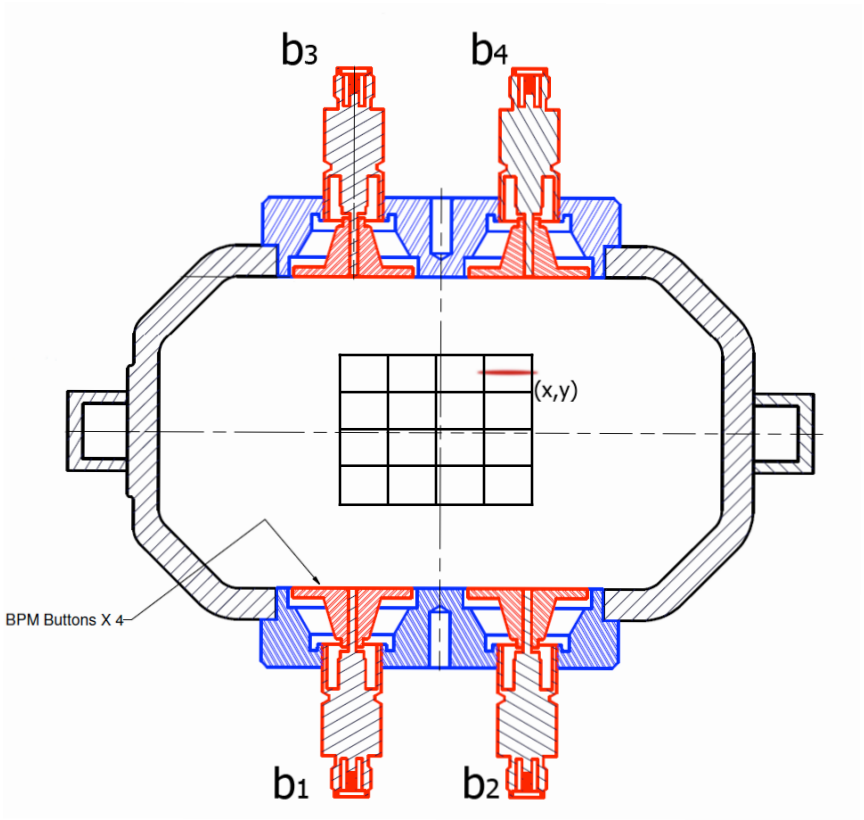

b3
b4
(x,y)
BPM Buttons X 4
b1
b2

Signal at each button depends on bunch current (k) and position (x,y)

$$B_1 = kf(x,y)$$

$$B_1 \approx k\left(f(0,0) + \frac{\partial f}{\partial x}x + \frac{\partial f}{\partial y}y + \frac{1}{2}\frac{\partial^2 f}{\partial x^2}x^2 + \frac{1}{2}\frac{\partial^2 f}{\partial y^2}y^2 + \frac{\partial^2 f}{\partial x \partial y}xy + \ldots\right)$$

$$B_1 \approx k(c_0 + c_1 x + c_2 y + c_3 x^2 + c_4 y^2 + c_5 xy)$$

Signals on the four buttons are related by symmetry

$$B_2 = kf(-x,y)$$
$$B_3 = kf(x,-y)$$
$$B_4 = kf(-x,y)$$

Combining sums and differences we find the following relationship, good to second order

$$B_1 - B_2 - B_3 + B_4 = \frac{1}{k}\left(\frac{c_5}{c_1 c_2}\right)(B_1 - B_2 + B_3 - B_4)(B_1 + B_2 - B_3 - B_4)$$

$$B(+--+) = \frac{c}{k}B(+-+-)B(++--)$$

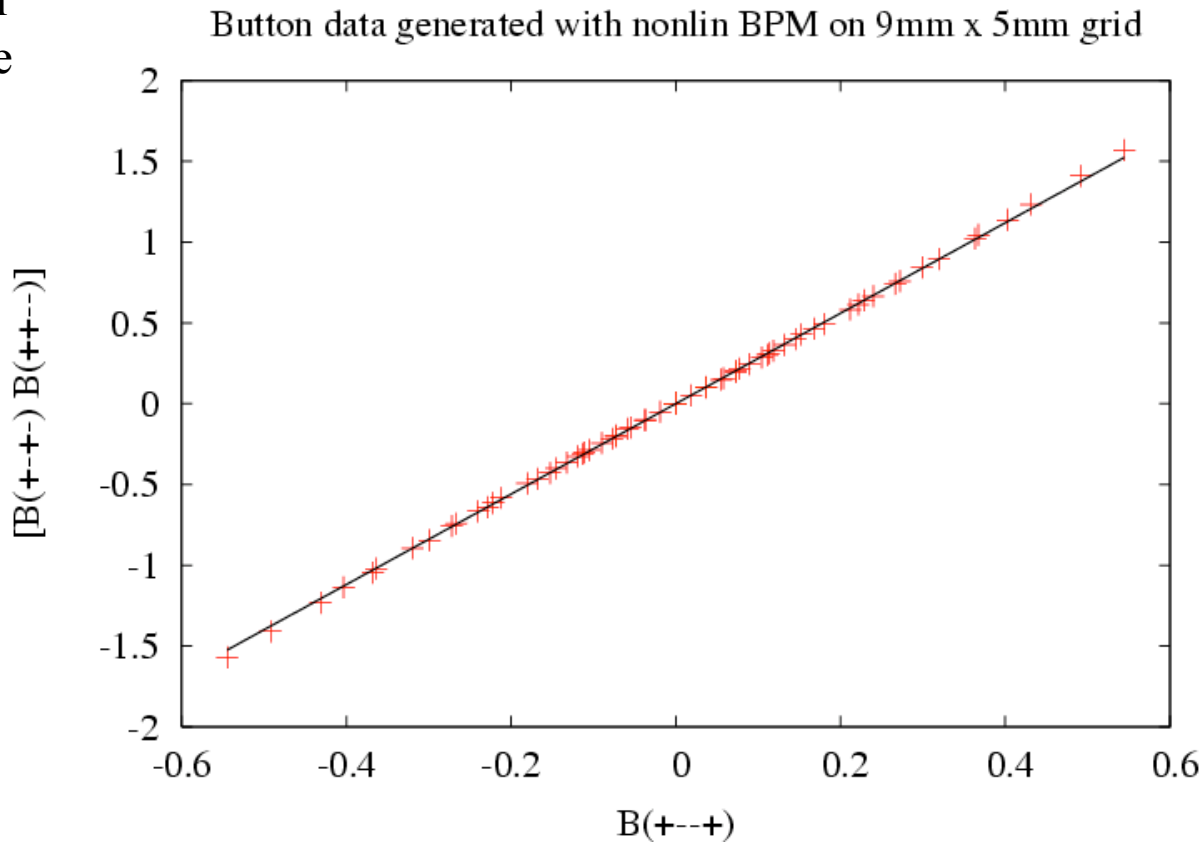
Simulation

$$B(+--+) = \frac{c}{k} B(+-+-) B(++--)$$

Using a map that reproduces the "exact" dependence of the button signals on the bunch positions we generate B_1,B_2,B_3,B_4 for each of 45 points on a 9mm x 5mm grid

In first order *c=0,* and therefore B(+--+) = 0. Evidently the first order approximation is not very good enough this range.

The small deviations from the straight line at large amplitudes is a measure of the higher than second order contributions.

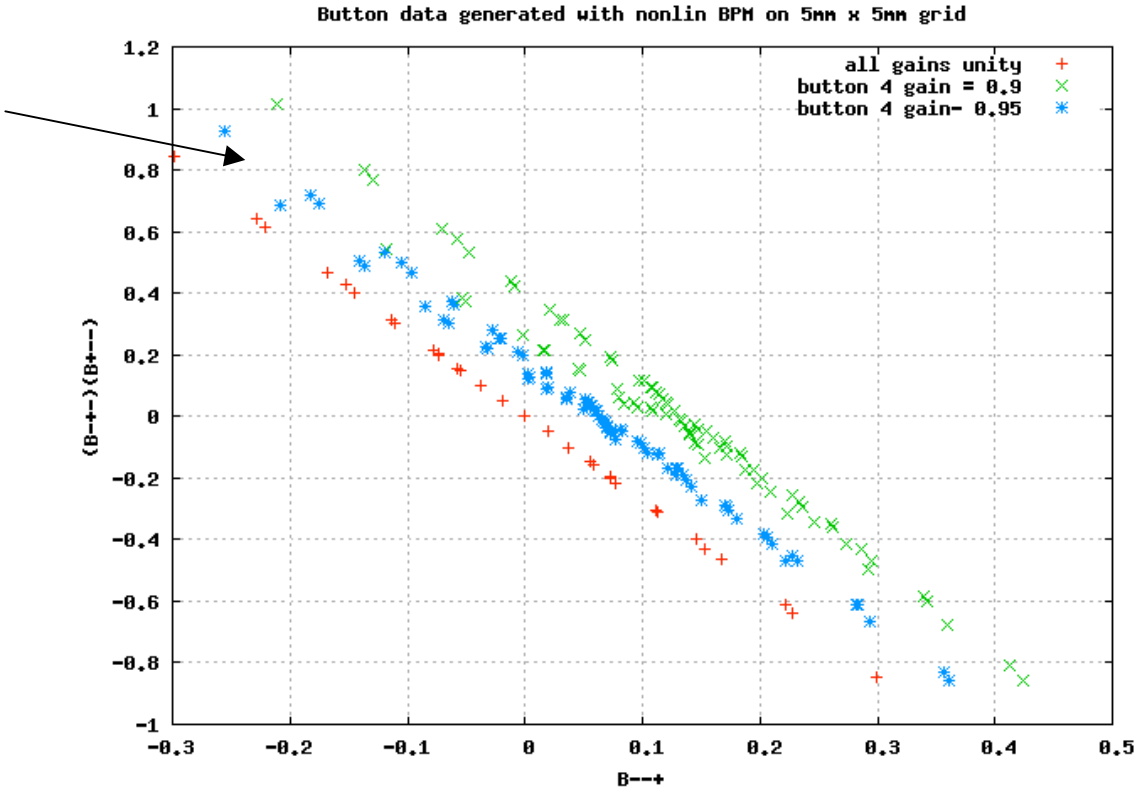


Simulation with gain errors

$$B(+-\,-+) = \frac{c}{k} B(+-\,+-) B(++\,--)$$

Introduce gain errors

Zero offset, nonlinearity, and multi - valued relationship i n is a measure of gain errors.

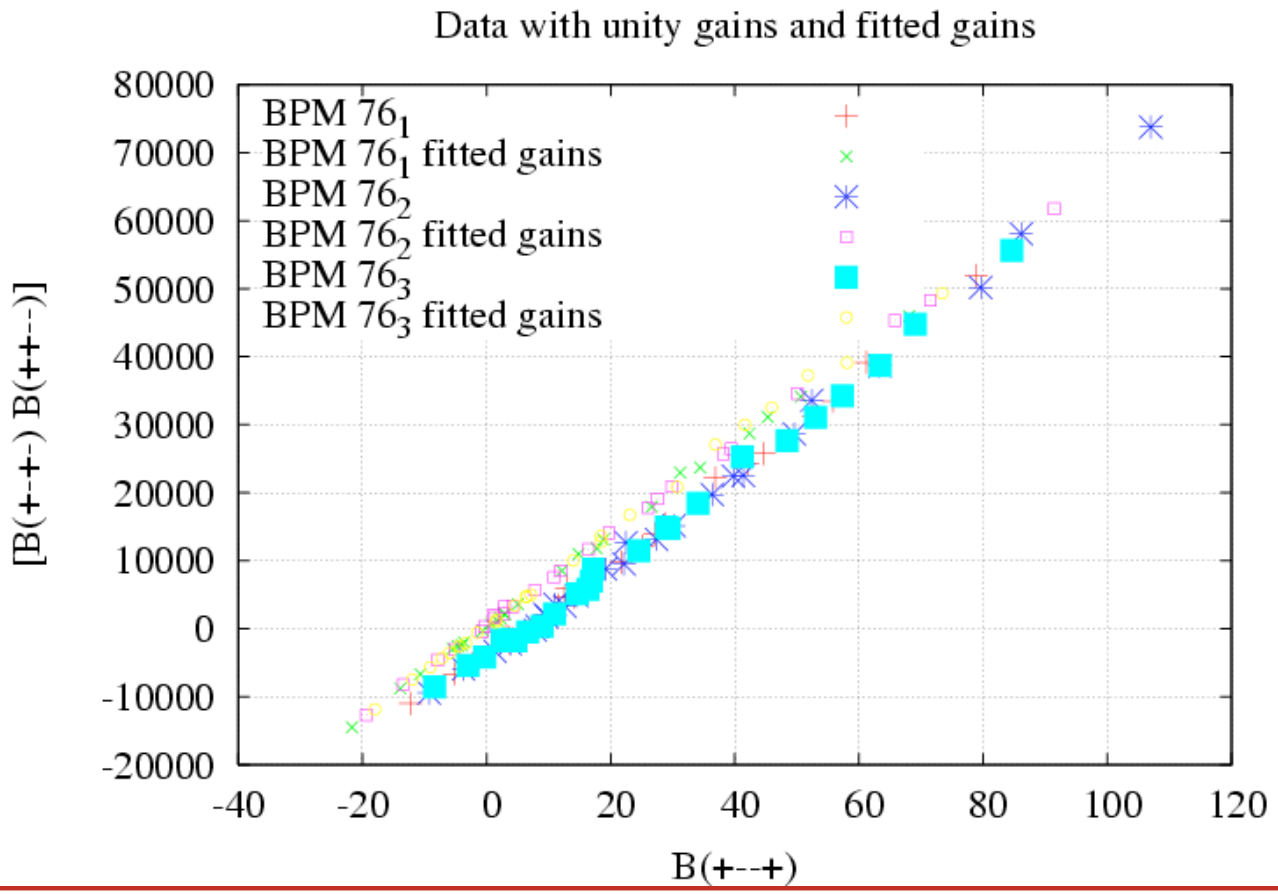


Orbit data collected on a grid

To fit for gains

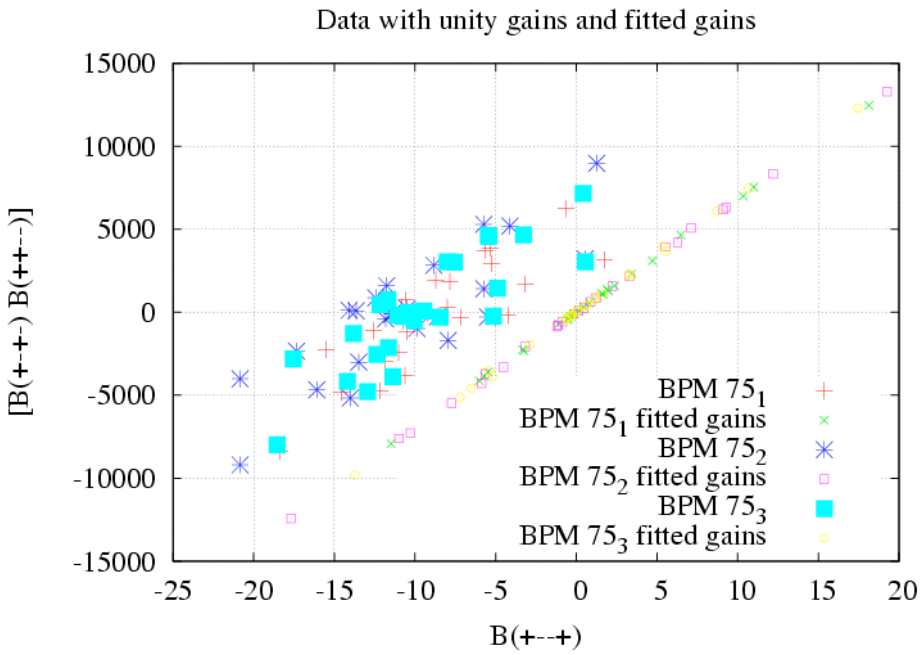
Fix g_1=1, and minimize with respect to g_2,g_3,g_4,c

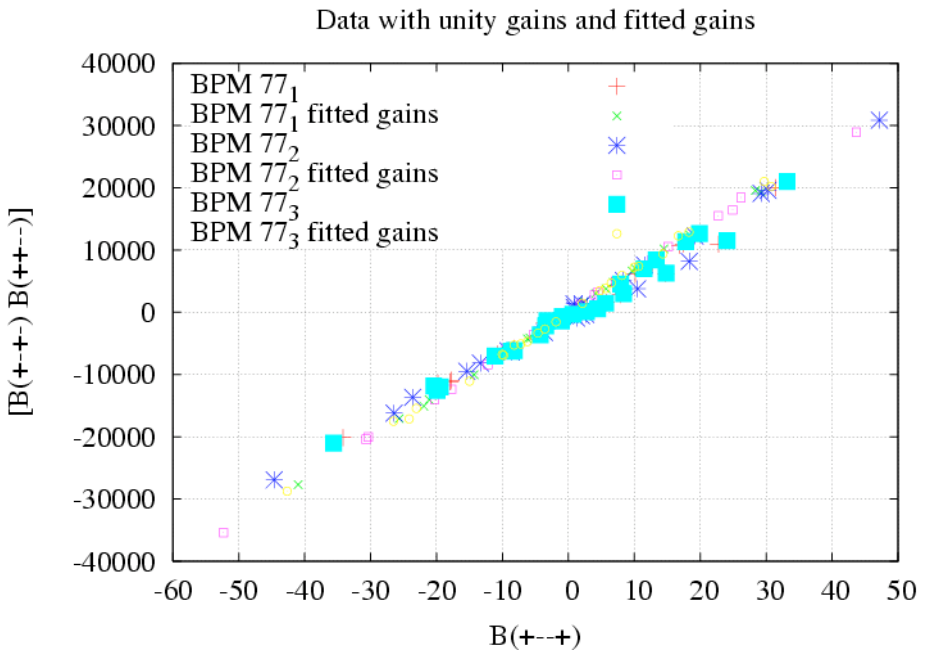
$$\sum_i \left[(g_1 B_1^i - g_2 B_2^i - g_3 B_3^i + g_4 B_4^i) - \frac{c}{I}(g_1 B_1^i - g_2 B_2^i + g_3 B_3^i - g_4 B_4^i)(g_1 B_1^i + g_2 B_2^i - g_3 B_3^i - g_4 B_4^i)\right]^2$$



Fitted gains = 1,0.95,0.96,0.97

Orbit data collected on a grid



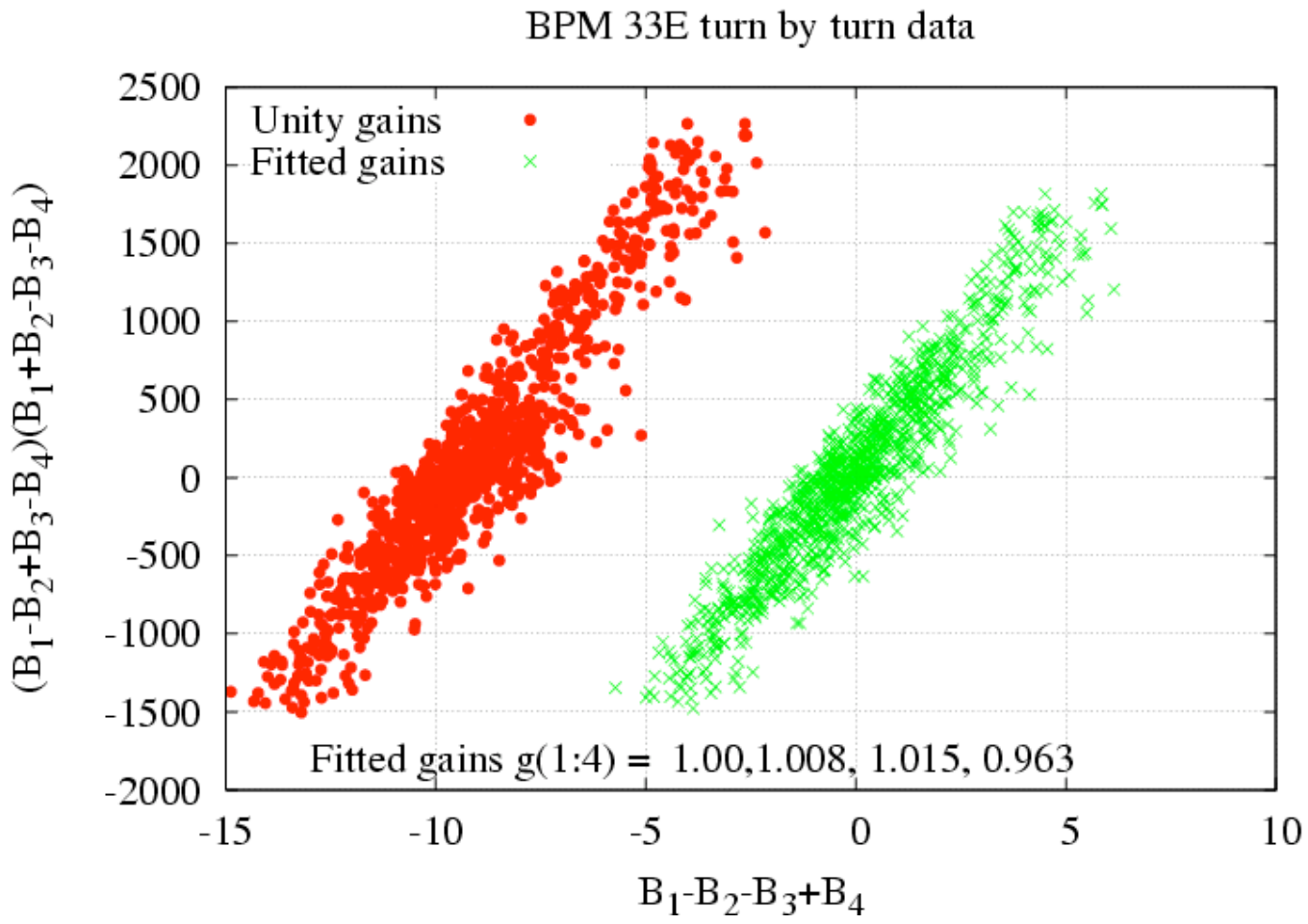


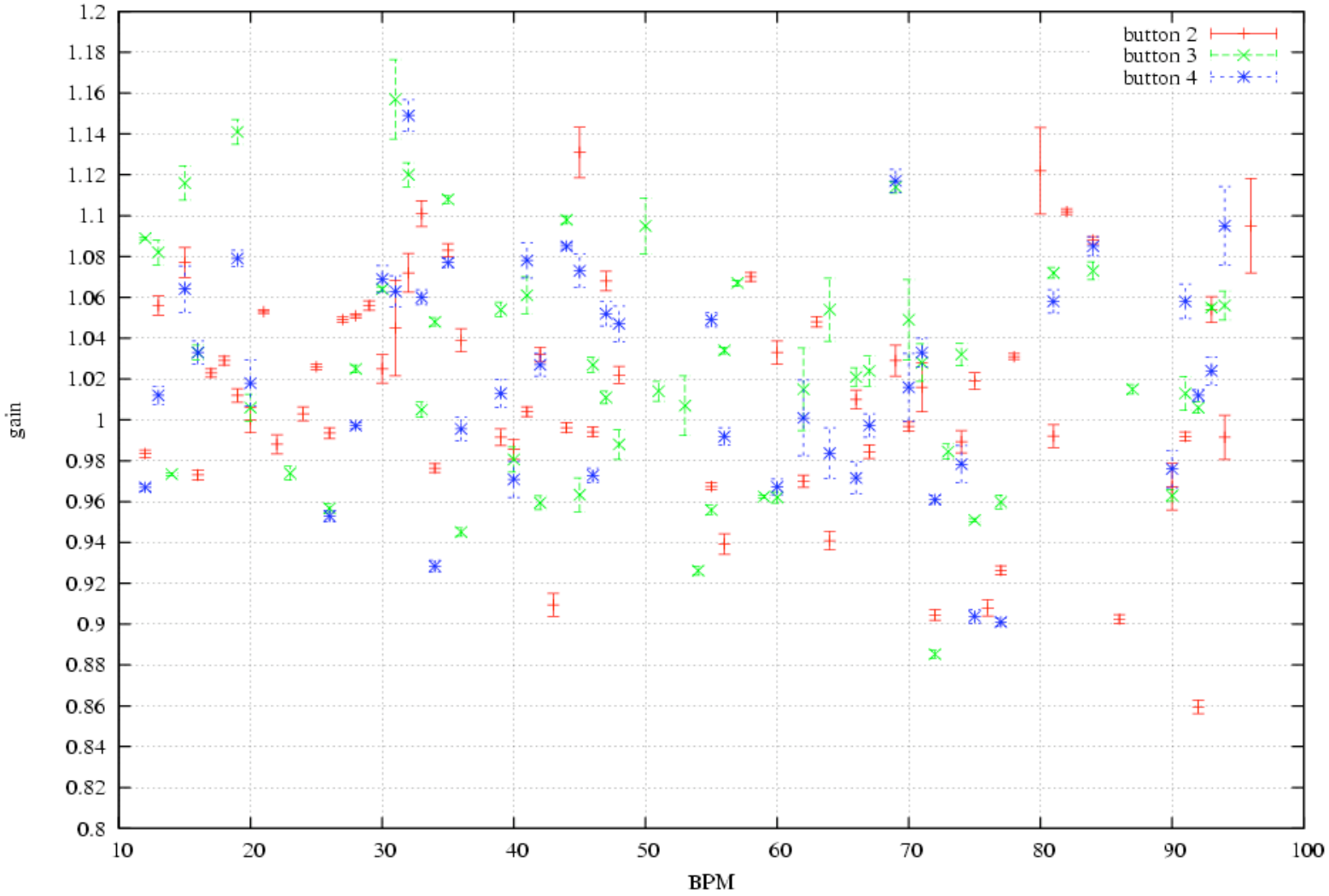
BPM 75 -
fitted gain = 1,1.02,0.96,0.91

BPM 77 -
fitted gain = 1,0.92,0.96,0.9

Fit typically reduces χ^2 by two orders of magnitude

Turn by turn data





Average gains computed for 5 turn by turn data sets

Normalized to unity gain on button 1

Error bar is the standard deviation of the 5