = \eta\delta$. The angle of the muon trajectory with respect to its closed orbit is

$$x'(s) = x_\delta \sin\varphi \cdot \frac{d\varphi}{ds} = \frac{x_\delta}{\beta}\sin\varphi = \frac{x_0 - \eta\delta}{\beta}\sin\varphi \quad (7)$$

and at $\varphi = \pi/2$

$$x'(s) = (x_0 - \eta\delta)/\beta \quad (8)$$

We apply a kick $\theta = -x'$ to direct the muon onto its closed orbit. The ideal kick angle depends on the energy

$$kick = kick_0/(1-\delta) \quad (9)$$

If-the kicker field is uniform and kicks the on energy muon by $\theta = x_0/\beta = 9.91$ mrad then the error in the kick angle for off energy particles will be

$$\Delta\theta(\delta) = \left(\frac{x_0 - \eta\delta}{\beta} - \frac{x_0}{\beta}\right)\frac{1}{1-\delta} = \frac{\eta\delta}{\beta}\frac{1}{1-\delta} \sim \frac{\eta\delta}{\beta} \quad (10)$$

to the lowest order in δ.

Table2. The kick vs energy

Fractional energy offset	kick [mrad]	$\Delta\theta$
0.545%	4.04	-5.84
0	9.91	0
−0.545%	18	5.84

In order to kick all energies on to their respective closed orbit we require that the kick have radial dependence

$$kick = kick_0 + \Delta\theta / x_{\max} \quad (11)$$

Then the kick compensates all energies if

$$kick(x) = 9.91\text{mrad} + 1.3[\text{mrad/cm}]\, x[\text{cm}] \quad (12)$$

We now suppose that the muon has nonzero angle x_0' exiting the inflector. Then at the kicker, $x(\pi/2) = x_0'\beta$. Evidently, there is no kick applied at $\varphi = \pi/2$ that will place the muon on axis. Indeed for an on energy muon, any kick except x_0/β will increase the residual betatron oscillation. We find that the optimum kicker field profile depends in detail on the energy and angular distribution of the muons exiting the inflector. As that distribution is anticipated to included substantial energy spread and angular divergence we choose to design kicker plates for a uniform field profile as that minimizes peak field (Fig.1).

To provide a kick α~10 mrad to ~3 GeV/c muon beam the required field integral is $\int Bdl = \alpha \cdot (BR) \cong 0.1 T\cdot m$ where $(BR) \cong 10 T \cdot m$ is the magnetic rigidity of muon beam at energy ~3 GeV. For the kicker length $L \cong 1.7m$ (the length of the old kicker section, 1/3 total), the magnetic field in the aperture of kicker should be $\int Bdl / L \cong 0.059T \approx 600G$. The height of the new plates is ~5cm (see Fig.2), so the feed current should be not less, than 2.5 kA. The presence of the surrounding conducting surfaces drastically reduces the field/current ratio. For a feed current of 1kA the field value at the center of aperture comes to ~80G, so the current required in a single 1.7 m-long kicker ~7.5 kA. In the E-821 experiment *three* ~1.7m long kickers [1], [5] were required to deliver the ~10mrad kick.

THE CONCEPT

Both the pulse generator and the kicker plate profile are redesigned for the new experiment. One technical complication is the necessity to accommodate the trolley that carries the 17 NMR probes for measuring the field distribution in the beam aperture vacuum. The outer diameter of the cartridge is ~90mm, with length ~500 mm, weight ~2kg. It travels on 18 wheels (total). This system allows magnetic field mapping with an accuracy better than 1.5ppm in vacuum during dedicated periods; while the ring is operating, the cartridge is moved into a docking position inside vacuum chamber. We chose the shape of the electrodes to allow free passage of the cartrige between the kicker plates (see Fig.1).

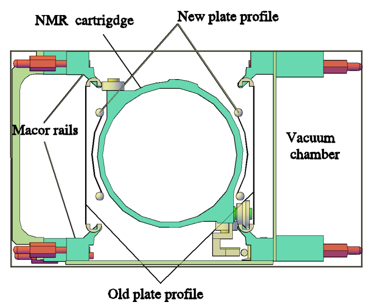

Figure 1. The old kicker plates and the new ones in comparison. The NMR cartridge profile in measuring position is shown also. Inner circle diameter is 88 mm.

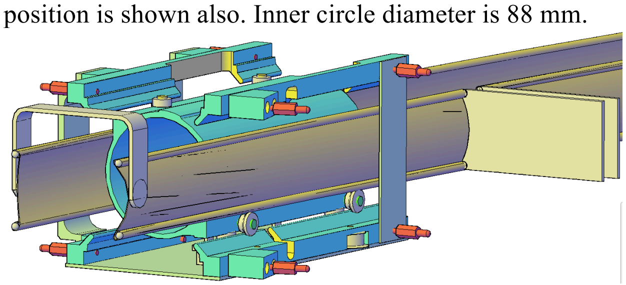

Figure 2. Fragment (slice) of NMR cartridge running on Macor© rails between the kicker plates. End jumper is seen at the left side too.

The current input is arranged in a break of the inner kicker plate, seen at the right of Fig.3. This was done as the generator (old and new) generates negative voltage at the output.

NUMERICAL MODEL

2D codes should be used cautiously for the time dependent problem modeling with well conducting materials when the skin depth is much less, than the thickness of the electrodes, as all processes in 2D while began, could not stop. 2D analyses allowed clarifying some details only which can improve the field/current ratio. So we launched full 3D time dependent analyses with FlexPDE. Severe limitations for the stray field level

<0.1 ppm, force careful modeling of the problem. Example of the output from 3D modeling is represented in Fig. 3.

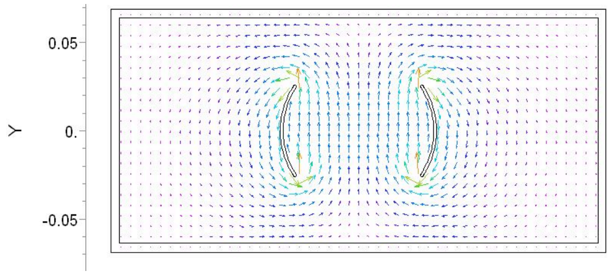

Figure 3. Magnetic field vectors in the new plates profile. Vertical scale Y in *cm*.

DESIGN CONCEPT

The Blumlein generator performs as a tri-axial line with one inside the other. Each coaxial pair has impedance Z_0=6.25Ω so the output impedance is 12.5 Ω. The coaxial cable RG-193/U matches this impedance. The outer diameter of this cable 2.1in allowing the possibility of bending it to a desirable shape. The Inner Aluminum ~4.4"OD tube is filled by Foamplast to reduce the oil volume. In the design we separate the HV transformer tank from the commutation unit, linking them with additional HV cable, see Fig.4.

The thyratron CX1193 is appointed as the commutator. It allows up to 160 kV peak forward anode voltage with maximal current up to 8 kA for repetition rate limited to 60 p.p.s and up to 60kA in a single shoot or crowbar service at 1 pulse per 10s.

Outer parameters of the prototype pulser are represented in Table 3. Due to specifics of Blumlein generator operation, the output voltage coincides with the charging voltage. The current running through the commutators is twice of the output current. The prototype Blumlein pulser is shown in Fig. 4.

Table3. Parameters of the prototype pulser.

Voltage	100kV
Current	8kA
Flat top	50ns
Matching impedance	12.5Ω

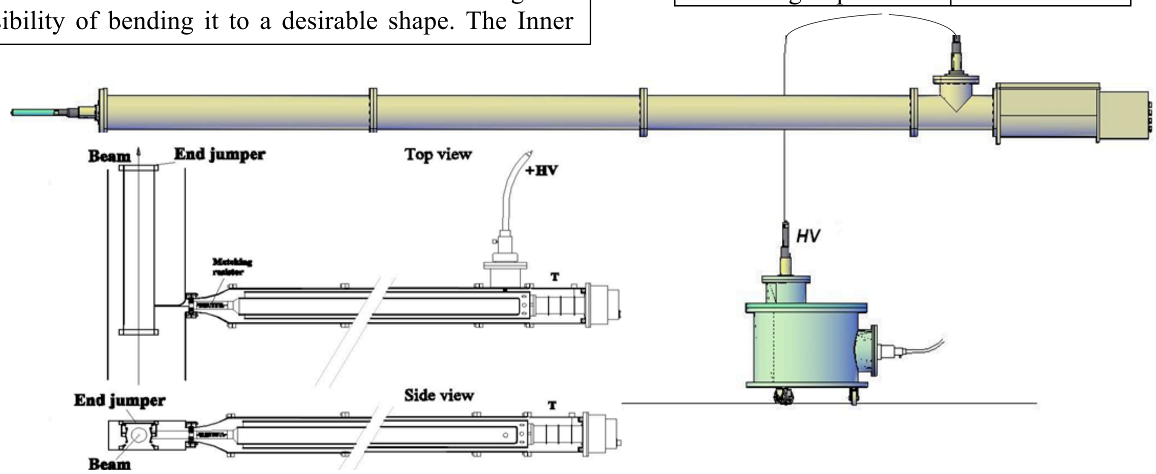

Figure 4. The Blumlein generator prototype. At the left: The Blumlein pulser as a three-coaxial line. Outer diameter of the Al tubing is 7", length of each section ~1.5 m. In the final model diameter will be reduced to 4" OD. Inner volume filled with oil. **T** stands for the thyratron.

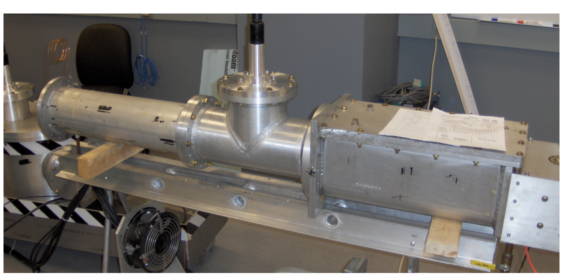

Figure 6. The thyratron housing with HV input.

SUMMARY

The kicker is a stripline with matched resistor installed at the input of the line. Usage of the Blumlein-type generator allows symmetric design of the generator with flat top pulse width up to ~100 ns.

The prototype developed at Cornell will allow ~ 8kA pulse with ~100kV and ~50ns pulse width. The new kicker electrodes shape allows ~50% increase of kick for the same feeding current as in the E-821 design. The vacuum chamber in the region of the kicker could be equipped with transverse grooves, for a further increase of the kick.

REFERENCES

[1] G.W.Bennet *et.al.,* " Final report of the Muon E821 Anomalous Magnetc Moment Measurement at BNL", arXiv:hep-ex/0602035v1, 20 Feb. 2006; Dated: August 9, 2009.

[2] X.Fei, V.W.Huges, R.Prigl, Nucl.Inst. and Meth. **A** **374**(1994), 118.

[3] R.Prigl *et.al*., Nucl.Inst. and Meth. **A** (1996) 349.

[4] The New (g-2) Experiment: A proposal to Measure the Muon Anomalous Magnetic Moment to ±0.14 ppm Precision, Submitted to DOE Office of High Energy Physics, April 5, 2010.

[5] E.Efstihadis, *et.al.*,"A Fast Non-Ferric Kicker for the Muon (g-2) experiment", Elsevier Sci., 15 July, 2002.