Inflector Acceptance

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We consider the acceptance through the inflector into the muon ring. Let's ignore the fact the the coil overlaps the ends of the inflector and that it has a physical aperture of $a_x = \pm 10$ mm and $a_y = \pm 28$ mm. We further assume that the admittance of the incoming beam is $\epsilon_x = \epsilon_y = 40$mm-mrad. The size in real space of the incoming beam is $\sqrt{\beta\epsilon}$. All of the muons in the beam will clear the walls of the inflector aperture if

$$\beta_x < \frac{a_x^2}{\epsilon_x} = 2.5 \text{ m}, \quad \text{and} \quad \beta_y < \frac{a_y^2}{\epsilon_y} = 19.6 \text{ m}$$

If $\beta_x = 2.1$ m halfway through the 1.72 m inflector then at the ends $\beta_x^{end} = \beta_{mid} + s^2/\beta_{mid} = 2.45$ m and $\alpha = \frac{1}{2}\beta' = s/\beta_{mid} = 0.41$.

We next step is to figure out how that β propagates through the ring. If we treat the ring as closed, rather than as a transfer line we know that $\beta_x = \frac{R}{\sqrt{n-1}} = 7.66$ m and $\beta_y = \frac{R}{\sqrt{n}} = 19.1$ m where we have used $n = 0.139$ where we have assumed that the focusing in the ring is uniform so that β_x and β_y are constant and $\alpha = 0$. The matrix that propagates beam in the ring looks like

$$\begin{pmatrix} \cos\phi & \beta_0 \sin\phi \\ -\frac{1}{\beta_0}\sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

The matrix for propagating twiss parameters is

$$\begin{aligned} N &= \begin{pmatrix} M_{22}^2 & -M_{22}M_{21} & M_{21}^2 \\ -2M_{22}M_{12} & (M_{11}M_{22} + M_{12}M_{21}) & -2M_{21}M_{11}^2 \\ M_{12}^2 & -M_{12}M_{11} & M_{11}^2 \end{pmatrix} \\ &= \begin{pmatrix} \cos^2\phi & \cos\phi\sin\phi/\beta_0 & \sin^2\phi/\beta_0^2 \\ -2\beta_0\cos\phi\sin\phi & (\cos^2\phi - \sin^2\phi) & 2\sin\phi\cos^2\phi/\beta_0 \\ \beta_0^2\sin^2\phi & -\beta_0\sin\phi\cos\phi & \cos^2\phi \end{pmatrix} \end{aligned}$$

where N propagates $\vec{\gamma}$ from point 1 to point 2, $N\vec{\gamma}_1 = \vec{\gamma}_2$ and $\vec{\gamma} = \begin{pmatrix} \gamma \\ \alpha \\ \beta \end{pmatrix}$

$$N = \begin{pmatrix} \cos^2\phi & \sin 2\phi/2\beta_0 & \sin^2\phi/\beta_0 \\ -\beta_0 \sin 2\phi & \cos 2\phi & \sin 2\phi\cos\phi/\beta_0 \\ \beta_0^2\sin^2\phi & -\frac{1}{2}\beta_0\sin 2\phi & \cos^2\phi \end{pmatrix} \tag{1}$$

We found above that if all of the beam is to clear the horizontal aperture of the inflector that at the exit of the inflector

$$\vec{\gamma} = \begin{pmatrix} 0.48 \text{ m}^{-1} \\ 0.41 \\ 2.45 \text{ m} \end{pmatrix}$$

We propagate $\vec{\gamma}$ through the ring using the matrix in Equation 1. The resulting β_x is shown in Figure 1.

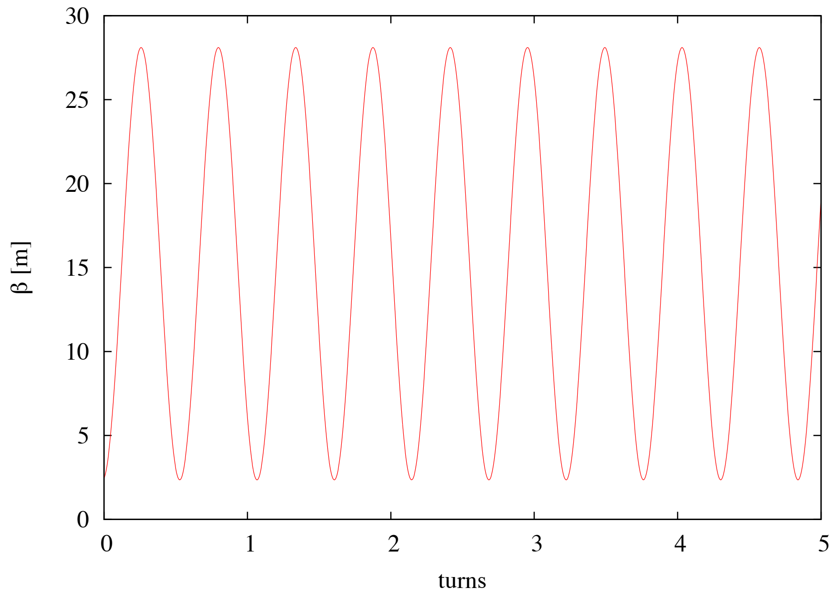

Figure 1: Horizontal β modulates as betatron phase advances

We find that if the beam is indeed focused so that all of the horizontal emittance clears the inflector aperture, that the mismatch results in a modulation of the horizontal beta in the ring with $\beta^{max} = 28$m and $\beta^{min} = 2.45$ m. The corresponding modulation of the horizontal size is $r = \sqrt{\frac{\beta^{max}}{\beta^{min}}} = 3.4$. Does this modulation contribute a systematic error ?

Things look much better in the vertical. We found that to clear the vertical aperture of the inflector, that $\beta_y^{inf} < 19.6$ m, which very nearly matches the optimal β_y in the ring which is ~ 19.1 m. It is clearly advantageous to match vertical beta at the exit of the inflector to the optimal ring vertical beta.

Note that it is possible, at least with the simplistic assumptions of perfect inflector with rectangular aperture 20 mm wide by 36 mm high, to transfer with zero losses and then store a muon beam with 40 mm-mrad into the ring. The penalty will be significant modulation of horizontal size of the stored beam.