CESR LATTICE DESIGN

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Abstract

New software has been developed at the Cornell Electron/positron Storage Ring CESR. An object–oriented subroutine library called BMAD has simplified the writing of programs by providing modular subroutines to read in lattice files, calculate Twiss parameters, etc. Built on top of this, the lattice optimization program designs the linear lattice to minimize problems with the pretzel orbit, and uses a novel strategy to quickly design sextupole distributions that maximize the dynamic aperture.

1 INTRODUCTION

In recent years a number of innovations has made it possible to reach ever higher luminosities at the Cornell Electron/positron Storage Ring CESR[1]. New hardware such as improved beam feedback, a fast luminosity monitor, etc. has been critical to machine performance. New software has aided in the study and design of the linear and nonlinear optics, orbits, etc. In particular, to be described in this paper, is the BMAD subroutine library which has become the basis for programs dealing with the CESR lattice. Also discussed here are the new optimization strategies used to design the linear and nonlinear optics for CESR.

2 BMAD

The basis for CESR simulation and analysis programs is an object–oriented subroutine library, written in Fortran 90, called BMAD. The goal of BMAD was to simplify the writing of programs, and reduce programming errors, by providing modular subroutines to read in lattice files, calculate Twiss parameters, do tracking, etc. This has significantly cut development time for new programs. For example, to read in a lattice and calculate the Twiss parameters is 3 lines of code.

The BMAD standard input format is modeled after that of MAD. BMAD defines the standard ring elements: drifts, bends, quadrupoles, etc. As in MAD, the layout of a ring is specified by an ordered list of elements. To provide flexibility, elements may be "superimposed" upon the layout; For example, if a solenoid element is superimposed on top of a section of the layout where there are quadrupoles and drifts then the quadrupoles will be converted to combination solenoid/quadrupole elements, etc. Elements that are at the edges of the superimposed element are split accordingly. To simplify the bookkeeping, extra elements outside of the layout are defined that represent the superimposed and unsplit elements. A bookkeeping routine links changes

in the attributes of these elements to the attributes of the superimposed elements. In addition, to be able to easily model the control system, controller elements may be defined that control the attributes of other elements.

3 LINEAR LATTICE DESIGN

There is an approximate mirror symmetry but no periodicity in the CESR lattice. Except for the permanent magnet final focusing near the Interaction Point (IP), all of the quadrupoles are independently powered so that there is considerable flexibility in the design of the storage ring optics. In addition to specific requirements for IP parameters, tunes, injection matching, etc., there are a number of constraints peculiar to the CESR pretzel separation scheme.

To design the linear lattice, the CESR design program uses as independent variables the 102 electromagnetic quadrupoles, the electrostatic separator voltages, and the IR quad rotation angles. The design program defines a Merit Function which is then minimized. The Merit Function depends on the lattice parameters: The tunes, β^*, η^*, emittances, etc., along with quantities associated with the pretzel separation of the trains of bunches. The pretzel related quantities are:

1. The maximum long range beam–beam tune shift of all the parasitic crossings.

2. The total long range beam–beam tune shift for each bunch and the spread from bunch–to–bunch that results from the uneven temporal spacing.

3. $B \sim \Sigma_i \epsilon_x \beta_{xi} \beta_{yi}/x_i^2$. This is a figure of merit for the long range beam–beam interaction. Here x_i is the separation at the ith parasitic crossing and ϵ_x is the horizontal emittance[3].

4. The minimum separation, in units of σ_x, between the bunches at the parasitic crossing points.

5. $\epsilon \equiv \min[(x_i/\sqrt{\beta_{xi}})/(x_{max}/\sqrt{\beta_{x,max}})]$. This is a measure of the "pretzel efficiency". Here x_{max}, $\beta_{x,max}$ are maximum pretzel amplitude and corresponding horizontal beta.

6. The pretzel dependence of the damping partition numbers.

7. The crossing angle at the interaction point.

The independent variables are chosen to minimize the Merit Function according to a weighted combination of the above criteria and subject to aperture constraints. Typical CESR lattice functions are shown in Figure 1.
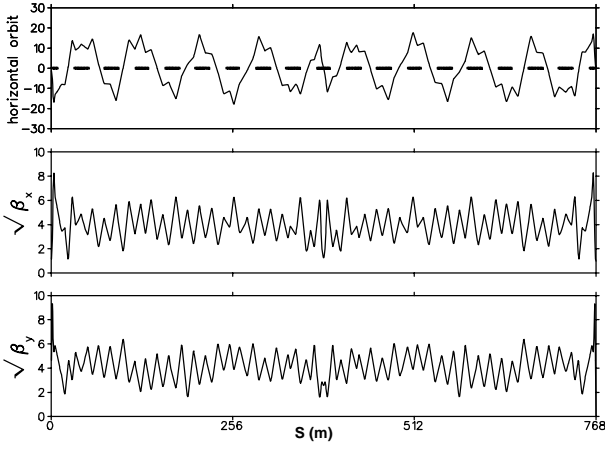
Figure 1: The CESR *beta* functions and closed orbits optimized for operation with 9 trains. The 5 bunches within each train are 14ns apart. The tic marks in the orbit plot indicate parasitic crossings.

4 SEXTUPOLE DESIGN

The principle requirement of the sextupoles is to provide for control of the energy dependence of the tune, both to damp the head–tail instability and to define the footprint in the tune plane.

The sextupoles in CESR are independently powered and there is a continuous set of sextupole strength distributions that will correct machine chromaticity. This flexibility is exploited by imposing additional constraints, the use of which have generally been found to give maximal dynamic aperture. In particular, it is desired to minimize the energy, the beam oscillation amplitude, and the pretzel dependence of the β-functions and coupling parameters. The dependencies are written to first order in sextupole strengths so that a solution can be determined analytically.

Consider the mapping of the phase space coordinates near the closed orbit. The Jacobian of the map can be computed numerically for arbitrary betatron amplitude. If the linear aperture is large then the Jacobian will be independent of the amplitude. The β function and betatron phases are based on the Jacobian, so the amplitude dependence of the Twiss parameters is, more or less, a measure of the dynamic aperture. The Jacobian can similarly be computed for arbitrary energy offset, and the energy dependence of the Jacobian, as reflected in the energy dependence of the Twiss parameters, indicates the energy aperture of the optics. The CESR lattice design program maximizes the aperture by choosing sextupoles that minimize the energy and amplitude dependence of the Twiss parameters. The specific contributions to the Merit Function are as follows:

Energy dependence of the β-function Both quadrupoles and sextupoles contribute to the energy dependence of the β-function. To first order in energy offset, the change in the β-function at location i depends on the strengths of the quadrupoles k_{1j}, and the sextupoles

k_{2n}.

$$\Delta\beta_i = \frac{\beta_i}{\sin 2\pi Q}\left[\Sigma_j \beta_j\, k_{1j}\, l_{qj}\cos(2\alpha_{ij}) + \right.$$
$$\left. \Sigma_n \beta_n\, \eta_n\, k_{2n}\, l_{sn}\cos(2\alpha_{in})\right]\delta_E, \quad (1)$$

where k_{1j} is the strength of the jth quadrupole, k_{2n} is the strength of the nth sextupole, with magnetic lengths l_{qj} and l_{sn} respectively. η_n is the dispersion in the sextupole, and $\alpha_{ij} \equiv \phi_i - \phi_j - \pi Q$. Finally, $\delta_E = \Delta E/E$ is the fractional energy offset. Note that the first term on the right side of Eq. (1) is independent of sextupole strength and the second term is a linear combination of sextupoles.

The betatron tune similarly depends on quadrupoles, dispersion and sextupole strength.

$$\Delta Q = \left[\Sigma_j\,\beta_j\, k_{1j}\, l_{qj} + \Sigma_n\,\beta_n\,\eta_n\, k_{2n}\, l_{sn}\right]\delta_E/4\pi \quad (2)$$

Dependence on closed orbit The horizontal displacement of the trajectory x in a sextupole of strength k_2 contributes a quadrupole error $\Delta k_1 = x\, k_2\, l_s$. The dependence of the β-function on the closed orbit is

$$\Delta\beta_i = \frac{\beta_i}{\sin 2\pi Q}\Sigma_j\beta_j x_j^{\text{closed}}\cos\alpha_{ij}\, k_{2j}\, l_{sj} \quad (3)$$

Tonality, the dependence of tune on closed orbit, is given by

$$\Delta Q = \frac{1}{4\pi}\Sigma_j\beta_j x_j^{\text{closed}}\, k_{2j}\, l_{sj} \quad (4)$$

Amplitude dependence of β-function The one turn map depends on the amplitude of both horizontal and vertical betatron motion. Associated with the horizontal motion is the effective quadrupole error due to displacement in the sextupoles. For free betatron oscillations the displacement in the sextupoles is turn dependent. In a single turn the perturbation to the $\beta-$function can be defined as if the oscillation represented a closed orbit. Any betatron oscillation can be described as a linear combination of sin-like and cos-like trajectories. If the sextupole distribution is chosen so that the perturbation to the β-functions due to both sin-like and cos-like trajectories is zero, then it is necessarily amplitude independent. We define amplitude dependence of β

$$\frac{\partial\beta_i}{\partial a} = \frac{\partial}{\partial a}\left(\frac{\beta_i}{\sin 2\pi Q}\Sigma_j\beta_j\, k_{2j}\, l_{sj}\, x_j^{\cos}\cos(2\alpha_{ij})\right)$$
$$\frac{\partial\beta_i}{\partial b} = \frac{\partial}{\partial b}\left(\frac{\beta_i}{\sin 2\pi Q}\Sigma_j\beta_j\, k_{2j}\, l_{sj}\, x_j^{\sin}\cos(2\alpha_{ij})\right) \quad (5)$$

The trajectories are

$$x_j^{\cos} = x_j^{\text{closed}} + a\,\sqrt{\beta^*\beta_j}\cos\phi_j$$
$$x_j^{\sin} = x_j^{\text{closed}} + a\,\sqrt{\beta^*\beta_j}\sin\phi_j \quad (6)$$

and finally

$$\frac{\partial\beta_i^{\cos}}{\partial a} = \frac{\beta_i}{\sin 2\pi Q}\Sigma_j\beta_j\, k_{2j}\, l_{sj}\cos\phi_j\cos(2\alpha_{ij})$$
$$\frac{\partial\beta_i^{\sin}}{\partial b} = \frac{\beta_i}{\sin 2\pi Q}\Sigma_j\beta_j\, k_{2j}\, l_{sj}\sin\phi_j\cos(2\alpha_{ij}) \quad (7)$$

Amplitude dependence of transverse coupling The transverse coupling introduced by the sextupoles is proportional to the vertical displacement of the trajectory from the magnet axis. The coupling measured at element j due to the sextupole n is proportional to the trajectory y_n. The field is equivalent to a skew quad with focal strength $1/f_n = k_{2n} l_{sn} y_n$. As in the case of horizontal motion we suppose that any such vertical trajectory can be constructed from sin-like and cos-like oscillations.

$$y_n^{\cos} = b\sqrt{\beta^* \beta_n} \cos \phi_n$$
$$y_n^{\sin} = b\sqrt{\beta^* \beta_n} \sin \phi_n \qquad (8)$$

If we define complex coupling coefficients[4]

$$a = \bar{c}_{11} - \bar{c}_{22} + i(\bar{c}_{12} + \bar{c}_{21})$$
$$b = \bar{c}_{11} + \bar{c}_{22} + i(\bar{c}_{21} - \bar{c}_{21}) \qquad (9)$$

then the coupling at j due to a skew quad at k is

$$a_j = \Sigma_n \rho_n \, e^{-i(\xi_{jn}^+ + \Sigma/2)}$$
$$b_j = \Sigma_n \chi_n \, e^{-i(\xi_{jn}^- + \Delta/2)} \qquad (10)$$

where

$$\rho_n = \frac{\sqrt{\beta_{xn}\beta_{yn}}}{tr(\mathbf{A}-\mathbf{B})\gamma}\left(\frac{2\sin Q_-}{f_n}\right)$$
$$\chi_n = \frac{\sqrt{\beta_{xn}\beta_{yn}}}{tr(\mathbf{A}-\mathbf{B})\gamma}\left(\frac{2\sin Q_+}{f_n}\right)$$
$$\xi_{jn}^{\pm} = \phi_{xj} - \phi_{xn} \pm (\phi_{yj} - \phi_{yn})$$
$$Q_{\pm} = (Q_x \pm Q_y)/2 \qquad (11)$$

Here γ and the $\bar{c}_{ij}$ are a measure of the local coupling and $\mathbf{A}$ and $\mathbf{B}$ are Normal mode matrices[4].

Sextupole uniformity In order to limit the magnitude of individual sextupole, we constrain the sum of the strengths of each adjacent pair of focusing and defocusing sextupole, weighted by the strengths that would characterize a two family distribution.

$$K_{tf}^h k_{2j} - K_{tf}^v k_{2j+1} = 0 \qquad (12)$$

K_{tf}^h and K_{tf}^v are horizontal and vertical sextupole strengths of a two family distribution designed to correct the chromaticity. k_{2j} and k_{2j+1} are strengths of adjacent horizontally and vertically focusing sextupoles.

Creating a sextupole distribution

We choose a set of sextupoles that minimize the dependence of the β-functions, coupling, and tunes, on energy, closed orbit, and betatron amplitude. Since each of the dependencies can be characterized as a linear combination of sextupoles we can abbreviate each as follows

$$D_i = B_i + A_{ij}k_{2j} \qquad (13)$$

where D_i is the dependent variable (i.e. $\partial\beta/\partial\delta$, etc.), B_i is the contribution from the lattice quadrupoles, and A_{ij}
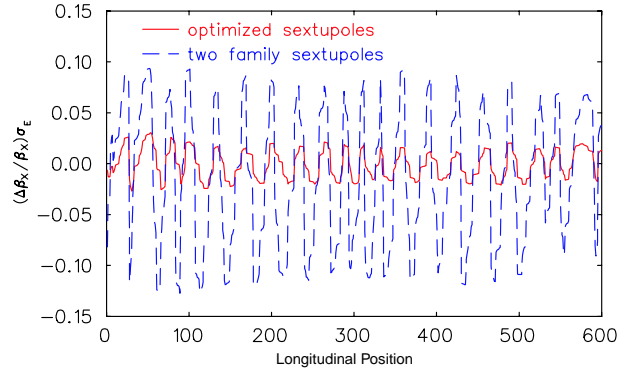

Figure 2: The energy dependence of β_x in the CESR lattice for optimized distribution and two family sextupole distribution. The chromaticity is zero for both distributions. The plot for β_y looks similar.

the coefficient of sextupole j with strength k_{2j}. In practice there are many more such equations than sextupoles and the system is over constrained. In particular, we would like to constrain the dependencies at every point in the lattice. We use a linear least square fitting procedure to find a set k_{2j} that minimizes $\chi^2 = \Sigma_i(W_i D_i)^2$ W_i is the weight attached to each constraint.

For a particular choice of weights, the sextupole distribution is uniquely defined. Finally, the quality of the distribution is measured by parameters that cannot be written as a linear combination of sextupole strengths and are computed in a separate step. These include:

1. Deviation from unity of the determinant of the full turn Jacobian as computed for various energies and amplitudes.

2. Discrepancy between first order and exact calculation of energy dependence of β.

3. Maximum sextupole strength.

4. Dynamic aperture, as determined by tracking.

The weights are subsequently adjusted until a satisfactory solution is achieved. As an example, a comparison of the energy dependence of β_x for the optimized sextupole distribution and for a simple two family distribution is shown in Figure 2. As seen in the figure, β with the optimized sextupoles has a much weaker energy dependence and tracking shows with the optimized sextupoles the dynamic aperture is much larger.

5 REFERENCES

[1] D. Rubin, et al., Proc. 1999 Part. Acc. Conf., 285 (1999).

[2] S. Belomestnykh, Proc. 1999 Part. Acc. Conf., 272 (1999).

[3] J. Welch and A. Temnykh, Cornell CBN 95-13, unpublished (1995).

[4] D. Rubin, "Handbook of Accelerator Physics and Engineering", World Scientific, A.Chao and M.Tigner, p. 271 (1999).