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THE MUON $(g-2)$ EXPERIMENTS

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INTRODUCTION

It is now 21 years since a group of experimental physicists at CERN under Leon Lederman started to study the problem of the muon g-factor. The magnetic moment is $g(e/2mc)(\hbar/2)$, where g is a dimensionless number. Since then, a number of measurements have been performed with higher and higher accuracy. At the same time a great deal of theoretical effort has been deployed to determine the theoretical value of g. If the muon obeys the simple Dirac equation for a particle of its mass (206 times heavier than an electron), then $g = 2$ exactly; but this is modified by the quantum fluctuations in the electromagnetic field around the muon, as specified by the rules of quantum electrodynamics (QED), making g

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larger by about 1 part in 800. It requires further correction for the very rare fluctuations, which include virtual pion states and strongly interacting vector mesons. At present, theory and experiment agree at the level of 1 part in 10^8, and the muon g-factor, together with that of the electron, R_∞, c, and the frequency of the hydrogen maser, are the most accurately known constants of nature.

A number of reviews both theoretical (Lautrup et al 1972, Calmet et al 1977, Kinoshita 1978) and experimental (Farley 1964, 1968, 1975, Picasso 1967, Bailey & Picasso 1970, Combley & Picasso 1974, Field et al 1979) have already been published. In this article we give an overview of the programme as a whole, trying to set each measurement in its historical perspective, in order to show how one developed from another, and to relate each to the contemporary thinking about the muon and about quantum electrodynamics.

THEORY

The gyromagnetic ratio is increased from its primitive value of 2, arising from the Dirac equation, to $g = 2(1+a_\mu)$, where $a_\mu \equiv (g-2)/2$ is defined as the anomalous magnetic moment or anomaly. In the QED theory the

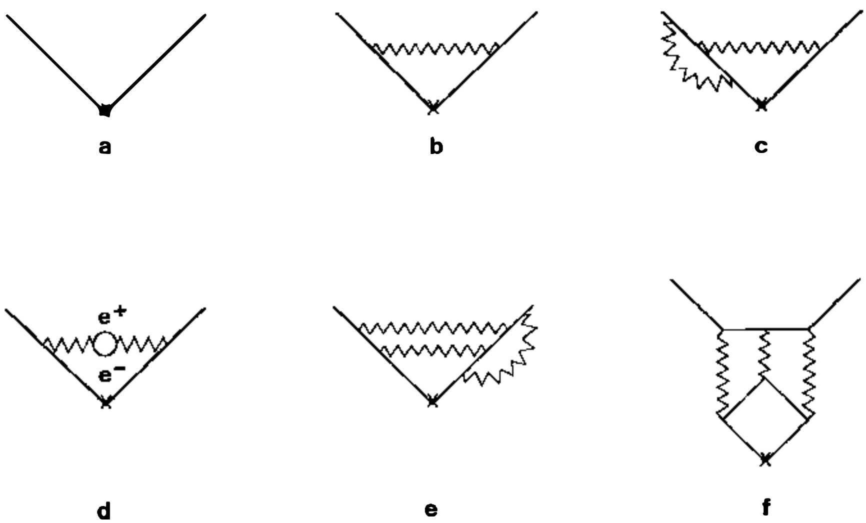

Figure 1 Feynman diagrams used in calculating a. The solid line represents the muon, which interacts with the laboratory magnetic field at X. The zigzag line represents a virtual photon, which is emitted and later reabsorbed. In (d) and (f) an e^+e^- pair, created and then annihilated, gives rise to the closed loop (*solid line*).

contributions to the anomalous moment are expressed as a power series in α/π

$$a_\mu^{\rm th} = A(\alpha/\pi) + B(\alpha/\pi)^2 + C(\alpha/\pi)^3 + \dots \qquad 1.$$

Typical Feynman diagrams, which contribute to the calculation of the theoretical value of a for the electron and muon, are shown in Figure 1, while a complete set of diagrams up to sixth order [terms in $(\alpha/\pi)^3$] is given by Lautrup et al (1972). These have all been calculated analytically, except for diagrams such as Figure 1*f*, including an electron loop with four electromagnetic vertices, which have been found by numerical integration. (Such diagrams are also involved in the scattering of light by light.) The coefficients in the expansion are listed in Table 1, together with rough estimates for the eighth- and tenth-order terms.

So far only the change in the gyromagnetic ratio due to the interaction of a particle with its own electromagnetic field has been mentioned. Any other field coupled to the particle should produce a similar effect, and the calculations have been made for scalar, pseudoscalar, vector, and axial-vector fields, using a coupling constant f (assumed small) to a boson of mass M (Berestetskii et al 1956, Cowland 1958). For example, for a vector field

$$\Delta a_\mu^{\rm V} = \left(\frac{1}{3\pi}\right)\left(\frac{f^2}{M^2}\right) m_\mu^2. \qquad 2.$$

A precise measurement of a_μ could therefore reveal the presence of a new field, but first all known fields, including the weak and the strong inter-

Table 1 Summary of theoretical contributions[a] to a_μ

QED terms	Muon	Numerical values ($\times 10^9$)	
2nd order: A	0.5	Total QED:	1 165 852 (1.9)
4th order: B	0.765 782 23	Strong interactions:	66.7 (8.1)
6th order: C	24.452 (26)	Weak interactions:	2.1 (0.2)
8th order: D	135 (63)	Total theory:	1 165 921 (8.3)
10th order: E	420 (30)		

[a] Whereas the first two terms are obtained exactly from analytical results, in the sixth order (α^3) there remain some diagrams that must be calculated by numerical integration implying a small residual error in the coefficient (Cvitanovic & Kinoshita 1974, Barbieri & Remiddi 1975, Calmet & Petermann 1975a, Levine et al 1976, Levine & Roskies 1976, Samuel & Chlouber 1976, Lautrup & Samuel 1977). The α^4 and α^5 values are estimated by inserting electron loops in the sixth-order diagrams (Lautrup 1972, Lautrup & de Rafael 1974, Samuel 1974, Calmet & Petermann 1975b, Chlouber & Samuel 1977). The figures in parentheses following any figure indicate the estimated error in the final digits.

actions, must be taken into account. Strongly interacting particles do not couple directly to the muon, but if they are charged, they couple to the photon. Thus they can appear in the inner loops such as Figure 1*d*, with, for example, a pion pair replacing the e^+e^- pair. Because of the high mass of the pion, one would initially expect such amplitudes to be small, but there are strong resonances in the $\pi^+\pi^-$ system that enhance the effect. Only a vector resonance can contribute, because it alone can transform directly into the virtual photon that must have $J^{PC} = 1^{--}$ (one unit of angular momentum, negative parity, and negative charge conjugation).

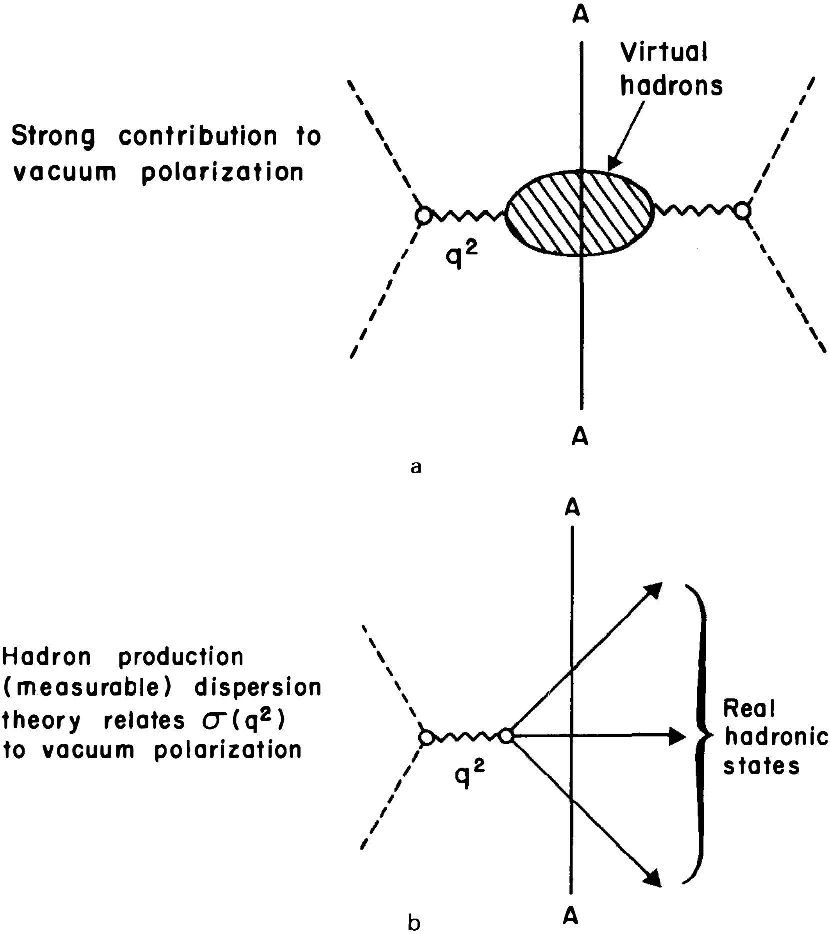

Figure 2 The photon propagator is modified by the creation of virtual hadrons (*a*). This is related by dispersion theory to real hadron production in e^+e^- collision (*b*).

To calculate this contribution it is necessary to specify the overall probability amplitude for a photon of a given q^2 to connect the two muon vertices shown in Figure 1*b*, with the effect of virtual hadron loops fully included. That is, one requires the propagator function of Figure 2*a*.

This cannot be calculated from theory, because not enough is known about hadrons. But fortunately the propagator Figure 2*a* can, in principle, be cut in half to obtain that of Figure 2*b*, which shows an e^+e^- pair annihilating to give real hadronic states. By using dispersion theory, the cross section for Figure 2*b* as a function of (total energy)^2, that is $\sigma(e^+e^- \rightarrow \text{hadrons})(t)$, can be related (Bouchiat & Michel 1961) to the propagator shown in Figure 2*a* and so to the anomalous moment arising from Figure 1*d* with hadron loops

$$\Delta a_\mu(\text{hadrons}) = (m_\mu^2/4\pi^3) \int_0^\infty \sigma(e^+e^- \rightarrow \text{hadrons})(t)\, g(t)\, dt, \qquad 3.$$

where $t = q^2$, and

$$g(t) = \int_0^1 \frac{x^2(1-x)\, dx}{m_\mu^2 x^2 + t(1-x)} \rightarrow \frac{1}{3t} \quad \text{at large } t. \qquad 4.$$

The process $e^+e^- \rightarrow$ hadrons has been extensively studied in electron-positron colliding beams and the cross section is rather well defined from near threshold up to about 3-GeV center-of-mass energy. Therefore the integration in Equation 3 can be carried out with fair confidence, the result being Δa_μ
et al 1976). This term is about 8 times the present experimental accuracy and its presence and order of magnitude have been confirmed.

The contribution of 4-fermion weak interaction is illustrated in Figure 3*a*. This is second order in the weak interaction and turns out to be negligibly small ($\sim 10^{-12}$). However, if the weak interaction is mediated by a charged intermediate boson $W^\pm$, the mechanism shown in Figure

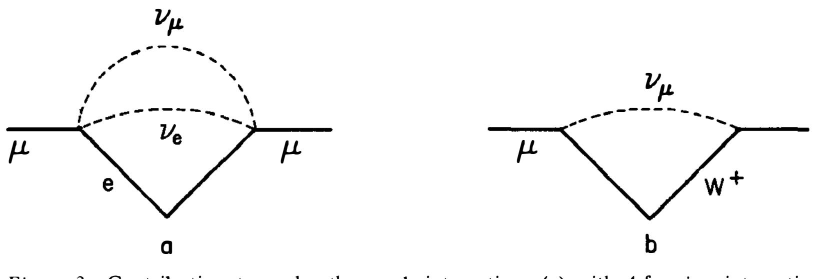

Figure 3 Contribution to a_μ by the weak interaction: (*a*) with 4-fermion interaction; (*b*) by the virtual production of an intermediate boson W^+.

3*b* will contribute. The new renormalizable theory of weak interactions then leads to $\Delta a_\mu(\text{weak}) \sim 2 \times 10^{-9}$ (Bardeen et al 1972, Bars & Yoshimura 1972, Fujikawa et al 1972, Jackiw & Weinberg 1972, Primack & Quinn 1972). This is a small effect, at present masked by the uncertainty in the strong interaction contribution, and indeed in the sixth-order QED term; so it is unlikely to be detected.

Finally we must consider the effect on a_μ^{th} of a modification of QED. If the muon is not completely point-like in its behavior, but has a form factor $F(q)^2 = \Lambda_\mu^2/(q^2+\Lambda_\mu^2)$, it can be shown that

$$\frac{\Delta a_\mu}{a_\mu} = \frac{-4m_\mu^2}{3\Lambda_\mu^2}, \qquad 5.$$

implying, for example, a reduction in a_μ of 24 parts per million (ppm) if $\Lambda_\mu = 25$ GeV/*c*. Similarly a modification[1] of the photon propagator by the factor $\Lambda_\gamma^2/(q^2+\Lambda_\gamma^2)$ implies

$$\frac{\Delta a_\mu}{a_\mu} = \frac{-2m_\mu^2}{3\Lambda_\gamma^2}. \qquad 6.$$

This result was first obtained by Berestetskii et al (1956), who emphasized the value of experiments on the muon; the high mass m_μ implies a significant correction to a_μ even when Λ_γ is large.

The theoretical predictions are summarized in Table 1, based on $\alpha^{-1} = 137.035\,987\,(29)$ (Cohen & Taylor 1973, Hansch et al 1974). The figures in brackets following any value indicate the estimated error on the last two digits. We must emphasize that the figures relate not to *g* but to the small anomaly a_μ, a quantity that would be zero in the absence of quantum fluctuations.

We now turn to the experimental problem of measuring the anomaly for the muon.

$(g-2)$ *Precession*

Let us first consider a particle, longitudinally polarized, moving at slow speed in a uniform static magnetic field. The momentum vector turns at the cyclotron frequency f_c with

$$2\pi f_c = \frac{eB}{mc}, \qquad 7.$$

while the spin precession frequency is the same as for a particle at rest,

[1] The violation of unitarity can be circumvented by introducing a negative metric (Lee & Wick 1969).

$$2\pi f_s = \frac{2\mu B}{h} = g\left(\frac{eB}{2mc}\right) = (1+a_\mu)\left(\frac{eB}{mc}\right). \quad 8.$$

If $g = 2$, then $f_s = f_c$ and the particle will always remain longitudinally polarized. But if $g > 2$ as predicted, the spin turns faster than the momentum vector. The laboratory rotation frequency f_a of the spin relative to the momentum vector is given by

$$2\pi f_a = 2\pi(f_s - f_c) = a\left(\frac{e}{mc}\right)B. \quad 9.$$

This is the basic equation for the $(g-2)$ experiments: if the particle is kept turning in a known magnetic field B, and the angle between the spin and the direction of motion is measured as a function of time t, then a can be determined. The value of (e/mc) is obtained from the precession frequency of muons at rest (Equation 8). In fact, for the same magnetic field B, one has $f_a/f_s = a_\mu/(1+a_\mu)$. In practice the fields are not the same in the two experiments, but are measured by proton magnetic resonance, and a proportional correction is applied.

Note that this gives a measure of the anomaly $a_\mu \equiv (g-2)/2$ instead of g itself; so the correction due to quantum fluctuations is measured directly. As $a_\mu \sim 1/800$, it follows from Equations 7 and 9 that the particle must take 800 turns in the field for the spin to make 801 turns, that is for the polarization to change gradually through 360°. Clearly, if this is to be measured with any accuracy, the particle should make thousands of turns in the field, so that several cycles of the anomalous precession can be studied: the more cycles it is possible to record, the more accurate will be the measurement of frequency.

The fundamental formula (Equation 9) has been derived only in the limit of low velocities but it proves to be exactly true at any speed. This was demonstrated by Mendlowitz & Case (1955) and Carrassi (1958) using the Dirac equation, and by Bargmann et al (1959) using a covariant classical formulation of spin motion. Other treatments have been given by Farley (1968) and Fierz & Telegdi (1970), and reviewed by Farago (1965). Note that the $(g-2)$ precession is not slowed down by time dilation even for high velocity muons.

PART I: 1958–1962

By 1958, QED was an established theory of some 10 years standing, corroborated by accurate measurements of the Lamb shift. The g-factor of the electron was known through electron spin resonance (Franken & Liebes 1956) to one part per million (ppm); Karplus & Kroll (1950) had shown how to calculate the higher order corrections to g and a numerical error in their results had recently been corrected by Petermann (1957a,b), Sommerfield (1957), and Suura & Wichmann (1957), bringing theory into line with the experiment at the level of $(\alpha/\pi)^2$.

For the free electron a direct determination of the anomalous magnetic moment $a_e \equiv (g-2)/2$ was in progress at the University of Michigan (Nelson et al 1959, Schupp et al 1961) using the recently discovered principle of $(g-2)$ spin motion explained above. Equation 9 had been proved to hold for relativistic velocities.

Turning to the muon, the bremsstrahlung cross section at high energies had been measured with cosmic rays and shown to agree with a spin assignment of $\frac{1}{2}$ rather than $\frac{3}{2}$ (Mathews 1956, Mitra 1957, Hirokawa & Komori 1958). A similar conclusion followed from data on neutron production by cosmic-ray muons (de Pagter & Sard 1960). The angular distribution of electrons from the decay of polarized muons agreed with spin $\frac{1}{2}$ (Bouchiat & Michel 1957) and was inconsistent with spin $\frac{3}{2}$ (Brown & Telegdi 1958). Experiments with cosmic-ray and accelerator-generated muons were in progress to compare the electro-magnetic scattering of muons and electrons by nuclei.

Thus evidence was accumulating that the muon behaves as a heavy electron of spin $\frac{1}{2}$. Berestetskii et al (1956) had emphasized that QED theory implied an anomalous magnetic moment a_μ for the muon of the same order as for the electron, but as the typical invariant momentum transfer involved was $q^2 \sim m^2$ an experiment for the muon would test the theory at much shorter distances. Feynman (1962) felt that the divergences in QED could be limited by a real energy-momentum cutoff Λ, and it seemed reasonable to expect Λ to be of the order of the nucleon mass. This would imply a 0.5% effect in a_μ. On the other hand, it was thought (Schwinger 1957) that the muon should have an extra interaction that would distinguish it from the electron and give it its higher mass. This could be a coupling to a new massive field, or some specially mediated coupling to the nucleon. Whatever the source, the new field should have its own quantum fluctuations, and therefore give rise to an extra contribution to the anomalous moment a_μ. The $(g-2)$ experiment was recognized as a very sensitive test of the existence of such fields, and potentially a crucial signpost to the μ-e problem.

At this stage there was no prospect of such an experiment, but in 1957 parity violation was discovered (Lee & Yang 1956, Wu et al 1957), muon beams were found to be highly polarized, and better still it was found that the angular distribution of the decay electrons could indicate the spin direction of the muon as a function of time (Garwin et al 1957, Friedman & Telegdi 1957). A wide variety of muon precession and spin-resonance experiments were carried out in the next few years (for reviews see Feinberg & Lederman 1963, Farley 1964). The ($g-2$) principle was invoked in the first paper on muon precession by Garwin et al (1957), who pointed out that g must be within 10% of 2.00, because although the muon trajectory had been deflected through 100° by the cyclotron magnetic field the muon polarization was still longitudinal.

The possibility of a ($g-2$) experiment for muons was envisaged, and groups at Berkeley, Chicago, Columbia, and Dubna started to study the problem (Panofsky 1958). If the muon had a structure that gave a form factor less than one for photon interactions, the value of a_μ should be less than predicted. Compared to the measurement for the electron, the muon ($g-2$) experiment was much more difficult because of the low intensity, diffuse nature, and high momentum of available muon sources. This implied large volumes of magnetic field; the lower value of (e/mc) made all precession frequencies 200 times smaller, but the time available for an experiment was limited by the decay lifetime, 2.2 μs. Hence large magnetic fields would be needed to give a reasonable number of precession cycles.

One solution was to scale up the method used at Ann Arbor for the electrons, using a large solenoid and injecting the muons spirally at one end (Schupp et al 1961). This was pursued at Berkeley and finally led to a 10% measurement (Henry et al 1969); see Table 2.

At CERN the work centered on the belief that it should be possible to store muons in a conventional bending magnet with a more or less uniform vertical field between roughly rectangular pole pieces. In a typical field of 1.5 T the muon orbit would make 440 turns during the lifetime of 2.2 μs. As $a_\mu \sim \alpha/2\pi \sim 1/800$, the angle between the spin and the momentum vector would develop 800 times more slowly, giving a change in beam polarization of about 180° to be studied.

The polarized muon beam from the CERN cyclotron could fairly easily be trapped inside a magnet. The particles were aimed at an absorber in the field; they lost energy and therefore turned more sharply and

Table 2 Experimental results[a] for a_μ

Charpak et al 1961a	μ^+	0.001 145 (22)
Charpak et al 1962, 1965	μ^+	0.001 162 (5)
Farley et al 1966	μ^-	0.001 165 (3)
Henry et al 1969	μ^+	0.001 060 (67)
Bailey et al 1968, 1972	$\mu^\pm$	0.001 166 16 (31)
Bailey et al 1975	μ^+	0.001 165 895 (27)
Bailey et al 1977a, 1979	$\mu^\pm$	0.001 165 924 (8.5)

[a] The error on the last digits is shown in parentheses.

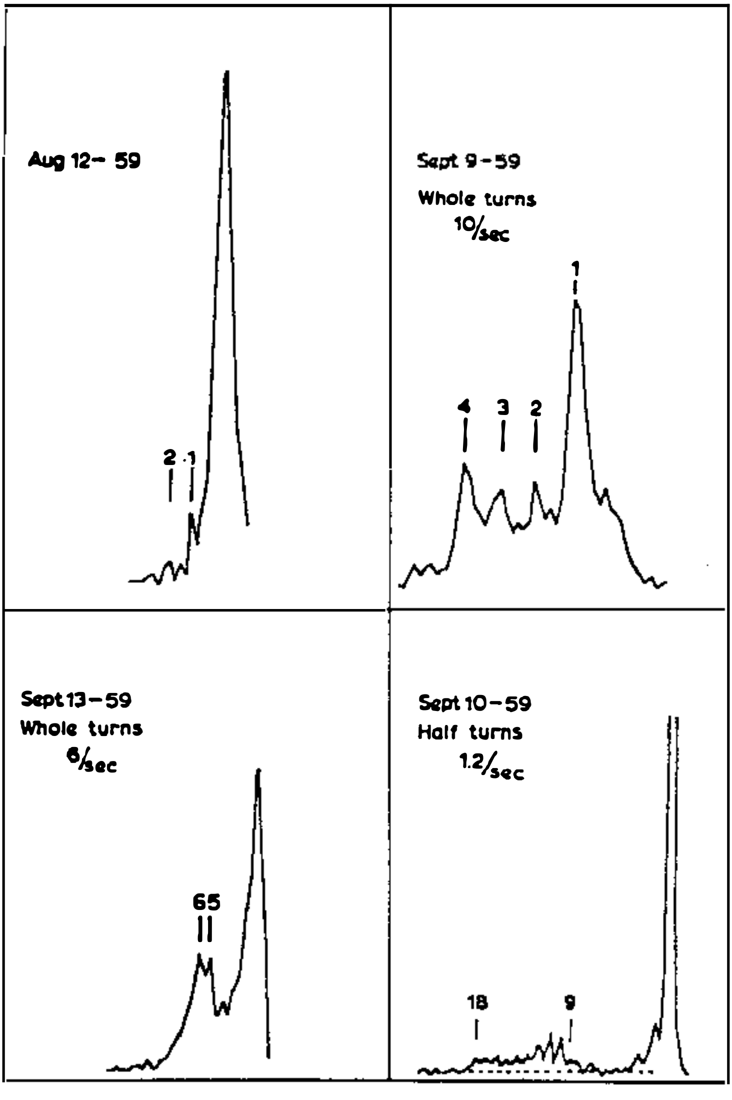

Figure 4 First evidence of muons making several turns in an experimental magnet. The time of arrival of the particles at a scintillator fixed inside the magnet is plotted horizontally (time increases to the left). The first (right-hand) peak coincides with the moment of injection. The equally spaced later peaks correspond to successive turns. Owing to the spread in orbit diameters and injection angles, some muons hit the counter after nine turns (lower right), while others take 18 turns to reach the same point (Charpak et al, unpublished).

remained inside the magnet. To prevent them reentering the absorber after one turn, a small transverse (y direction) gradient of the magnetic field was introduced, causing the orbits to drift sideways perpendicular to the gradient (x direction). Vertical focusing was added by means of a parabolic term in the field.

If the field is of the form

$$B_z = B_0(1+ay+by^2), \tag{10.}$$

where a and b are small, an orbit of radius ρ moves over in the x direction a distance $s = a\pi\rho^2$ per turn (called the step size). On average, the wavelength of the vertical oscillations is $2\pi/b^{1/2}$. Figure 4 is of historical interest. It shows the first evidence of particles turning several times inside a small experimental magnet. These results gave the laboratory sufficient confidence to order a very long magnet for the experiment.

An overall view of the final storage system (Charpak et al 1961a,b, 1962, 1965) is shown in Figure 5. The magnet pole was 6-m long, 52-cm wide, and the gap was 14 cm. Muons entered on the left through a magnetically shielded iron channel and hit a beryllium absorber in the injection part of the field. Here the step size s was 1.2 cm. Then there was a transition to the long "storage region" where $s = 0.4$ cm with field gradient $a = (1/B)(\mathrm{d}B/\mathrm{d}y) = 3.9 \times 10^{-4}\ \mathrm{cm}^{-1}$. Finally, a smooth transition was made to the ejection gradient, where $s = 11$ cm per turn. After ejection the muons fell onto the "polarization analyzer" (Figure 6), where they were stopped and decayed to e^+. The time t a muon spent in the field was determined by recording the coincidences in counters 123 at the input, and counters 4566′7 at the output. The interval was measured with respect to a 10-MHz crystal.

The shimming of this large magnet to produce the correct gradients was a *tour de force*. This was assisted by the theorem that in weak gradients the flux through a wandering orbit is an invariant of the motion. Therefore, if the field along the center line of the magnet was constant, unwanted sideways excursions would be avoided, and this could be checked more exactly by moving a flux coil of the same diameter as the orbit all along the magnet.

However, the constant flux theorem implied that once the particle was trapped inside the magnet it would never emerge. This was seen as a major difficulty, because the final spin direction could only be measured in a weak or zero magnetic field: otherwise one would lose track of the spin direction, while waiting for the muon to decay. For weak gradients and slowly walking orbits, calculations of the orbit confirmed these doubts and some participants lost faith in the project. Fortunately it was found

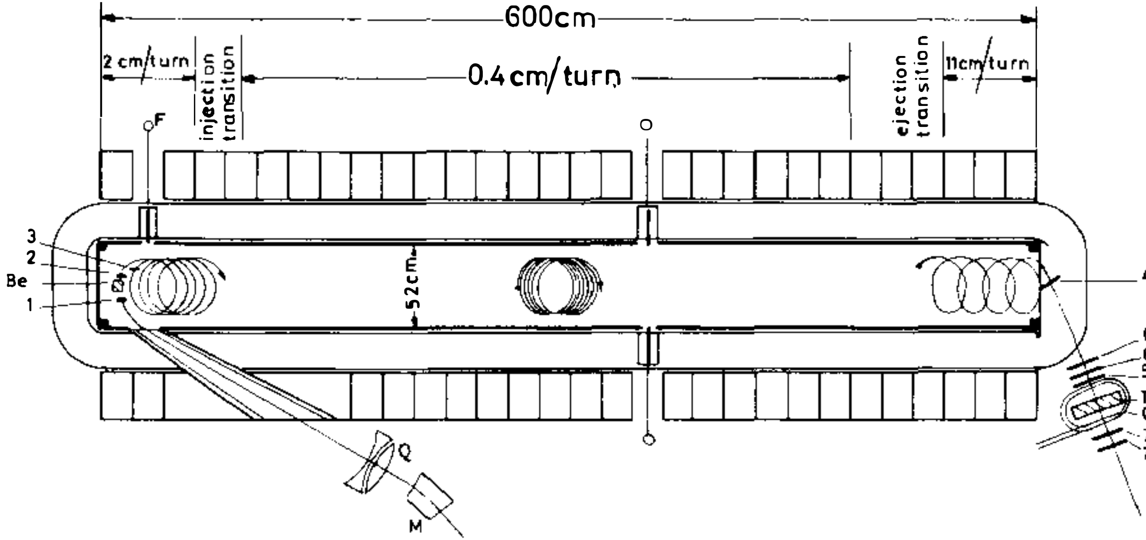

Figure 5 Storage of muons for up to 2000 turns in a 6-m bending magnet. The field gradient makes the orbit walk to the right. At the end a very large gradient is used to eject the muons so that they are stopped in the polarization analyzer. Coincidences 123 and $466'5\bar{7}$, respectively, signal an injected and ejected muon. The coordinates used in the text are x (the long axis of the magnet), y (the transverse axis in the plane of the paper), and z (the axis perpendicular to the paper).

that in large gradients of order $\pm 12\%$ over the orbit diameter the particles were ejected successfully.

The muons were trapped in the magnet for times ranging from 2 to 8 μs depending on the location of the orbit center on the varying gradient given by Equation 10. About one muon per second was stopped finally

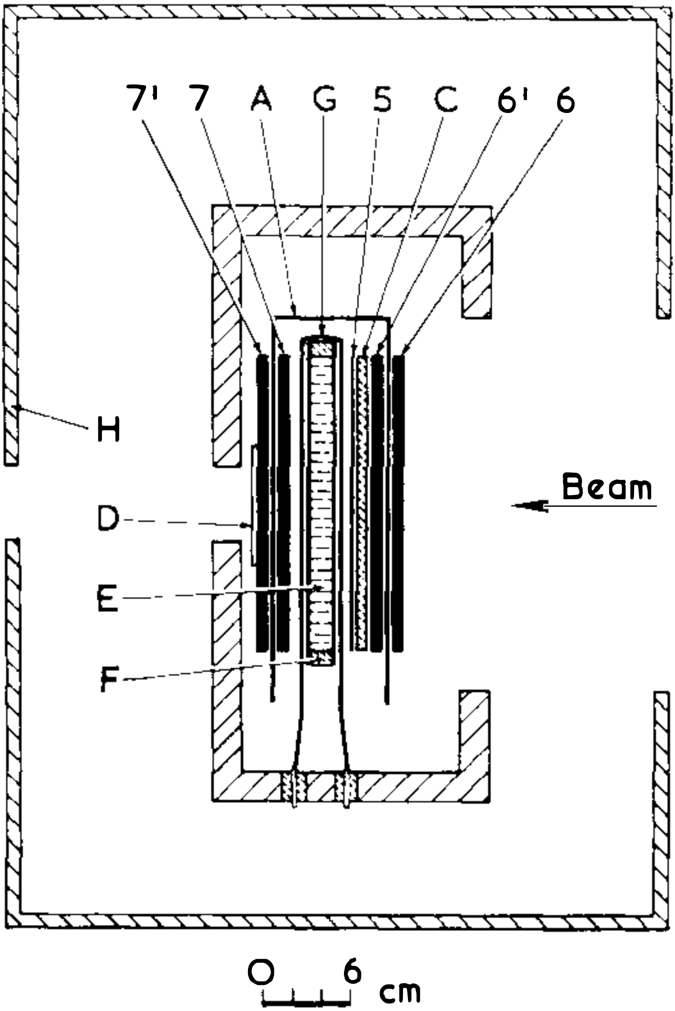

Figure 6 Polarization analyzer. When a muon stops in the liquid methylene iodide (*E*) a pulse of current in coil *G* is used to flip the spin through $\pm 90°$. Backward or forward decay electrons are detected in counter telescopes 66′ and 77′. The static magnetic field is kept small by the double iron shield and the mumetal shield *A*.

in the polarization analyzer, and the decay electron counting rate was 0.25 per second.

The spin direction can, in principle, be obtained from the ratio of two counting rates measured in different directions. But if two counter telescopes are used (say one forward and one backward relative to the direction of the arriving muons), it is not easy to ensure that they have equal efficiencies and solid angles. Therefore it is more reliable to use only one set of counters, but to move the muon spin direction after it has stopped. This can be done with a small constant magnetic field (cf muon precession at rest), but it is more efficient to turn the spin rapidly to a new position by applying a short sharp magnetic pulse, created by applying a pulse of current to a solenoid wound round the absorber in which the muon is stopped. This flipping was accomplished within 1 μs, before the gate that selected the decay electrons was opened.

In the apparatus shown in Figure 6, the electron counts c_+ and c_- in the forward telescope 77′ were recorded in separate runs with the spin flipped through $+90°$ and $-90°$, respectively. The asymmetry A of these counts, defined as $(c_+ - c_-)/(c_+ + c_-)$, was then related to the initial direction θ_s of the muon spin (before flipping) relative to the mean electron direction subtended by telescope 77′:

$$A \equiv \frac{(c_+ - c_-)}{(c_+ + c_-)} = A_0 \sin \theta_s. \qquad 11.$$

By flipping instead through 180° and 0°, another ratio proportional to $A_0 \cos \theta_s$ was measured; so θ_s could be determined completely. Similar, but independent, calculations were made for the telescope 66′, which detected decay electrons emitted backwards.

This polarization analyzer was first used to study the muon beam available for injection. For muons that had been through the magnet the analyzer recorded the asymmetry A as a function of the time t the particle had spent in the field. This showed a sinusoidal variation due to the $(g-2)$ precession in the magnet. Using Equations 9 and 11 it follows that

$$A = A_0 \sin \theta_s = A_0 \sin \left\{ a_\mu \left(\frac{e}{mc} \right) Bt + \phi \right\}, \qquad 12.$$

where ϕ is an initial phase determined by measuring the initial polarization direction and the orientation of the analyzer relative to the muon beam.

The experimental data are given in Figure 7, together with the fitted line obtained by varying A_0 and a_μ in Equation 12. Full discussion of the precautions necessary to determine the mean field B seen by the muons,

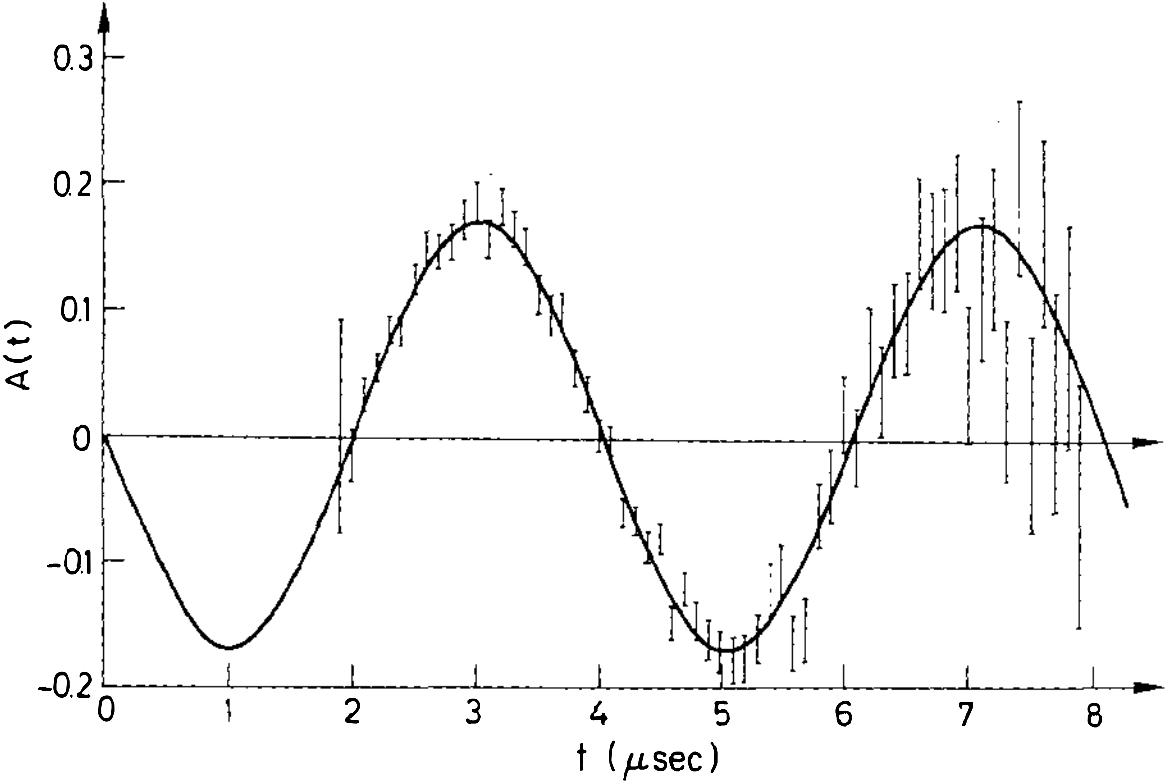

Figure 7 Asymmetry A of observed decay electron counts as a function of storage time t. The sinusoidal variation results from the $(g-2)$ precession; the frequency is measured to $\pm 0.4\%$.

and to avoid systematic errors in the initial phase ϕ, are given in Charpak et al (1962, 1965).

The results of this experiment are given in Table 2. Preliminary runs gave $\pm 2\%$ accuracy in a_{μ}, and this was later improved to $\pm 0.4\%$. The figures agreed with theory within experimental errors. The corresponding 95% confidence limit for the photon propagator cutoff (see Equation 6) was $\Lambda_{\gamma} > 1.0$ GeV, and for the muon vertex function (Equation 5) $\Lambda_{\mu} > 1.3$ GeV.

This was the first real evidence that the muon behaved so precisely like a heavy electron. The result was a surprise to many, because it was confidently expected that g would be perturbed by an extra interaction associated with the muon to account for its larger mass (Schwinger 1957, Kobzarev & Okun 1961). When nothing was observed at the 0.4% level, the muon became accepted as a structureless point-like QED particle, and the possibility of finding a clue to the μ-e mass difference seemed suddenly much more remote.

PART II: 1962–1968 MUON STORAGE RING I

By now the CERN Proton Synchrotron (PS) and Brookhaven Alternating Gradient Synchrotron (AGS) were operating, and the distinct properties of the two neutrinos ν_e and ν_{μ} had been established, further emphasizing the parallel but dual behavior of muon and electron (Danby et al 1962).

Muon-pair production by 1-GeV gamma rays on carbon was measured by Alberigi-Quaranta et al (1962) in agreement with theory. With this and the $(g-2)$ data, the evidence for point-like behavior was now much better for the muon than for the electron. The scattering of muons by lead and carbon (Masek et al 1961, 1963, Kim et al 1961, Citron et al 1962) agreed with the form factors deduced for electron scattering. Logically this was the best evidence for the point-like behavior of the electron, but was generally seen as another contribution to our knowledge of the muon. Knock-on electrons from 8-GeV muons confirmed the picture (Backenstoss 1963). Muonium formation in high pressure argon had been observed by Hughes et al (1960) and the hyperfine splitting of the ground state confirmed the theoretical picture to one part in 2000 (Ziock et al 1962). For this and subsequent muonium experiments the $(g-2)$ result was an essential input, not only for the g-factor, but also to deduce the muon mass from the precession frequency at rest, now determined to 16 ppm by Hutchinson et al (1963), see Equation 9.

The $(g-2)$ experiment was now the best test of QED at short distances. For this reason, and to search again for a new interaction, it was desirable to press the accuracy of the experiment to new levels. It would be essential to increase the number of $(g-2)$ cycles observed, either by increasing the field B or by lengthening the storage time. With the CERN PS available it was attractive to see what could be done by using high energy muons with relativistically dilated lifetimes. As there is no factor $\gamma \equiv (1-v^2/c^2)^{-1/2}$ in Equation 9, the $(g-2)$ precession frequency would not be reduced and more cycles would be available before the muons decayed. But to store muons of GeV energy in a magnetic field and measure their polarization required totally new techniques. Farley (1962) proposed to measure the anomalous moment using a muon storage ring. As in the cyclotron, if the primary target was placed inside the magnet, muons would be produced by π-μ decay in flight and some of them would remain trapped in the field. With a pulsed accelerator, such as the PS, there should be no continuous background following the injection. Transfer of a pulse of protons from the PS to the muon ring could be achieved with the fast ejected beam already developed for the neutrino experiment. Estimates of the stored muon intensity and polarization looked favorable.

To determine the muon spin direction, decays in flight would be observed. The decay electrons would emerge on the inside of the ring and the detectors would respond only to the high energy particles emitted more or less forward in the muon rest frame. Thus as the spin rotated the electron counting rate would be modulated at the $(g-2)$ frequency.

It was later realized that at injection the muons would be localized in azimuth (injection time 10 ns, rotation time about 50 ns), so the counting rate would also be modulated at the mean rotation frequency. This would enable the mean radius of the stored muons to be calculated, leading to a precise knowledge of the corresponding magnetic field.

On the evening of October 21, 1963, a significant chance coincidence of time and place influenced the development of the project. The present authors, having first met that morning, found themselves filling a vacant evening in the same bar of the Hawthorne Hotel, Bristol, drifted into discussing physics, and thus initiated a 16-year collaboration.

The first Muon Storage Ring, Figure 8, (Bailey et al 1972) was a weak-focusing ring with $n = 0.13$, orbit diameter 5 m, a useful aperture of 4 cm $\times$ 8 cm (height $\times$ width), beam momentum 1.28 GeV/c corresponding to $\gamma = 12$ and a dilated muon lifetime of 27 μs. The mean field at the central orbit was $B = 72.852\,7\,(36)$ proton MHz (1.711 T).

The injection of polarized muons was accomplished by the forward decay of pions produced when a target in the magnetic field was struck by 10.5-GeV/c protons from the CERN PS. The proton beam consisted of either two or three radiofrequency bunches (fast ejection), each $\sim$10-ns wide and spaced at $\sim$105 ns. As the rotation time in the ring was 52.5 ns, these bunches overlapped exactly inside the ring. Approximately 70% of the protons interacted, creating among other things pions of 1.3 GeV/c that started to turn around the ring. The pions made on an average four turns before hitting the target again, and in one turn about 20% of the pions decayed. The muons created in the exactly forward decay, together with undecayed pions and stable particles from the target, eventually hit the target and were lost. However, the decay of pions at small forward angles gave rise to muons of slightly lower momentum, and some of these fell into orbits that missed the target and remained permanently stored in the ring. Thus the perturbation, essential for inflection into any circular machine, was here achieved by the shrinking of the orbit arising from the change of momentum in π-μ decay and to some extent by the change in angle at the decay point, which could leave the muon with a smaller oscillation amplitude than its parent pion. The muons injected in this way were forward polarized, because they came from the forward decay of pions in flight. About 200 muons were stored per cycle of the PS. The muon injection was accomplished in a time much shorter than

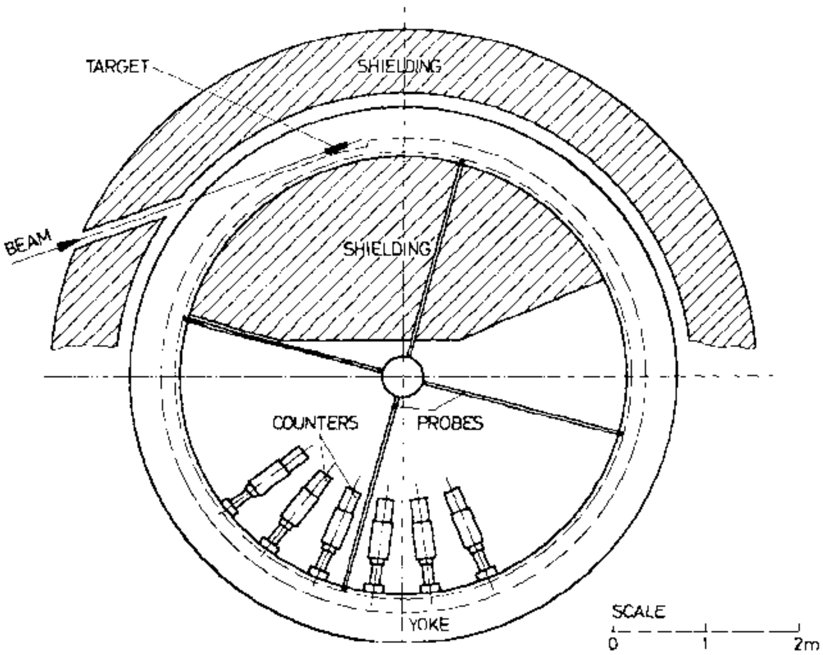

Figure 8 Muon Storage Ring I, diameter 5 m, muon momentum 1.3 GeV/c, time dilation factor 12. The injected pulse of 10-GeV protons produces pions at the target, which decay in flight to give muons.

both the dilated muon lifetime (27 μs), and the precession period of the anomalous moment (3.7 μs).

Unfortunately this simple injection system created a blast of particles inside the ring. Some of them were the desired pions, trapped for a few turns, but there were many more pions of higher momentum. Each had only a small probability of launching a muon into the storage aperture, but the overall contribution was significant. These muons were emitted at large angles in the pion rest frame so the average longitudinal polarization was around 26% compared to the 95% expected.

The method of injection used had the advantage of being very simple, but had the following disadvantages:

1. low muon polarization due to muons from a wide range of pion momenta;
2. high general background;
3. contamination by electrons at early times;
4. low average trapping efficiency.

For some time a magnetic horn was used around the target to concentrate pions of the correct energy in the forward direction. This gave a good muon polarization, but because of increased background was not finally adopted.

The muon precession was recorded by observing the decay in flight of muons in the ring magnet. The detectors responded to decay electrons of energy greater than a minimum value E_{min} (750 MeV). To obtain this high an energy in the laboratory, the electron had to be (*a*) near the top end of the β-spectrum in the muon rest frame [high asymmetry parameter (Bouchiat & Michel 1957)], and (*b*) emitted more or less forward in the muon rest frame. A counter with high energy threshold in the laboratory was equivalent, in the muon rest frame, to a telescope observing a small angular interval around the direction of motion. Therefore, as the muon spin rotated relative to its moment vector according to Equation 9, the observed counting rate (Figure 9) was modulated according to

$$N(t) = N_0 e^{-t/\tau}\{1 - A \sin(2\pi f_a t + \phi)\} \qquad 13.$$

and the frequency f_a could be read from the data.

To calculate a_μ from the data using Equation 9 the value of $(e/mc)B$ is required. This was obtained from the magnetic field measurement in terms of the proton resonance frequency f_p and the known ratio $\lambda = f_s/f_p$ for muon and proton spin precession in the same field (Hutchinson et al 1963). From Equations 8 and 9,

$$\frac{a_\mu}{(1+a_\mu)} = \frac{f_a}{f_s} = \frac{f_a}{\lambda f_p(1+\varepsilon)}, \qquad 14.$$

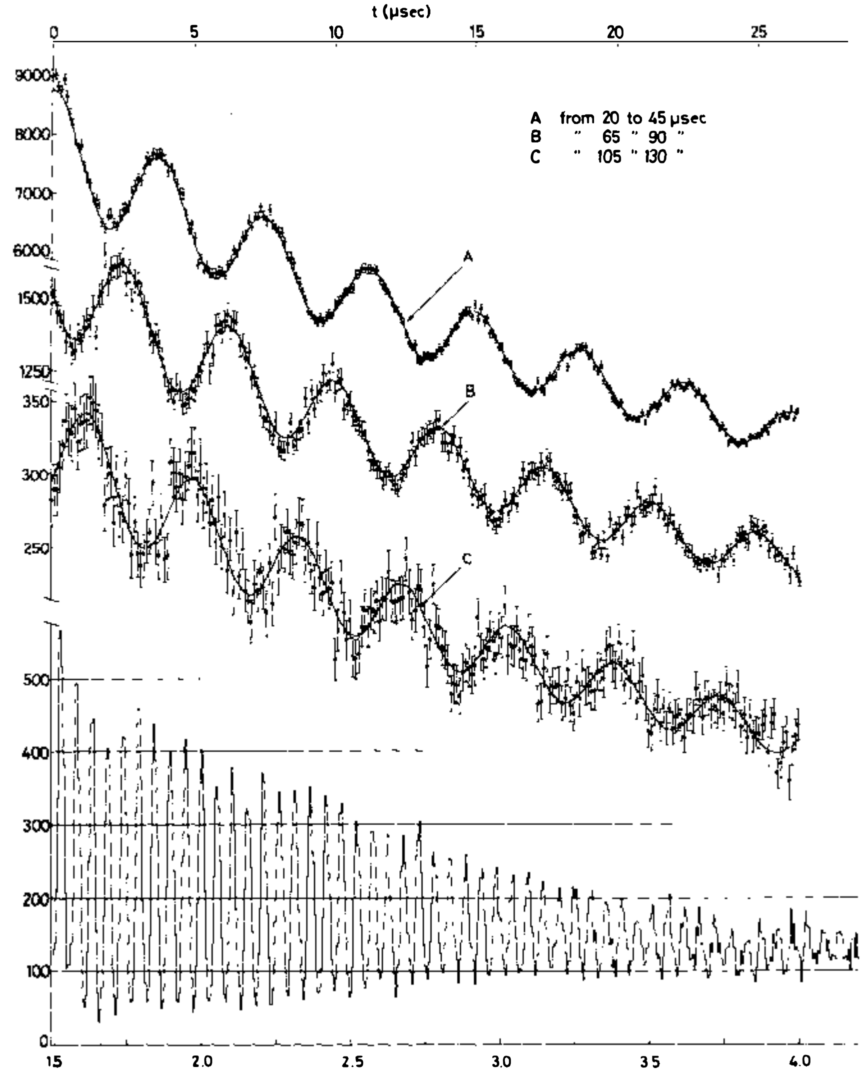

Figure 9 Muon Storage Ring 1: decay electron counts as a function of time after the injected pulse. The lower curve from 2 to 4.75 μs (*lower time scale*) shows 19-MHz modulation due to the rotation of the bunch of muons around the ring. As it spreads out the modulation dies away. This is used to determine the radial distribution of muon orbits. Curves *A*, *B*, and *C* are defined by the legend (*upper time scale*); they show various sections of the experimental decay (lifetime 27 μs!) modulated by the $(g-2)$ precession. The frequency is determined to 215 ppm, $\bar{B}$ to 160 ppm leading to 270 ppm in a_μ.

where ε is the small diamagnetic shielding correction (~ 26 ppm) to correct the measured field in water to the field in vacuum seen by the muons.

The magnetic field was surveyed in terms of the corresponding proton spin-resonance frequency f_p; measurements were taken at 288 azimuthal settings at each of 10 radii.

The radial magnetic gradient necessary for vertical focusing implied a field variation of $\pm 0.2\%$ over the full radial aperture (8 cm). Hence a major problem was to determine the mean radius of the ensemble of muons that contributed to the signal. This was obtained from measurements of the rotation frequency f_r of the muons. The injection pulse was only 5–10 ns long, and the rotation period of the muons, $T = 2\pi r/\beta c$, was about 52.5 ns, so the counting rate was initially modulated at the rotation frequency. The bunches of muons spread out uniformly around the ring after about 5 μs owing to the spread in radii so this modulation gradually diminished in amplitude and disappeared (see Figure 9). The analysis of the modulated record yielded the mean radius $\bar{r} = 2494.3$ (2.7) mm. Figure 10 shows the reconstructed number of muons as a function of radius compared with a theoretical prediction.

Unfortunately the determination of the mean radius was subject to systematic troubles. In the time interval during which the mean radius

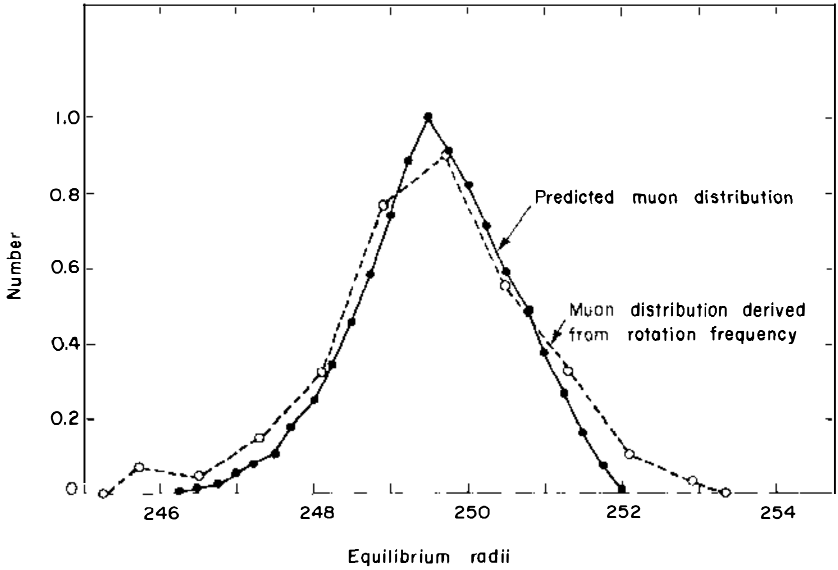

Figure 10 The distribution of muons in radius (horizontal axis, cm) derived from the analysis of the decay electron events at early time. The muon rotation frequency has been analyzed from 1.8 μs to 5.5 μs.

could be determined, there was an excess of counts, caused partly by the fact that some muons were lost later, and partly by a nonrotating background produced by neutrons and other background created by the injection system. So numerous checks were needed to establish the validity of the radius measurement. A measurement of the rotation frequency was made, with reduced intensity, to minimize some of the systematic errors mentioned above. This experiment gave a value of the mean radius at very early times of 0.6–1.6 μs, $\bar{r} = 2492$.
good agreement with the value given above. Checks were also made to show that the mean radius did not change with time by more than ± 1.1 mm between 3 μs and 50 μs. A conservative overall error in the mean radius of ± 3 mm was assigned, implying an error of 160 ppm in the value of a_μ.

The statistical error in a_μ arising from the fit of Equation 13 to the counting data was $\pm 23 \times 10^{-8}$, and the fluctuations of the results of eight different runs about the mean gave $\chi^2 = 7.84$, compared to 6.35 expected. The error in the magnetic field corresponding to ± 3-mm uncertainty in radius contributed $\pm 19 \times 10^{-8}$ to a_μ. The two errors, combined in quadrature, gave the overall error in a_μ of $\pm 31 \times 10^{-8}$. The experimental result was

$$a_\mu = 1\ 166\ 16\ (31) \times 10^{-8}\ (270\ \text{ppm}). \tag{15}$$

Initially this was nearly two standard deviations higher than the theoretical value, a sign perhaps that there was more to discover about the muon. In fact the discrepancy resulted from a defect in the theory. Theorists had originally speculated that the contribution of the photon-photon scattering diagrams (Figure 1*f*) to the $(\alpha/\pi)^3$ term in a_μ^{th} might be small or perhaps cancel exactly. The experimental result stimulated Aldins et al (1969, 1970) to examine this more carefully, obtaining a coefficient of 18.4! The situation then was

$$a_\mu^{\text{exp}} - a_\mu^{\text{th}} = 28\ (31) \times 10^{-8} = 240 \pm 270\ \text{ppm}. \tag{16}$$

For the photon propagator cutoff this implied $\Lambda_\gamma > 5$ GeV, and for the muon vertex $\Lambda_\mu > 7$ GeV. The Einstein time dilation was confirmed to 1%.

PART III: 1969–1976 MUON STORAGE RING II

By 1969 an electron-electron colliding-beam experiment had demonstrated the point-like nature of the electron ($\Lambda_\gamma > 4$ GeV, $\Lambda_e > 6$ GeV) (Barber et al 1966), and e^+e^- storage rings were giving useful data on vector meson production (Augustin et al 1969, 1975, Auslander et al 1968). A comparison of ep and μp scattering, and experiments on e^+e^- and $\mu^+\mu^-$ pair production, wide-angle bremsstrahlung, and muon tridents were all in accord with theory. (For reviews see Lederman & Tannenbaum 1968, Brodsky 1969, Farley 1970, Picasso 1970, Brodsky & Drell 1970).

The pure quantum effects were less satisfactory. The Lamb shift data (Robiscoe 1968a,b, Robiscoe & Shyn 1970) were consistently higher than theory, but this was resolved by a recalculation of a small theoretical term by Appelquist & Brodsky (1970). The electron $(g-2)$ data of Wilkinson & Crane (1963) had been rediscussed by Farley (1968), Henry & Silver (1969), and Rich (1968), who concluded $a_e^{exp}-a_e^{th} = -(79 \pm 26)$ ppm.[2] This discrepancy was to be resolved in a new measurement by Wesley & Rich (1970, 1971). Thus QED was doing well, but in early 1969, a_μ, a_e, and the Lamb shift all showed uncomfortably large departures from theory. It could have been the beginning of something new.

The major motivations for carrying out a third measurement were therefore as follows:

1. to look for departures from standard QED;
2. to detect the contribution of strong interactions to a_μ through hadron loops in the vacuum polarization (see Table 1);
3. to search for new interactions of the muon.

The Third $(g-2)$ Experiment

A major difficulty in the previous experiment was the radial magnetic gradient necessary to provide the vertical focusing; this implied a magnetic field variation of $\pm 0.2\%$ over the aperture in which the muons were stored, and a corresponding radial dependence of f_a. Even if the mean radius was determined precisely after injection, uncertainties in radius would arise from uncontrolled muon losses. The central question for a new experiment was: Can the dependence of f_a on r be removed without destroying the vertical focusing? The answer is yes. The forces that hold the muon in its orbit and give focusing for small deviations from equilibrium arise from what appears in the muon rest frame as an electric field, while the spin precession arises from what appears there as a magnetic field. These two fields may be varied independently by applying suitable magnetic and electric fields in the laboratory frame.

The advantages of this method may be appreciated by writing the classical relativistic equations of motion of a charged particle with an anomalous magnetic moment in laboratory fields **B** and **E** (using cgs

[2] For later work on the pitch correction see Granger & Ford (1972, 1976), Farley (1972a), and Field & Fiorentini (1974).

units) (Bailey et al 1969, Bailey & Picasso 1970, Combley & Picasso 1974, Farley 1975, Jackson 1975):

$$\mathrm{d}\boldsymbol{\beta}/\mathrm{d}t = \omega_c \times \boldsymbol{\beta}, \quad \mathrm{d}\boldsymbol{\sigma}/\mathrm{d}t = \boldsymbol{\omega}_s \times \boldsymbol{\sigma}$$

with

$$\omega_c = \left(\frac{e}{mc}\right)\left[\frac{\mathbf{B}}{\gamma} - \left(\frac{\gamma}{\gamma^2 - 1}\right)\boldsymbol{\beta} \times \mathbf{E}\right]$$

and

$$\omega_s = \left(\frac{e}{mc}\right)\left[\frac{\mathbf{B}}{\gamma} - \left(\frac{1}{\gamma + 1}\right)\boldsymbol{\beta} \times \mathbf{E} + a_\mu(\mathbf{B} - \boldsymbol{\beta} \times \mathbf{E})\right]$$

Here we have assumed that $\boldsymbol{\beta} \cdot \mathbf{B} = \boldsymbol{\beta} \cdot \mathbf{E} = 0$ (muon charge is $-e$). The precession of the spin relative to the velocity vector is

$$\omega_a \equiv \omega_s - \omega_c = \left(\frac{e}{mc}\right)\left[a_\mu \mathbf{B} + \left(\frac{1}{\gamma^2 - 1} - a_\mu\right)\boldsymbol{\beta} \times \mathbf{E}\right] \quad 17.$$

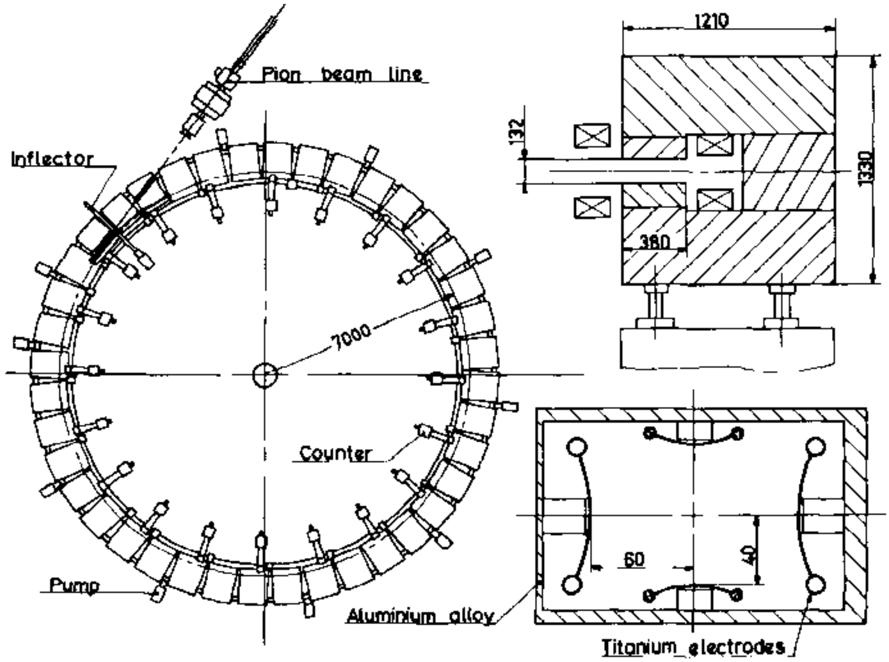

Figure 11 Muon Storage Ring II: diameter 14 m, muon momentum 3.094 GeV/c (magic energy), time dilation factor 29.3. The magnetic field is uniform and vertical focusing is by electric quadrupoles (shown on right). A pulse of 3.1-GeV/c pions is inflected to make nearly one turn. Their decay in flight leaves polarized muons stored in the field.

Thus the effect of the electric field on the precession of the polarization can be made zero if the particle energy satisfies $\gamma = [1+(1/a_\mu)]^{1/2} = 29.304$. For muons this is equivalent to a momentum of 3.094 GeV/c and this was the value chosen for the new storage ring (Figure 11) (Bailey et al 1969). The main idea, therefore, was to provide the vertical focusing by an electrostatic quadrupole field, to work at the "magic" value of $\gamma_0 = 29.304$, corresponding to a muon momentum $p_\mu = 3.094$ GeV/c, and to use a uniform magnetic field. As the magnetic field was to be independent of radius, and the electric field had no effect on spin, it would no longer be necessary to know the radius of the muon orbit.

This cancellation of the effect of the electric field on the spin motion would occur only for the central momentum of the muon sample; for the other equilibrium orbits a small correction (~ 1.5 ppm) would be necessary. The value of the electric field was chosen to give appropriate focusing, but was not needed for calculating a_μ.

The system for injecting muons into the ring was designed to give maximum muon polarization, minimum background, and as large an intensity as possible. A high value of the longitudinal polarization can be achieved by starting with a momentum-selected pion beam and only accepting those decay muons whose momenta lie in a narrow band close to that of the pion beam.

It was therefore decided to locate the primary target outside the Muon Storage Ring, and prepare a momentum-selected pion beam to be guided into the ring by a pulsed inflector. Because of the size of the inflector structure, the pions would only make one turn in the ring, and the useful aperture of the inflector would be very small. Loss of intensity due to these factors could however, be compensated by using special beam optics, which collected pions over a large solid angle and matched them to the acceptance of the storage ring.

The injector was in the form of a coaxial line in which a 10-μs current pulse of peak value 300 kA produced the required field of about 1.5 T between the inner and outer conductors. The great technical difficulty of this method of injection was outweighed by the increased pion flux and the high longitudinal polarization of the stored muons (95%). This was borne out by the large observed modulation of the decay electron spectra. The polarization direction was independent of the muon equilibrium radius and consequently any possible asymmetric muon losses could cause no significant shift in the measured spin precession frequency f_a. Finally the background was reduced considerably with respect to the previous experiment, in which the copper target was located in the ring, and therefore the electron detectors could be located all around the ring.

Other improvements can only be mentioned briefly in this review. The magnetic field was very stable, very reproducible, and uniform. Figure 12 shows a contour plot of the magnetic field strength in the muon aperture. This map was obtained by averaging a three-dimensional map in azimuth. The interval between the contours of equal field strength is 2 ppm or 3 μT. The stability and reproducibility of the magnetic field were achieved by very careful mechanical design of the yoke and magnet coils (Drumm et al 1979). Furthermore each of the 40 magnet blocks was separately stabilized (Borer 1977) with a nuclear magnetic resonance probe and a pick-up coil as sensors. The signals were used to determine automatically the current through additional compensating coils, which were wound around the yoke of the individual magnets close to each pole tip.

It is worth mentioning here the extreme insensitivity of the average value of the magnetic field ($\bar{B}$) computed for different assumed radial distributions of muons. Even in extreme cases the average magnetic field was the same within less than 2 ppm, compared with the 160 ppm uncertainty in $\bar{B}$ in the previous experiment.

To achieve the accuracy reached in this last experiment many technical problems had to be solved. The ability to find these solutions constituted part of the beauty of the experiment. A particular solution that deserves mention is the construction and understanding of the electric quadrupole

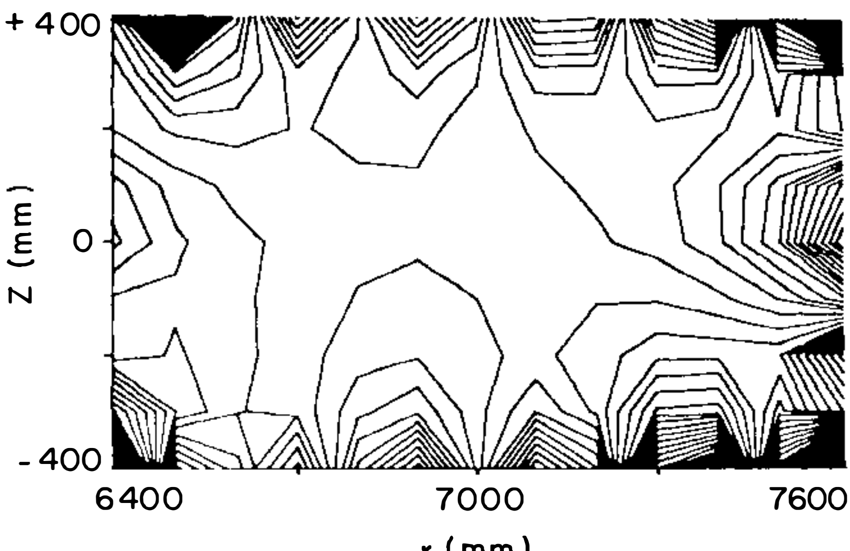

Figure 12 A contour line plot of the magnetic field strength in the muon storage aperture. This map is obtained by averaging a three-dimensional map in azimuth. The interval between the contours of equal field strength is 2 ppm or 3 μT.

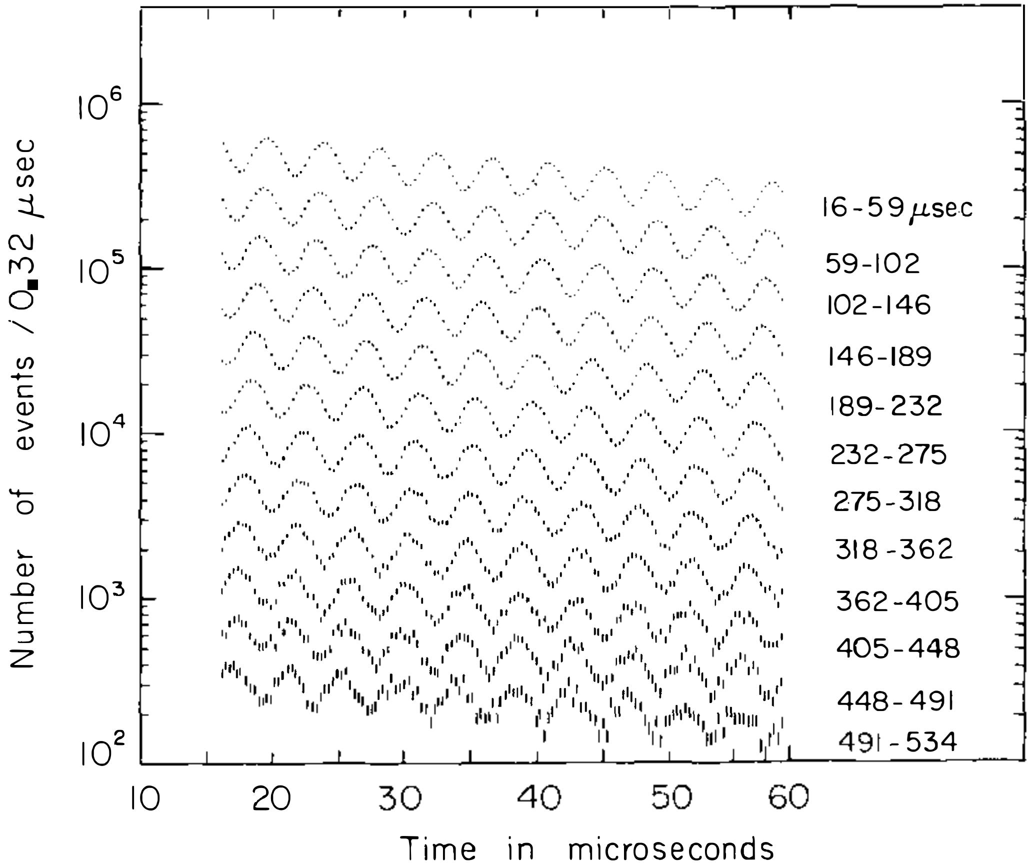

Figure 13 Muon Storage Ring II: decay electron counts versus time (in microseconds) after injection. Range of time for each line is shown on the right (in microseconds).

field in the presence of the high magnetic field. We invite the reader to consult the final report on the CERN Muon Storage Ring for these details (Bailey et al 1979; see also Flegel & Krienen 1973). Figure 11 shows the ring and focusing system, while Figure 13 gives a summary of the counting data.

EXPERIMENTAL RESULTS Nine separate runs were made over a period of two years to measure the $(g-2)$ precession frequency f_a, the field being determined in terms of the proton resonance frequency f_p (Bailey et al 1975, 1977a, 1979). The ratio $R = f_a/f_p$ showed good consistency ($\chi^2 =$ 7.3 for 8 degrees of freedom). The overall mean value is the principal result of the experiment:

$$R = f_a/f_p = 3.707\,213\,(27) \times 10^{-3}\ (7\ \text{ppm}). \qquad 18.$$

Equation 14 allows us to calculate $a_\mu = R/(\lambda - R)$ if $\lambda = f_s/f_p$ is known.

The magnitude of λ has now been determined to about 1 ppm, directly from measurements of muon precession at rest (Crowe et al 1972, Camani et al 1978) and indirectly from the hyperfine splitting in muonium (Casperson et al 1977). The weighted average value of these measurements is

$$\lambda = 3.183\,343\,7\,(23).$$

This leads to the following results for the anomalous moment:

$$a_{\mu^+} = 1\,165\,911\,(11) \times 10^{-9}\ (10\ \text{ppm}), \qquad 19.$$

$$a_{\mu^-} = 1\,165\,937\,(12) \times 10^{-9}\ (10\ \text{ppm}), \qquad 20.$$

and for μ^+ and μ^- combined

$$a_\mu = 1\,165\,924\,(8.5) \times 10^{-9}\ (7\ \text{ppm}). \qquad 21.$$

COMPARISONS BETWEEN THEORY AND EXPERIMENT The main six conclusions that can be drawn from this last measurement of the anomalous magnetic moment of the muon are the following:

1. The QED calculations of the muon anomaly are verified up to the sixth order, the experimental uncertainty being equivalent to 1.2×10^{-5} in A, 3.5×10^{-3} in B, or 4.7% in C (see Equation 1).

2. The hadronic contribution to the anomaly is confirmed to an accuracy of 20%. The existence of hadronic vacuum polarization has thus been established at the level of five standard deviations.

3. There is no evidence for a special coupling of the muon. The experimental range of possible values of an extra contribution to the moment is

$$-20 \times 10^{-9} < \Delta a_\mu < 26 \times 10^{-9} \qquad 22.$$

to 95% confidence. The limits implied for unknown boson fields then depend on the nature of the coupling and are given in Figure 14 (Bailey et al 1979).

4. With the advent of renormalizable gauge theories unifying the weak and the electromagnetic interactions, the calculation of the weak interaction contribution to the muon anomaly has become reliable. In general the weak contribution depends upon the parameters of the theory, such as the masses of the Higgs and intermediate vector bosons. To the extent that we do not yet know the correct form of the weak interaction Hamiltonian, the above results for a_μ (and also the result for a_e) can be uscd to rcstrict the range of possible models. Only in the simplest of such theories, that of Weinberg (1967) and Salam (1968), are the parameters sufficiently well determined experimentally to give a firm prediction of the expected value of the weak anomaly. Kinoshita (1978) has recently reviewed this subject. If the arguments of Weinberg on the Higgs boson are accepted, and the current limits on $\sin^2 \theta_W$ are taken into account, we obtain:

$$1.9 \times 10^{-9} \leqq a_\mu \text{ (weak)} \leqq 2.3 \times 10^{-9}.$$

Clearly the precision of even the latest experiment is inadequate for testing this prediction.

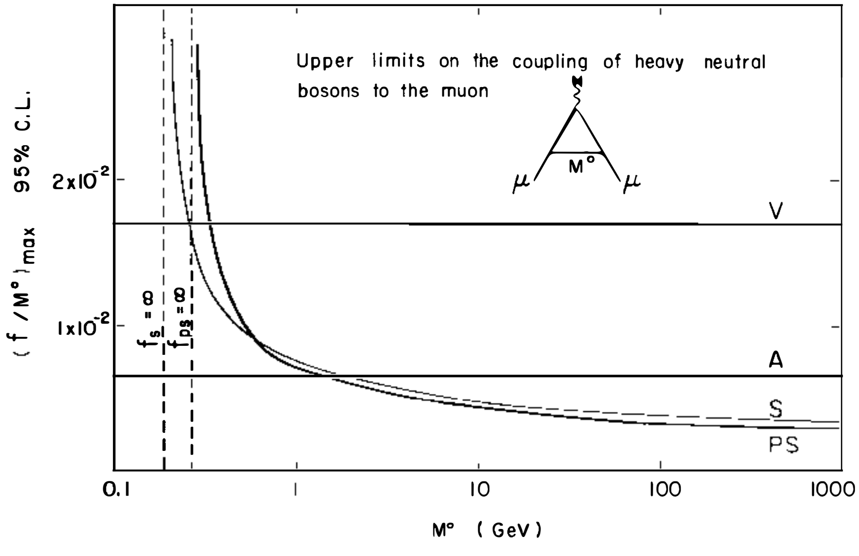

Figure 14 The upper limits on the coupling constant f of the muon to a heavy neutral boson of mass M^0, for vector (V), axial vector (A), scalar (S), and pseudoscalar (PS) coupling.

5. The range defined by the inequalities (Expression 22) may be used to set limits on single contributions to the muon anomaly from other sources including various models for breaking QED as discussed in the theoretical section. The following limits apply to 95% confidence:

(*a*) The muon may not behave like a point charge, but instead have a finite size, in analogy with the proton. This would show up as a form factor. The limit imposed on Λ_μ would be $\Lambda_\mu > 36$ GeV.
(*b*) The modification in the photon propagator leads to $\Lambda_\gamma > 20.7$ GeV.
(*c*) From the latest experiment it is possible to set a limit to the modification of the muon propagator (Kroll 1966) by a factor $(1-q^4/\Lambda^4_{\text{prop}})$. The value is $\Lambda_{\text{prop}} > 1.5$ GeV.
(*d*) A possible new, undiscovered lepton of mass M_{L} would contribute to the vacuum polarization through a mechanism such as diagrammed in Figure 1*d*. The value of the anomaly would depend on the ratio M_{L}/m_μ. The 95% confidence limit sets the limit $M_{\text{L}} \gtrsim 210$ MeV/c^2, which is not very interesting. In passing, the recently discovered heavy lepton τ (Perl et al 1975), with mass 1.8 GeV/c^2, gives a contribution,
$$\Delta a_\mu(\tau) \simeq 0.4 \times 10^{-9},$$
well below the present sensitivity.

6. Recently Kadyshevsky (1978) has given a new gauge formulation of the electromagnetic interaction theory, containing a "fundamental length" $\boldsymbol{l}$ as a universal scale constant as important as $\hbar$ and c. This new hypothetical constant $\boldsymbol{l}$, together with $\hbar$ and c, is expected to regulate all microscopic phenomena. The quantity $M = \hbar/\boldsymbol{l}c$ plays the role of a fundamental mass. In the new approach the electromagnetic potential becomes a 5-vector associated with the de Sitter group O(4,1). Among the various predictions given are the value of the anomalous moment for a lepton of mass m_{l}

$$a_{\text{lepton}} \simeq \frac{m_{\text{l}}^2}{2M^2}$$

and the electric dipole moment (EDM)

$$\mathbf{d}_{\text{lepton}} \simeq \frac{e\boldsymbol{l}}{2}.$$

From the present experimental result one then obtains an upper bound for the fundamental length: $\boldsymbol{l} < 2.6 \times 10^{-17}$ cm.

Electric Dipole Moment

An upper limit for the electric dipole moment (EDM) of the muon has been measured directly in the CERN Muon Storage Ring (Bailey et al

1978). For a particle with both magnetic and electric dipole moments the electromagnetic interaction Hamiltonian contains a term $(\boldsymbol{\mu}\cdot\mathbf{B}-\mathbf{d}\cdot\mathbf{E})$, where $\mathbf{B}$ and $\mathbf{E}$ are the magnetic and electric field strengths and $\boldsymbol{\mu}$ and $\mathbf{d}$ are the magnetic and electric dipole moment operators. Treating the electric dipole moment analogously to the magnetic dipole moment we can write

$$\boldsymbol{\mu} = g\left(\frac{e}{2mc}\right)\left(\frac{\hbar\boldsymbol{\sigma}}{2}\right) = g\mu_0\left(\frac{\boldsymbol{\sigma}}{2}\right)$$

$$\mathbf{d} = f\left(\frac{e}{2mc}\right)\left(\frac{\hbar\boldsymbol{\sigma}}{2}\right) = f\mu_0\left(\frac{\boldsymbol{\sigma}}{2}\right),$$

where μ_0 is the muon Bohr magneton $e\hbar/2mc$.

It is well known that the expectation value of the electric dipole moment $\mathbf{d}$ must be zero for a particle described by a state of well-defined parity. However, Purcell & Ramsey (1950) stressed that the existence of an EDM for particles should be treated as a purely experimental question, and they suggested possible physical mechanisms that could lead to a nonvanishing EDM. After the discovery of parity violation in the weak interactions, it was pointed out by Landau (1957) that even if P is violated, the existence of an EDM is still forbidden by T invariance, i.e. the existence of a nonvanishing EDM for a particle implies that both P and T are violated. See Field et al (1979) and Jackson (1977) for comprehensive reviews of the subject.

The technique used to measure the muon electric dipole moment follows from a suggestion originally made by Garwin & Lederman (1959). They pointed out that in the $(g-2)$ precession experiments using magnetic mirror traps, the electron (or the muon) will experience in its rest frame an electric field proportional to the particle velocity, as a result of the Lorentz transformation of the laboratory magnetic field. This electric field is perpendicular to the magnetic field. If the EDM is not zero, the spin precession frequency relative to the momentum will pick up a component $\mathbf{f}_{\mathrm{EDM}}$ along the electric field direction in addition to the normal $(g-2)$ frequency $\mathbf{f}_a$ along the magnetic field direction (Figure 15). The observed $(g-2)$ frequency is then $f_a' = f_a[1+\beta^2 f^2/4a^2]$. As a further consequence of this new precession component, the decay electrons from the muon will show a time-varying up-down asymmetry perpendicular to the plane of the orbit. Such an effect was explored in the first CERN muon $(g-2)$ experiment (Charpak et al 1961b) in which the muons were brought to rest in a polarimeter. The value measured was

$$d_\mu = |\mathbf{d}| = (0.6 \pm 1.1) \times 10^{-17}\ \mathrm{e\cdot cm}.$$

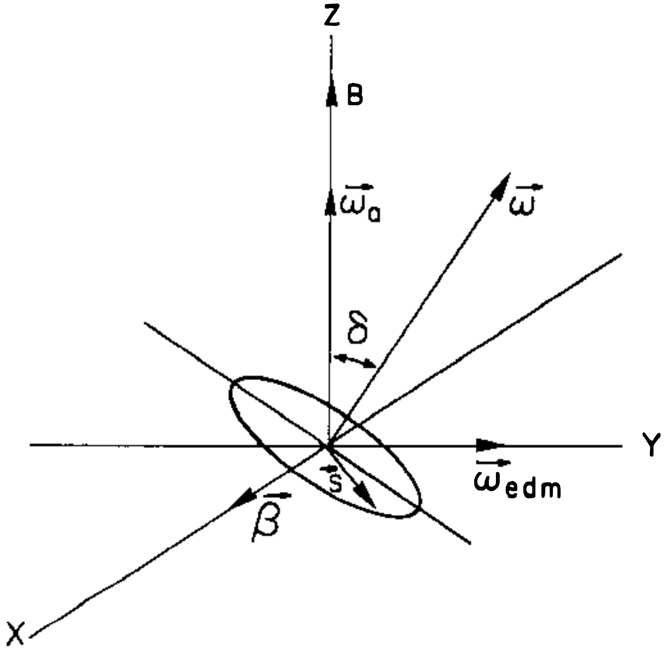

Figure 15 Precession of the spin relative to the momentum resulting from the combination of an anomalous magnetic moment and an electric dipole moment. The plane of precession is tilted through the angle $\delta = \beta f/2a$, see text.

A similar technique was used in the most recent Muon Storage Ring experiment by detecting separately the electrons emitted upwards and downwards from muon decay in flight. Separate measurements on μ^+ and μ^- (Bailey et al 1978) gave:

$$d_{\mu^+} = (8.6 \pm 4.5) \times 10^{-19}\ \text{e}\cdot\text{cm}$$
$$d_{\mu^-} = (0.8 \pm 4.3) \times 10^{-19}\ \text{e}\cdot\text{cm}.$$

Assuming opposite EDMs for the particle and antiparticle, the combined result was

$$d_\mu = (3.7 \pm 3.4) \times 10^{-19}\ \text{e}\cdot\text{cm}.$$

For comparison the current upper limits for the electron, proton, and neutron in units e·cm (Pais & Primack 1973) are electron $\lesssim 3 \times 10^{-24}$, proton $\lesssim 2 \times 10^{-20}$, and neutron $\lesssim 1 \times 10^{-23}$. That these limits are much lower than the limit of the muon largely reflects the fact that, unlike the muons, they are studied in neutral systems. The fundamental length l of Kadyshevsky can therefore not be greater than 2×10^{-18} cm (muon evidence) or 10^{-23} cm (electron evidence).

Muon Lifetime in Flight

Accurate measurements of the muon lifetime in a circular orbit provide a stringent test of Einstein's theory of special relativity. As a bonus it sheds light on the so-called twin paradox, gives an upper limit to the granularity of space time, and tests the CPT invariance of weak interaction.

The muon is an unstable particle, and can therefore be regarded as a clock and used to measure the time dilation predicted by special relativity. The existence of cosmic-ray muons at ground level supports the idea of time dilation, for, if the muon lifetime was not lengthened in flight, they would all decay in the upper atmosphere (Rossi & Hall 1941). Experiments verifying the time dilation in a straight path have also been made with high energy accelerators (see Bailey et al 1977b).

Recently Hafele & Keating (1972) loaded cesium atomic clocks onto a commercial aircraft on an around-the-world trip and verified the time dilation at low velocity with an accuracy of about 10%.

In the CERN Muon Storage Ring, the muon performs a round trip and so when compared with a muon at rest the experiment mimics closely the twin paradox already discussed in Einstein's first paper (Einstein 1905). The circulating muons, although they return again and again to the same place, should remain younger than their stay-at-home brothers. It is indeed observed that the moving muons live longer, in agreement to one part in a thousand with the predictions of special relativity. The stationary twin's time scale is given by the muon decay rate at rest determined in a separate experiment.

An accurate measurement of the muon lifetime in a circular orbit at $\gamma \simeq 29.3$ requires high orbit stability in a short time interval (a few hundred microseconds), for any loss of muons will set a limit to the accuracy of the measurement. The reported stability was achieved by using a scraping system that shifted the muon orbits at early times in order to "scrape off" those muons most likely to be lost.

The experiment consisted of measuring the decay electron counting rate $N(t)$ (see Equation 13) and the fitting procedure gave the value of $\tau = \gamma\tau_0$. The rotation frequency f_r of the muons obtained from the counting record at early times (see Figure 16) gave $\gamma = \lambda\overline{f_p}/(1+a)f_r =$ 29.327 (4). The best value for the lifetime at rest is 2.197 11 (8) μs (Balandin et al 1974), which then gives $\tau_{th} = 64.435$ (9) μs, compared with the experimental result $\tau_{exp} = 64.378$ (26). Thus the transformation of time is validated to an accuracy of $-(0.9 \pm 0.4) \times 10^{-3}$ (Bailey et al 1977b).

In the actual experiment, corrections were made for a residual small loss of stored muons, for variations of the photomultiplier gain accompanying the recovery from the initial flash, and for background counts due to stored protons (in the case of μ^+). In order to measure the muon lifetime with an accuracy of 0.1% it was necessary to study carefully these three effects, which could systematically distort the recorded time spectrum.

Another check on relativity theory can be obtained by comparing $(g-2)$ measurements carried out at different values of γ. For the electron this has been argued by Newman et al (1978), and discussed by Combley et al (1979) for both e and μ. Inevitably the conclusions are model dependent, but one can make a plausible case that these results confirm the relativistic transformation laws for magnetic field and mass, as well as for time.

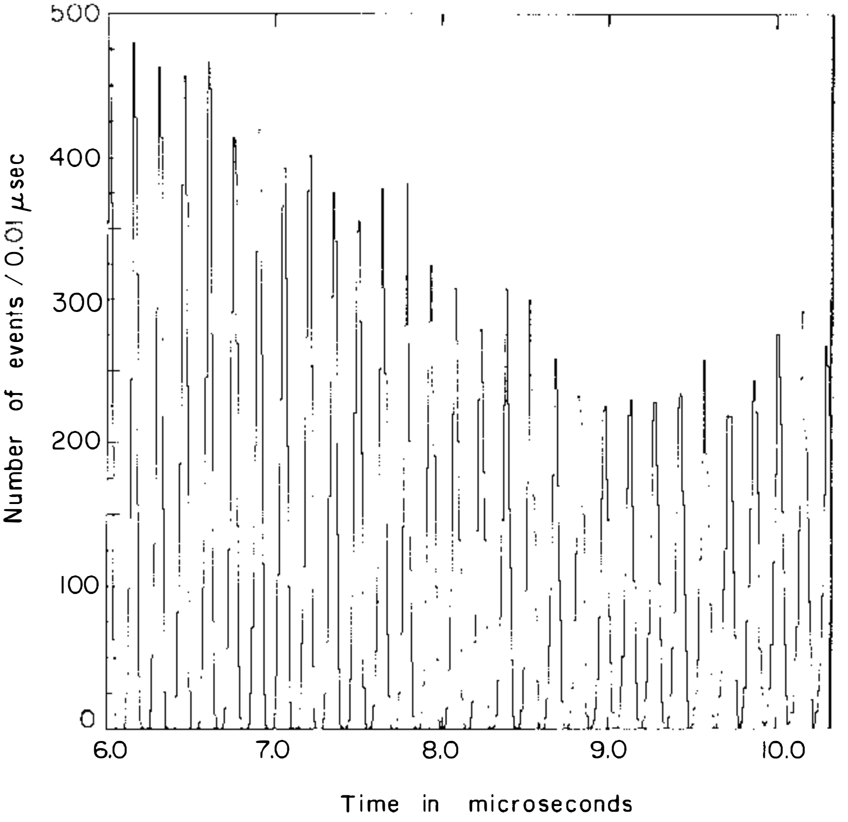

Figure 16 The fast rotation pattern. This is the count rate at early time which clearly shows the muon bunch rotating around the ring with a period of 147 ns.

Verification of the CPT Theorem

From CPT it follows that $g_{\mu^+} = g_{\mu^-}$. The measurements in the CERN Muon Storage Ring gave to 95% confidence

$$7 \times 10^{-9} > \frac{g_{\mu^+} - g_{\mu^-}}{g_\mu} > -58 \times 10^{-9}.$$

From CPT it follows also that $\tau_{\mu^+} = \tau_{\mu^-}$. The experimental data on the μ^+ and μ^- lifetime in flight give the best test of this equality (as τ_{μ^-} cannot be measured at rest because of muon capture). In this connection it should be noted that the Lorentz γ-factor is the same for μ^+ and μ^- to a much higher precision than the quoted lifetime errors. The limits are

$$3.0 \times 10^{-3} > \frac{\tau_{\mu^+} - \tau_{\mu^-}}{\tau_\mu} > -1.4 \times 10^{-3}.$$

Thus the theorem is validated for muons to very high accuracy for the electromagnetic interaction, and rather less accurately for the weak interaction.

CONCLUSION: THE SITUATION TODAY

The anomalous magnetic moment of the electron has now been measured to 0.2 ppm (Van Dyck et al 1977) in agreement with calculations to the order of $(\alpha/\pi)^3$ again confirming the QED series expansion. At present the theory is only good to 0.1 ppm. When this is improved the experiment will provide the most accurate and clearest measurement of the fine structure constant α (Kinoshita 1978). Any modification to the photon propagator or new coupling common to e and μ would imply a perturbation to a_μ a factor $(m_\mu/m_e)^2$ greater than for a_e. Therefore, barring possible couplings peculiar to the electron, the muon result ensures that a_e is a "pure QED quantity" to the order of three parts in 10^{10}. Another good route to α is via the hyperfine splitting in muonium. Here again the results for a_μ and a_e ensure that muonium is a "pure QED system" (Farley 1972b).

A major stride forward in QED at high energy has been made with e^+e^- colliding beams. All experiments agree with theory (except in the neighborhood of the new resonances Υ, ψ, ψ') and the corresponding cut-off limits are $\Lambda_e > 21$ GeV, $\Lambda_\mu > 27$ GeV (Hofstadter 1975) and $\Lambda_\gamma > 38$ GeV (Barber et al 1979). (See also Schwitters & Strauch 1976, Cords 1978, Hughes & Kinoshita 1977.)

Small parity-violating effects due to the weak interaction have been detected in the scattering of polarized electrons at high energies (Prescott et al 1978) in agreement with the predictions of the new unified theory.

The discovery of a new lepton of mass 1.8 GeV (Perl et al 1975) has changed the μ-e problem without providing any answers. Mass splittings in the lepton family are much larger than other mass splittings between similar particles. Although the $(g-2)$ result for the muon has shed no light on the problem, it nevertheless provides a serious constraint on the fantasies of theorists.

In first-order QED the self-mass $m = (m_0/137) \ln (\lambda_c/L)$ becomes significant when the cutoff distance L is of order 10^{-69} cm or less. The only other physical length of this order is the Schwarzchild radius of the electron ($\sim 10^{-54}$ cm). In principle any photons originating closer to the particle than this will not be able to reach the outside world, so in this sense gravitation provides a natural cutoff for QED. We already have a unified theory of the weak and electromagnetic interactions (Weinberg 1967, Salam 1968). If the attempt to include gravitation is successful it may in the end offer an explanation for the lepton masses (Isham et al 1971).

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All the experimcnts have been a team effort, with each member listed in the references making his unique personal contribution. We offer this summary as a souvenir of our shared endeavor. In particular John Bailey has been a constant and stimulating colleague through all the storage ring experiments.

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