

The Single Bunch Emittance Preservation in TESLA Main Linac

V. Tsakanov

CANDLE / YerPHI

TESLA Specific

- **Small transverse wakefields**

$$W_x(\sigma) \sim 20 \text{ V/ pC/ m}^2$$

- **Correlated energy spread kept at the min level**

$$\sigma_{\text{cor}} = 3 \cdot 10^{-4}$$

- **To use "weak" focusing lattice**

4 accel modules per FODO cell
Cell length $L_{\text{cell}} = 65 \text{ m}$
Phase advance/ cell **60 degree**

- **Easy transverse autophasing solution**

1 Linear Optics and Cell Arrangement

Table 1

Parameters	TESLA
Energy $E(\text{GeV})$	5
Charge per bunch $Q(\text{nC})$	3.2
Bunch RMS length $\sigma_z(\text{mm})$	0.3
Initial uncorrelated RMS energy spread σ_ϵ	2.5%
Normalized emittance hor./vert. $\epsilon_{x,y}(\text{mm} \cdot \text{mrad})$	1/ 0.03

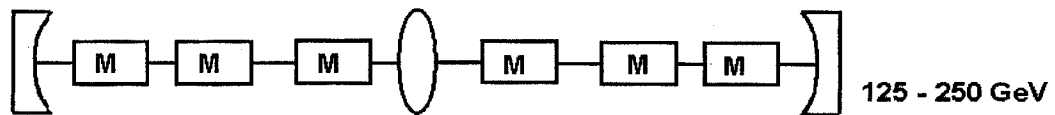
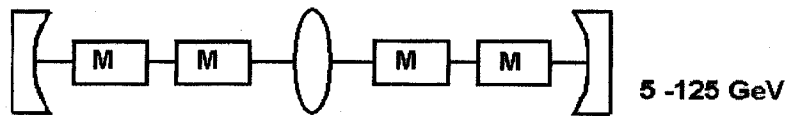


Fig.1 Quadrupole spacing (in unit of cryomodules) for TESLA.

TABLE 2. Parameters of TESLA focusing lattice.

Quadrupole, effective length (m)	0.6
Quadrupole, geometrical length (m)	0.9
Cell length 5-125 GeV (m)	65
Phase advance per cell μ_T	$45^\circ, 60^\circ$
Cell length 125-250 GeV (m)	96.6
Phase advance per cell 125-250 μ_T	60°
Maximum gradient of quadrupoles at 400 GeV (T/m)	47

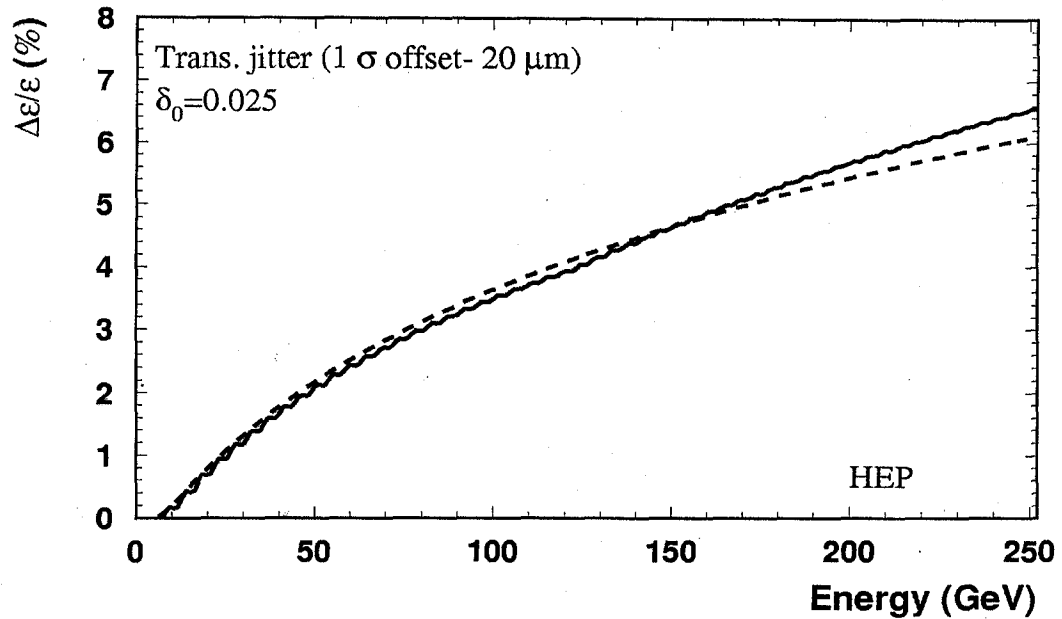


Fig. 8 Dispersive emittance dilution of the HEP beam caused by an injection offset and an initial uncorrelated energy spread. The solid line shows the particle tracking simulation, and the dashed line the analytical prediction for a constant FODO lattice.

For initial jitter 0.5-6

$$\frac{\Delta\epsilon}{\epsilon} \sim 1.7\%$$

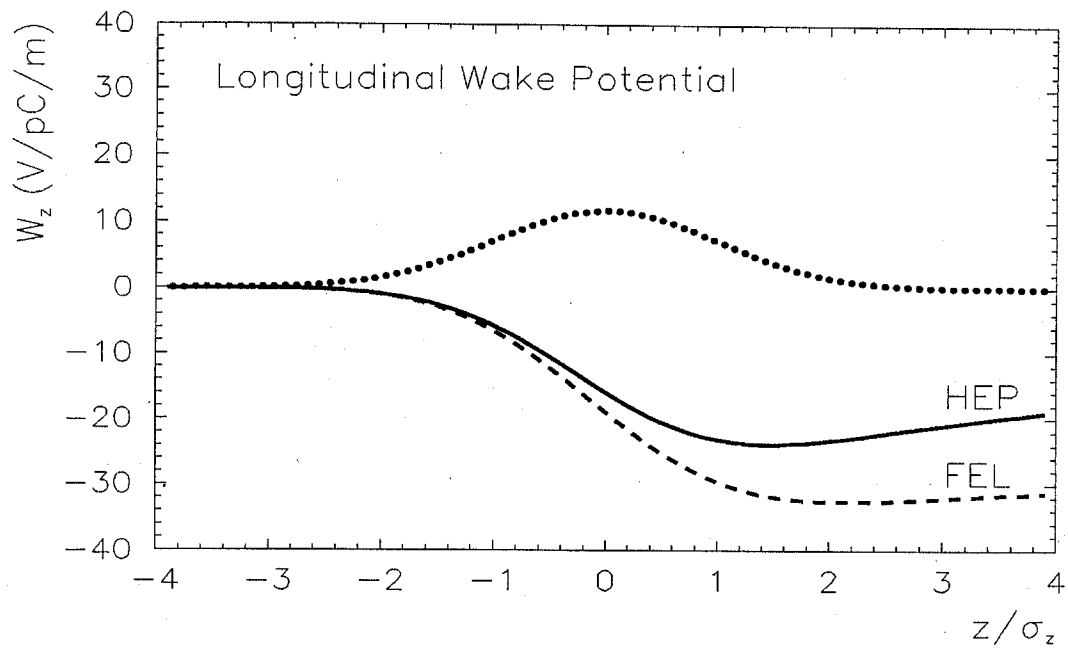


Fig.5 Longitudinal monopole wake potentials for HEP (solid line) and FEL (dashed line) beams.

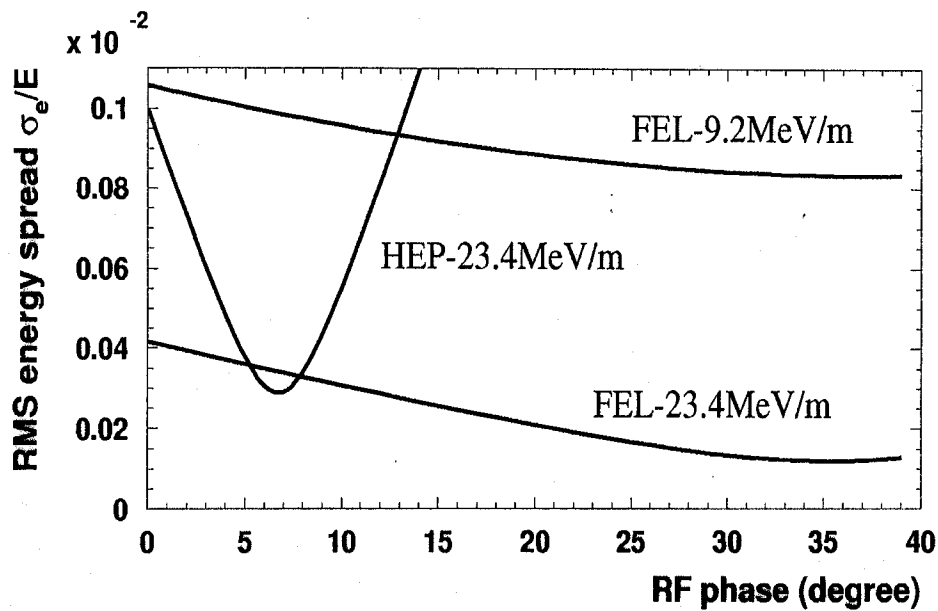


Fig.6 The RMS energy spread for the HEP and the FEL beam versus the accelerating RF phase.

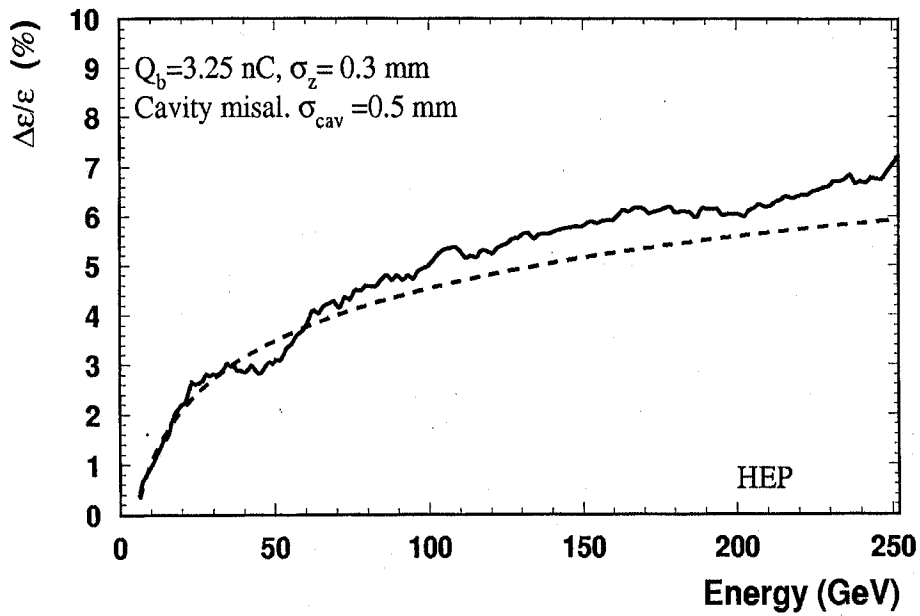


Fig. 13 *Rms emittance dilution of the HEP beam due to the cavity misalignment ($\sigma_{cav} = 0.5$ mm). The dashed line is an analytical prediction ($\mu = 45$ degree).*

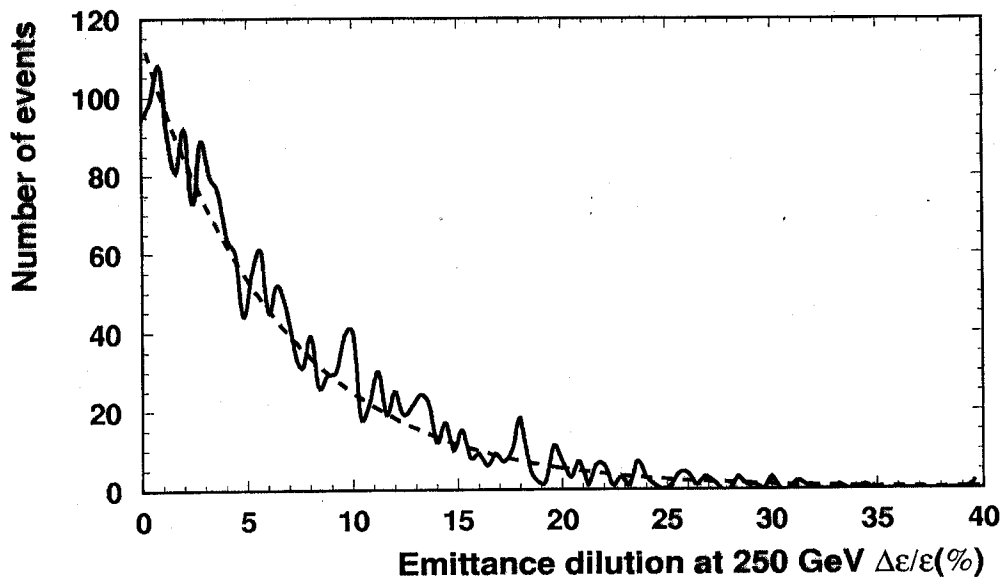


Fig. 15 *The probability of the rms emittance dilution of the HEP beam due to the cavity misalignment ($\sigma_{cav} = 0.5$ mm, 2000 random seeds).*

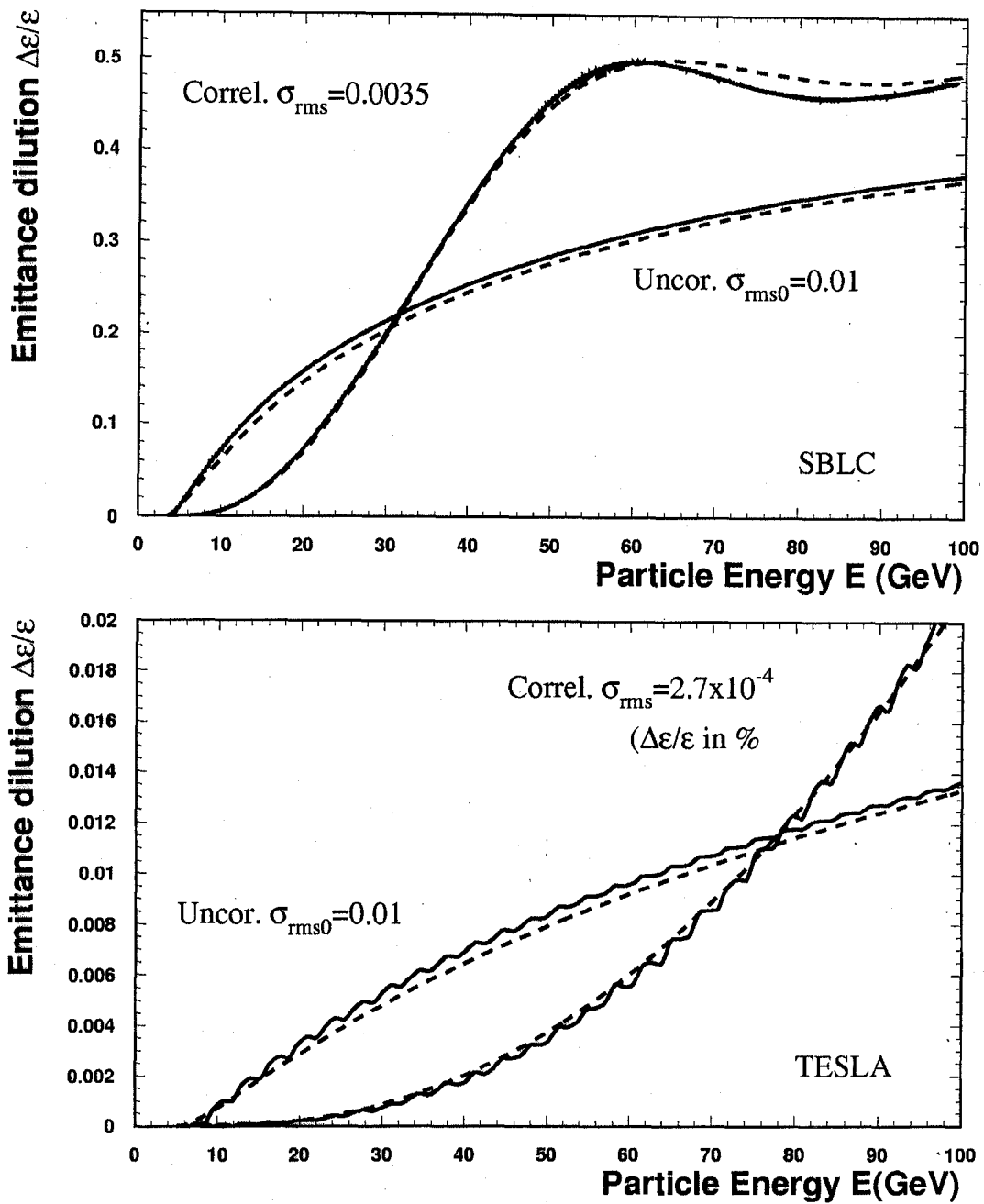


Fig.1 The dispersive emittance dilution caused by injection jitter, initial uncorrelated and actual correlated energy spread. The results of particle tracking (solid line) and analytical prediction (dashed line) for high dispersive SBLC (top) and low dispersive TESLA (bottom) machines.

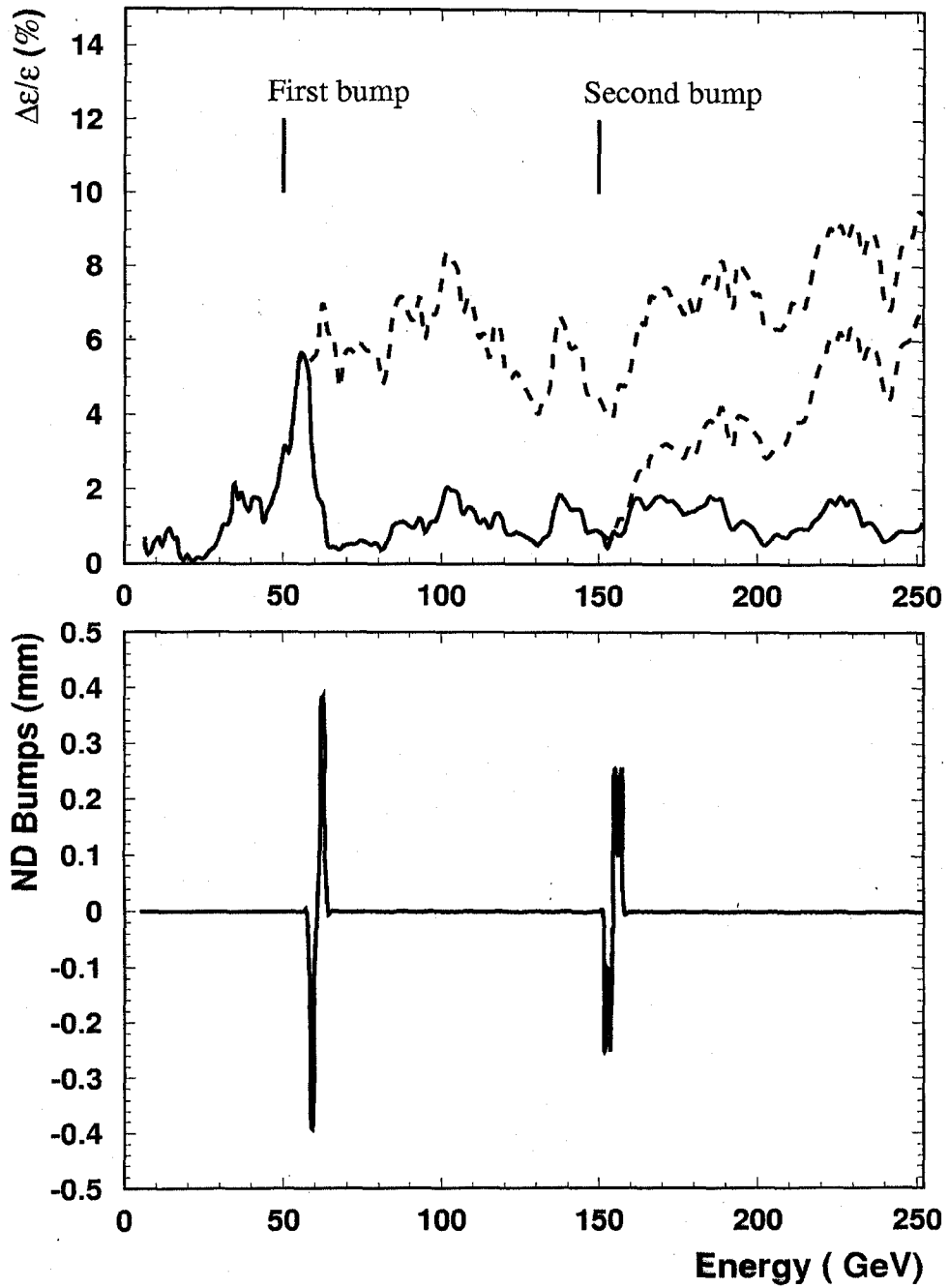


Fig. 17 The ND Bumps and the reduction of the emittance dilution by cavity misalignment. ($\sigma_{cav} = 0.5\text{mm}$).

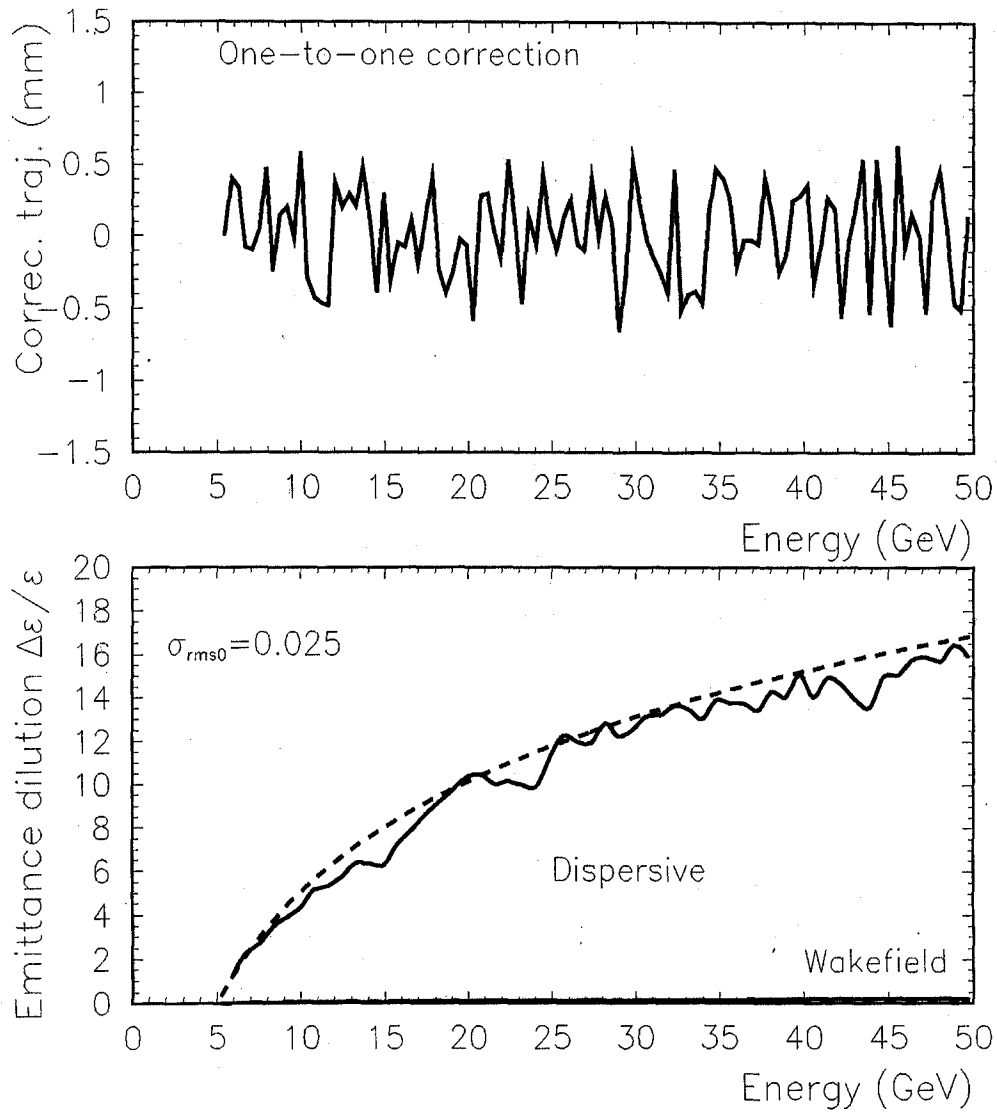


Fig. 18 The one-to-one corrected trajectory of the HEP beam in the TESLA main linac (top) and the rms dispersive and wakefield emittance dilution of the beam ($\sigma_q = 300\mu\text{m}$, $\sigma_b = 100\mu\text{m}$, $\sigma_r = 10\mu\text{m}$). The dashed line is the analytical prediction.

$$\sigma_{\text{quad}} = 300\mu\text{m}$$

$$\sigma_{\text{BPM}} = 100\mu\text{m}$$

$$\sigma_{\text{res}} = 10\mu\text{m}$$

DF Correction (T. Raubenheimer, ...)

BPM measurements for two energies (betatron phase advance)

$$\begin{aligned} & m_{1k} (r_{1k}, b_k, \theta_{1k}, E_1) \\ & m_{2k} (r_{2k}, b_k, \theta_{2k}, E_2) \end{aligned}$$

Difference orbit

$$\Delta m_k = [r_k(t_2) - r_k(t_1)] + \Sigma \theta_i [M_{12}^{(1)} - M_{12}^{(2)} E_1/E_2]$$

The trajectory equation

$$y_1(z) - \int_0^z K_2(z') y_1(z') M_{12}^{(2)}(z', z) dz' = \frac{E_2}{\Delta E} \bar{r}(z)$$

In smooth focusing model

$$y_1(z) - k_2 \int_0^z y_1(z') \sin[k_2(z - z')] dz' = \frac{E_2}{\Delta E} \bar{r}(z)$$

Trajectory in n-th quadrupole

$$y_n = \frac{E_2}{\Delta E} \left[\bar{r}_n + k_2 D^2 \sum_{k=1}^{n-1} (n-k) \bar{r}_k \right]$$

The rms trajectory grow

$$y_n \sim \frac{E}{\Delta E} \sigma_r \mu^2 N_{quad}^{3/2}$$

For multi-stage correction

$$y_n \sim \frac{E}{\Delta E} \sigma_r \mu^2 N_{quad}^{3/2} \frac{1}{m^{1/2}}$$

For TESLA

$$E/\Delta E = 1.4, \quad \mu = 60 \text{ degree}, \quad \sigma_r = 10 \mu\text{m}$$

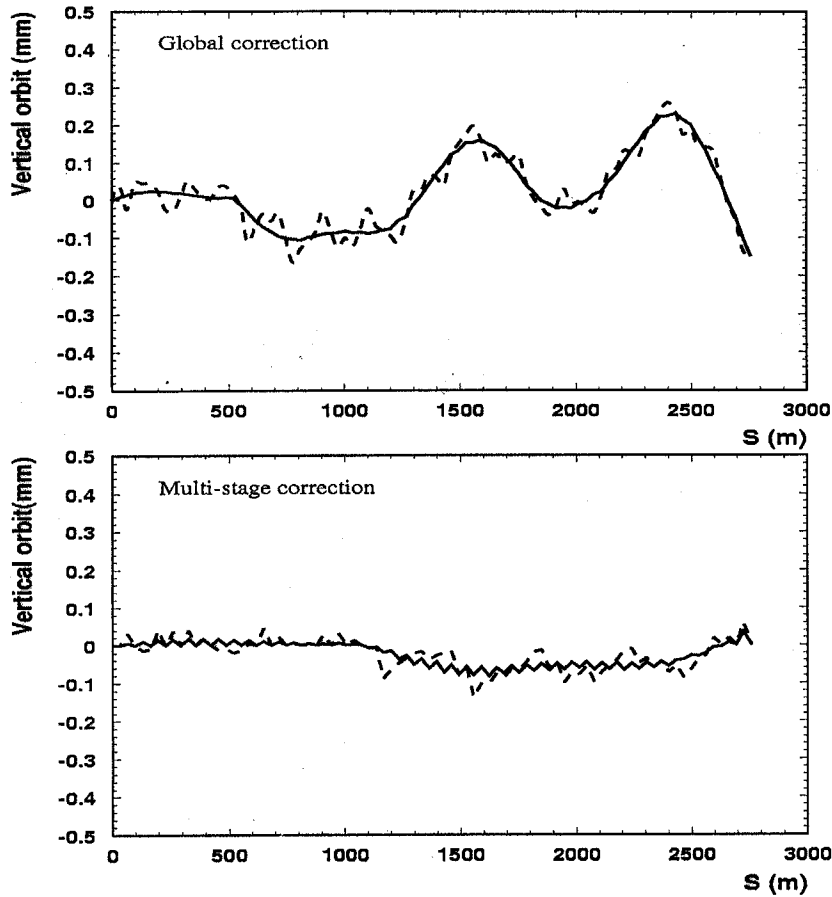
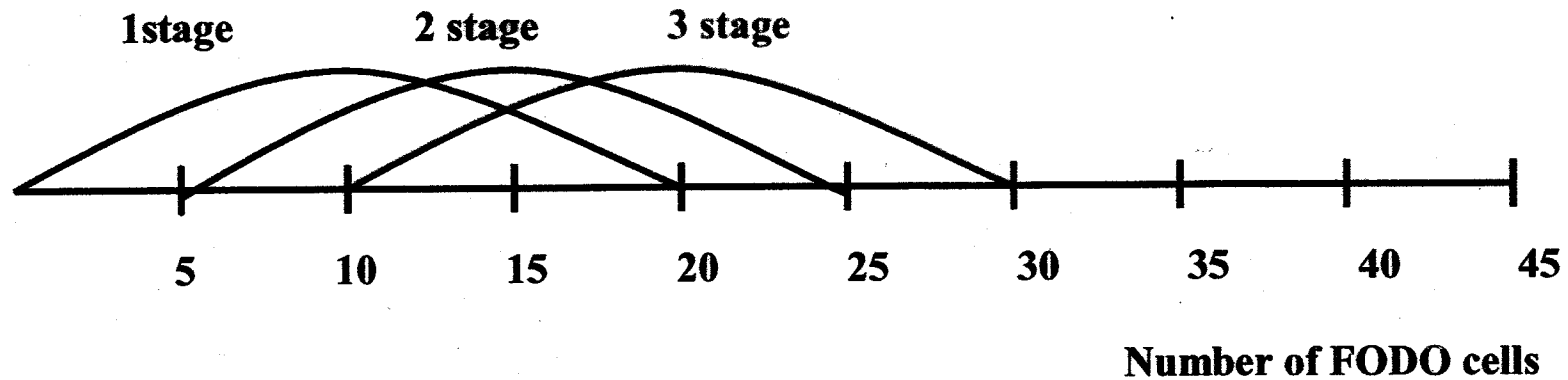


Fig. 19 Two beam based trajectory correction for TESLA. The trajectories of the HEP beam (solid line) and the FEL beam are shown after a global correction (top) and after performing a multi-stage correction (bottom). ($\sigma_q = 0.3\text{mm}$, $\sigma_b = 0.1\text{mm}$, $\sigma_r = 10\mu\text{m}$)

Minimization procedure

$$\sum_k \frac{(m_k + y_k)^2}{\sigma_r^2 + \sigma_b^2} + \frac{(\Delta m_k + \Delta y_k)^2}{2\sigma_r^2}$$

Multi- Stage DF correction



1-th stage - minimization over 1-20 cells → value of dipole correctors in 1-5 cells fixed

2-nd stage - minimization over 5-25 cells → value of dipole correctors in 6-10 cells fixed

3-d stage - minimization over 11-30 cells → value of dipole correctors in 6-10 cells fixed

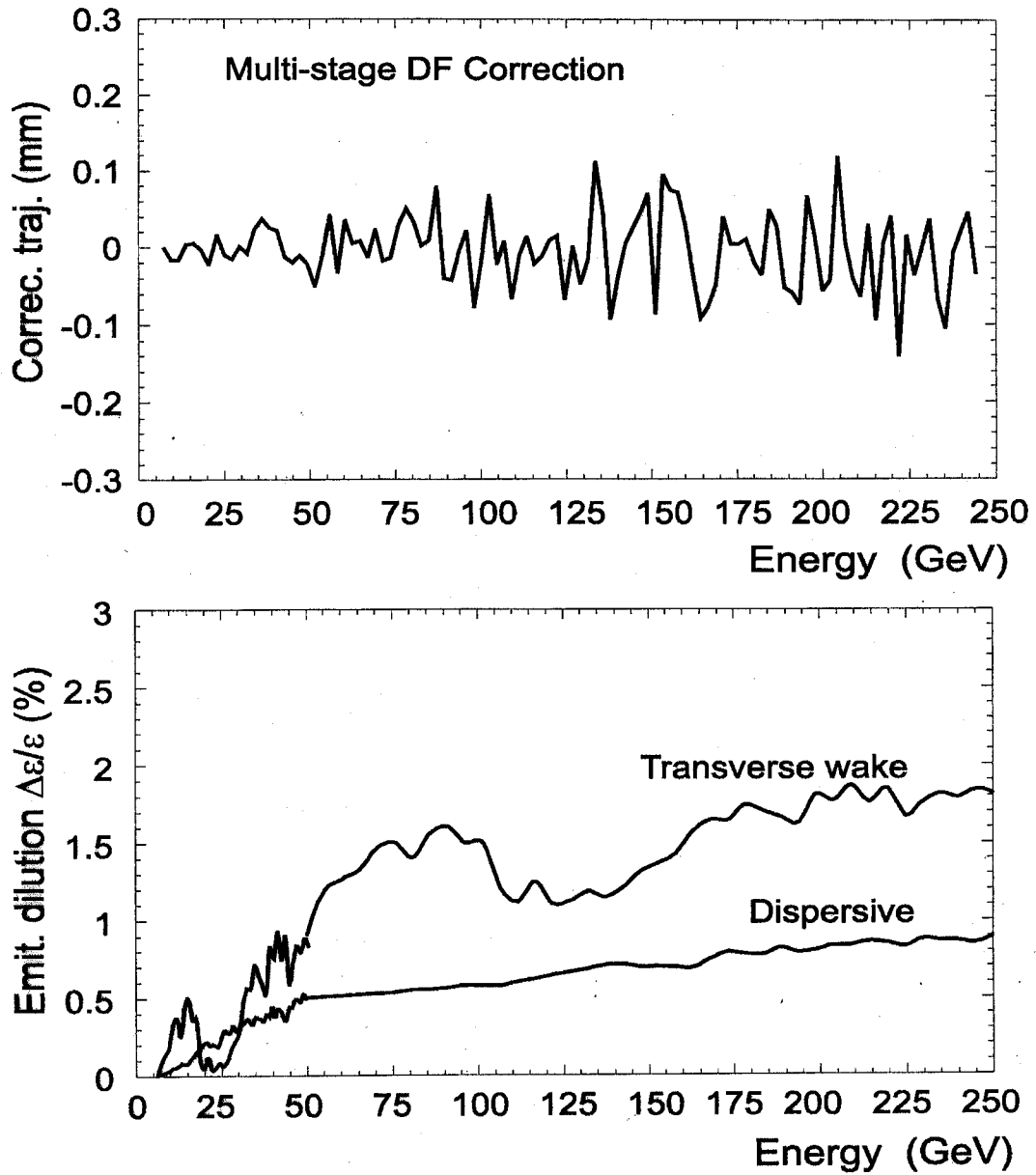
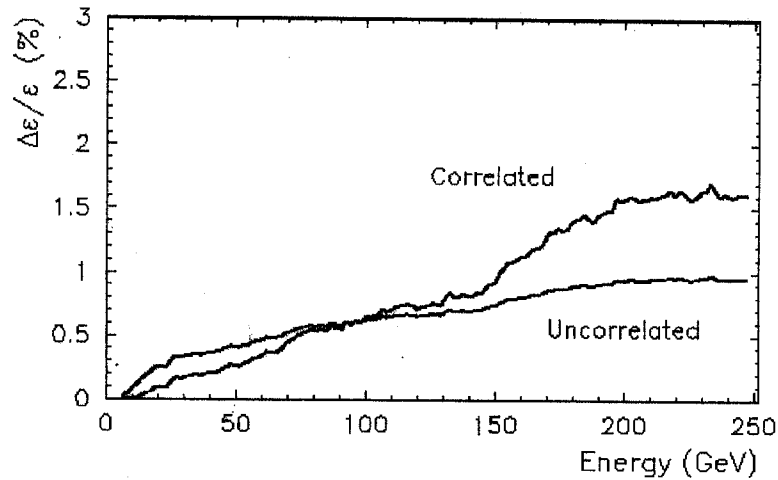


Fig. 21 Beam orbit (top) and emittance dilution (bottom) along the entire linac after a multi-stage dispersion free correction.

Beam Based Traject. Correction



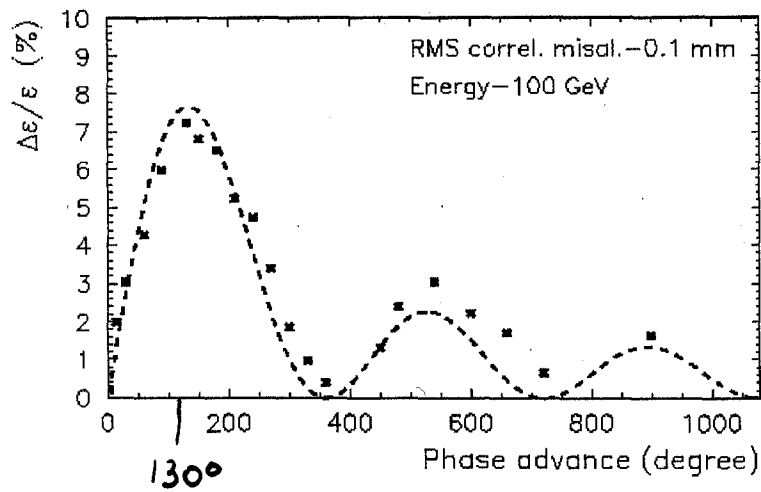
Drift:

$$\sigma(n) \sim \left(1 - \frac{M_1}{M_2}\right)^{-1} \sigma_r K_1^2 D^2 \frac{n^{3/2}}{N^{1/2}}$$

Two trajectories:

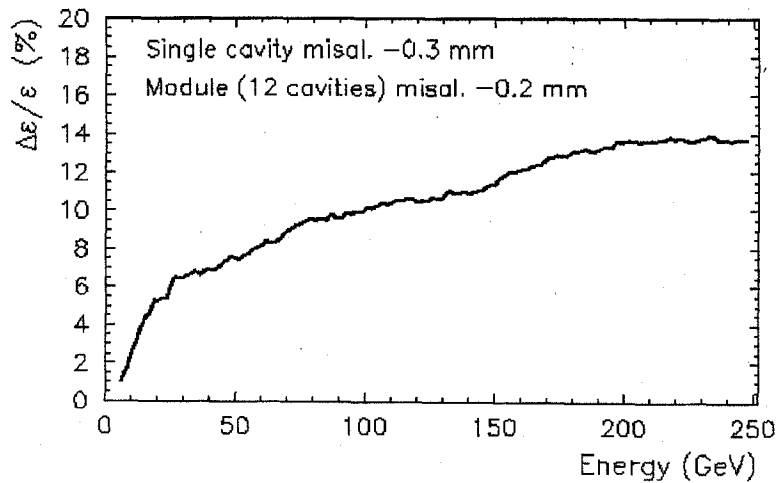
$$M_1 = 60^\circ ; M_2 = 90^\circ$$

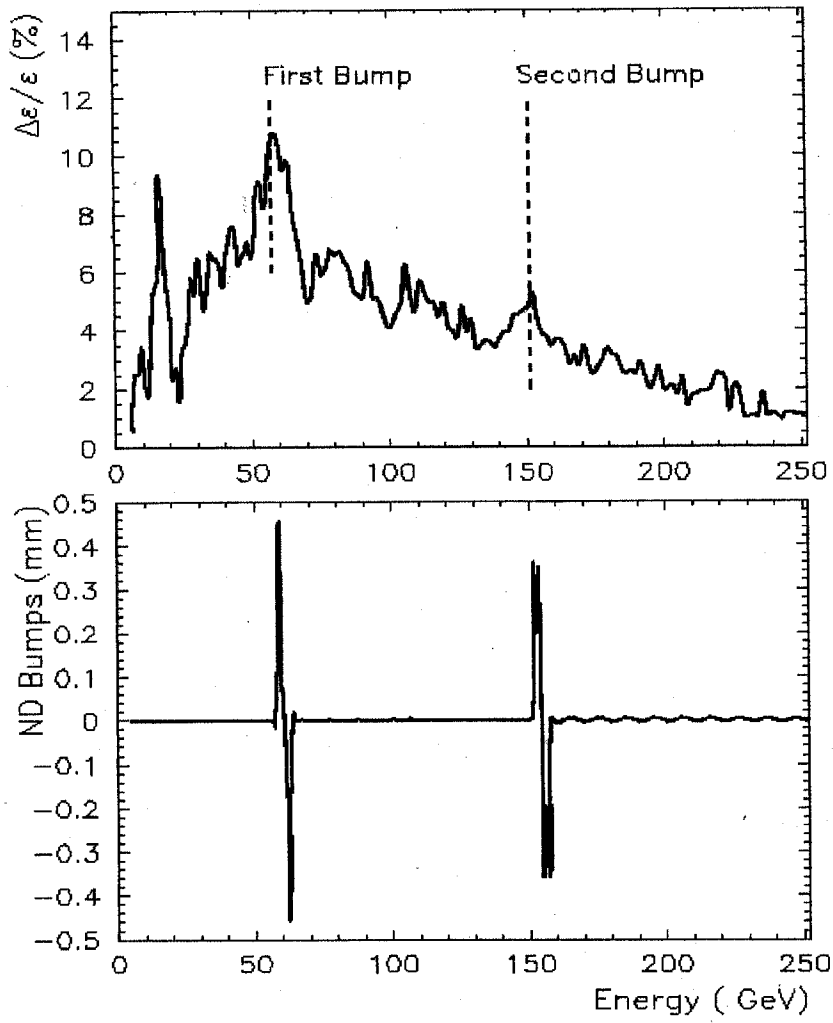
Cavities correlation



$$\frac{\Delta\varepsilon}{\varepsilon} = 2 \frac{\sigma_c^2}{\varepsilon_0} \left(\omega^2 \frac{\Delta\delta}{\delta_0} \frac{L_{col}}{\mu \sin \mu} \left(\frac{\sin^2 \phi/2}{\phi} \right) \cdot \ln \frac{\delta}{\delta_0} \right) \quad \text{--- correl. term}$$

Realistic alignment





Summary

Coherent oscillations' (15)

Uncorrel. emit. dilution - 6.5%

Correl. emit. dilution - 2.5%

Autophasing - 0.4%

Static misalignment

Quadrupoles

One-to-one correction - 1800%

Beam-based correction - 1.5-2%

Cavities & Modules

No correction - 14%

ND bumps' - 2-3%