Analysis of Phase Space Matching with RF Quadrupole

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1 Introduction

Young-Im Kim, Seung Pyo Chang, Martin Gaisser, Uiryeol Lee, Soohyung Lee, and Yannis Semertzidis propose to superimpose an RF voltage on an electrostatic quadrupole in the g-2 ring to match the phase space of the injected muon distribution to the twiss parameters characteristic of the storage ring. The goal is to effect matching of the phase space by modulation of the focal length of a single ring quad on each of the first 150-200 turns. A tracking simulation, based on the BMAD model of the g-2 ring is used to test the concept, and in particular the effect of the nonlinearities of the quadrupoles, the finite energy and temporal spread of the muon distribution, and the displacement of the centroid of the distribution.

2 Phase Space Matching

The result of the mismatch of β-functions and horizontal dispersion at the inflector exit is shown in Figures 1. Nominal closed ring twiss parameters are at left and effective twiss parameters that result from the mismatch at right.

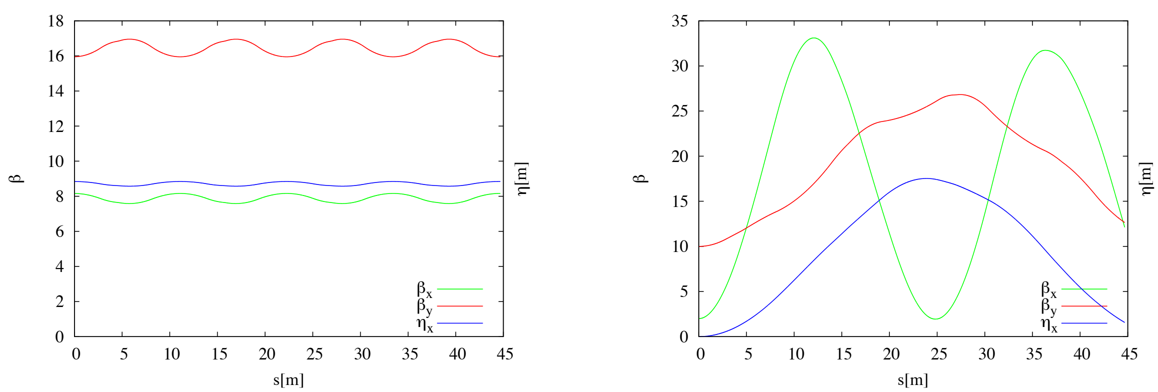

Figure 1: Closed ring β, η at left. Modulation of β and dispersion η on first turn due to mismatch at inflector exit at right.

The β-error of the magic momentum muon distribution that results from mistmatch of inflector into ring, propagates at twice the betatron frequency, as in Equation 1.

$$\Delta\beta(s)_{x.y} = A_{x,y}\cos(2(2\pi Q_{x,y}) + \phi_{x,y}) \tag{1}$$

The proposal of Kim et al. is to similarly modulate the voltage of a quadrupole, that is at twice the respective horizontal and vertical betatron frequencies. The RF voltage applied to one of the ring quadrupoles, is (Equation 2)

$$V = A_x\cos(2\omega_x t + \phi_x) + A_y\cos(2\omega_y t + \phi_y) \tag{2}$$

where $\omega_{x,y} = 2\pi f_{rev}(2Q_{x,y})$ and f_{rev} is the cyclotron frequency of the magic momentum muon, and $Q_{x,y}$ the horizontal or vertical betatron tune.

3 Horizontal

Consider modulation of the horizontal beam size due to mismatch of β at the inflector exit to the closed ring values (as given in Fig 1 left). A muon distribution is tracked from the inflector exit through 200 turns ($\sim 30\mu$s). The phase space of the 1000 particle distribution, corresponds to that defined by the design twiss parameters at the inflector exit, namely, $\beta_h = 2$m, $\beta_y = 10$m, and $\alpha_x = \alpha_y = 0$ and as shown in Figure 1. The horizontal and vertical displacement of the centroid of the distribution is initially set to zero, so that there is no coherent betatron oscillation. Initially the energy spread of the distribution is also set to zero so that all of the particles have the magic momentum. Finally, the quad multipoles are set to zero, (except of course for the linear term). Figure 2 shows the width (standard deviation of the horizontal position) of the distribution as a function of circumferential coordinate in terms of number of turns.

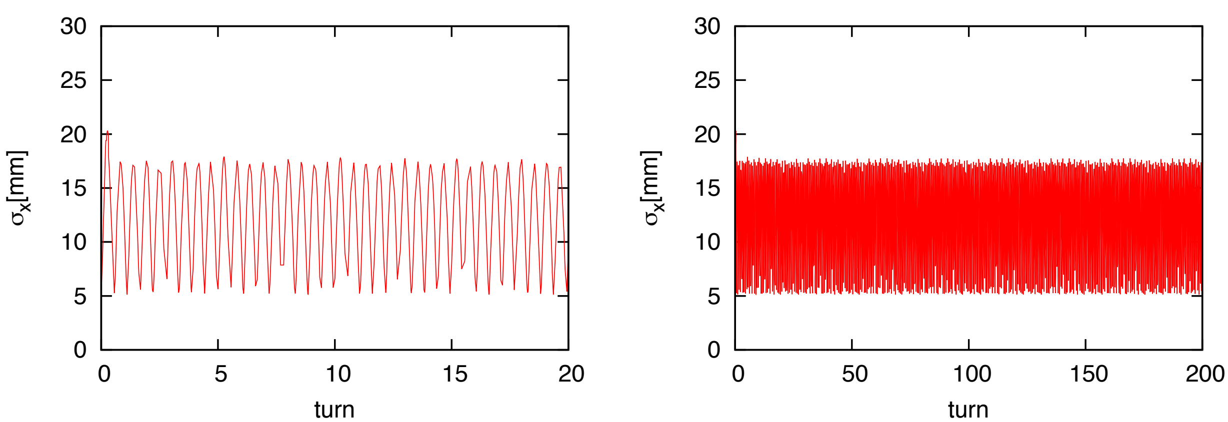

Figure 2: Beam width versus turn. The modulation of the width propagates at $2Q_x$(Number of turns), as is evident in the plot at left ($Q_x = 0.90386$). The range is extended to 200 turns at right. The amplitude of the modulation is unchanged over at least the first 200 turns.

Next apply an RF signal to one of the four ring quads, namely, Quad 1. The RF frequency $f_{RF} = 2(1 - Q_x)f_{rev}$ where $Q_x = 0.90386$, is the horizontal tune. The RF voltage and phase are adjusted to achieve a good match within the first 25μs. As shown in Figure 3, with $V = 1.6$kV, and $\phi = 0.27(2\pi)$, the mismatch is essentially eliminated, with final width about half the peak width of the mismatched beam.

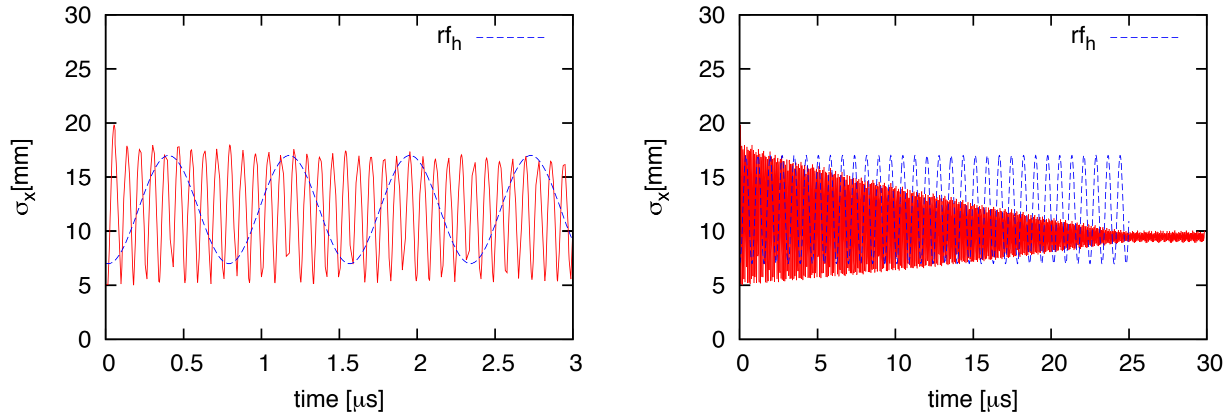

Figure 3: The RF signal is superimposed on the plot of width vs time. The RF frequency $f_{RF} = 2(1 - Q_x)f_{rev}$.

3.1 Quad multipoles

The quad nonlinearities introduce a dependence of betatron tune on particle amplitude. The result is that the distribution decoheres with some characteristic time. If the decoherence time is short compared to the matching time (25μs), the RF modulation will be ineffective. The effect of the multipoles on the width of the distribution is shown in Figure 4 (left). The RF matching (Figure 4 right) is seen to be almost entirely unaffected by the presence of the quad multipoles.

3.2 Finite energy spread

Because there is some spread of momenta in the distribution, the modulation of the horizontal beam size is due to mismatch of dispersion (η) as well as β_h. The dispersion is nominally zero at the exit of the inflector. As a result of the mismatch, the amplitude of the peak dispersion is nearly double that of the closed ring value, as can be seen by comparing Figures 1 left and right. That is $\Delta\eta \sim \eta_{ring}$. We investigate the effect on the dispersion mismatch by introducing the characteristic energy spread to the muon distribution, $\sigma_E/E \sim 0.112\%$.

The dispersion error, $\Delta\eta$ propagates at the betatron tune, (rather than at twice the tune as for the $\Delta\beta_h$). The modulation of the quadrupole voltage that effects the matching β_h from inflector into ring, will in general increase the dispersion mismatch, as is indeed evident from Figure 5. At best the dispersion matching will require a different phase than the β matching. An additional effect of the spread in energies is the associated variation in the period of revolution $\Delta\tau_{rev} = \frac{\sigma_E}{E}\tau_{rev}$, resulting in an accumulating phase error. The phase error increases with time at the rate

$$\frac{d\phi}{dt} = \omega_{RF}\frac{\sigma_E}{E} \sim 0.009 \text{ rad}/\mu\text{s}$$

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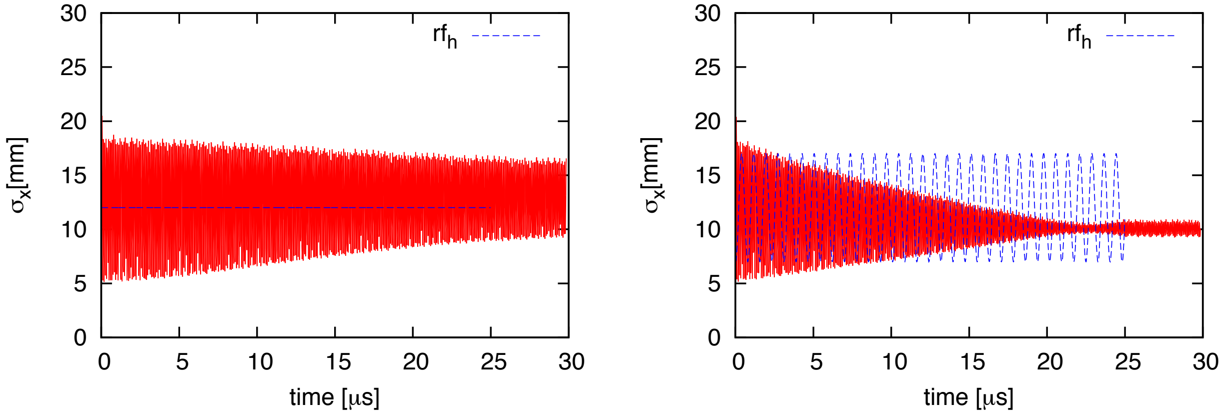

Figure 4: Quad multipoles are included. In the plot at left, with the RF signal off, the effect of the quad nonlinearities is to reduce the depth of the modulation (as compared to Fig. 2 right), without reducing the peak width. In the plot at right, the RF is on and beam width is reduced within 25μs as before. The remaining modulation of the width due to quad nonlinearities is small.

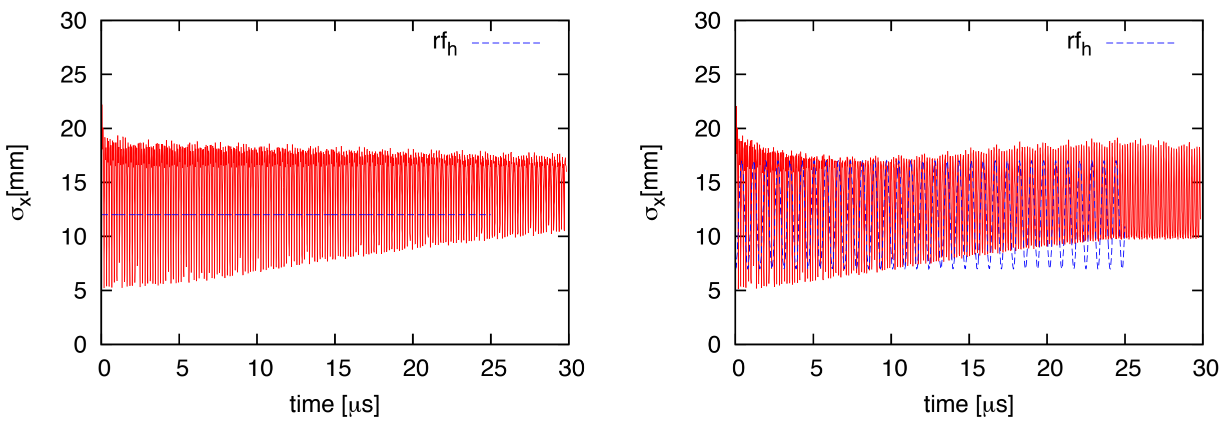

Figure 5: Quad multipoles are included. Energy spread of the muon distribution is 0.112%. At left, with the RF signal off, the effect of the quad nonlinearities and finite energy spread is to reduce the depth of the modulation, and very slightly reduce the peak width. At right, with RF on, beam width is increased. The RF effectively increases the dispersion mismatch.

3.3 Finite bunch length

The muons do not all arrive in the ring at the same time. The phase of the RF voltage will vary by $\omega_{RF}\tau = \pm 0.49$rad from the head of the 120ns long bunch to the tail. We test the sensitivity to the finite length by including the temporal distribution expected for the Fermilab muon bunch in the simulation. The effect is small, as shown in Figure 6

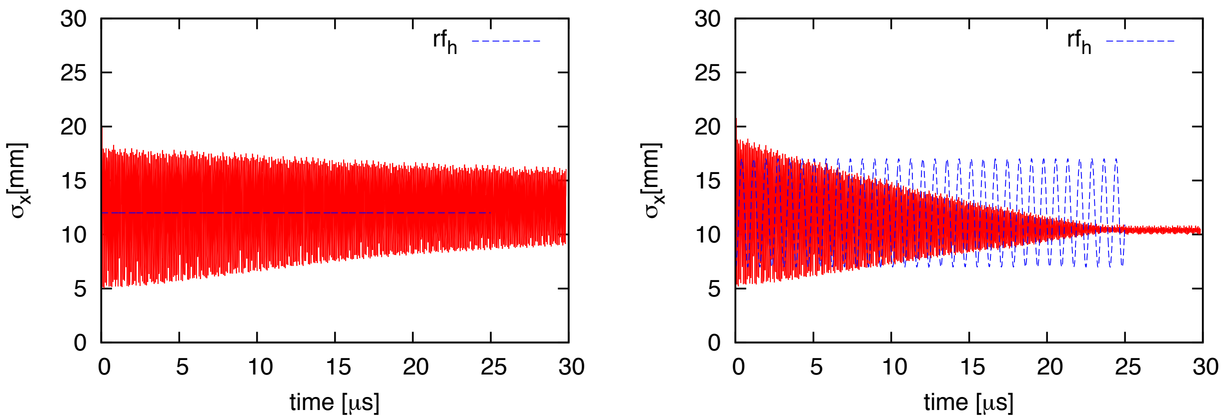

Figure 6: Quad multipoles are included. Length of the muon distributon is increased to 120 ns. Energy spread is set to zero. At left, with the RF signal off, the width is unaffected by the temporal extent of the distribution. At right the response to the RF is very nearly unaffected by the finite length.

3.4 Centroid motion

What if we include centroid motion? The coherent betatron motion propagates at the betatron tune, namely half the frequency of the β error. In general the phase of the RF signal that shrinks the beam width, will increase centroid motion. Figure 7 shows the motion of a distribution with centroid displaced 10mm from the magic radius at the inflector exit. The energy and temporal spread of the distribution are both near zero. In Figure 8 the RF is on, with the same phase and voltage as in Figure 4. Finally, in Figure 9 put all of the pieces together. The muons have temporal and energy spread as expected for the distribution at Fermilab, and a 10mm centroid offset. The RF drive has phase and amplitude that were effective when the distribution had zero energy spread and centroid offset.

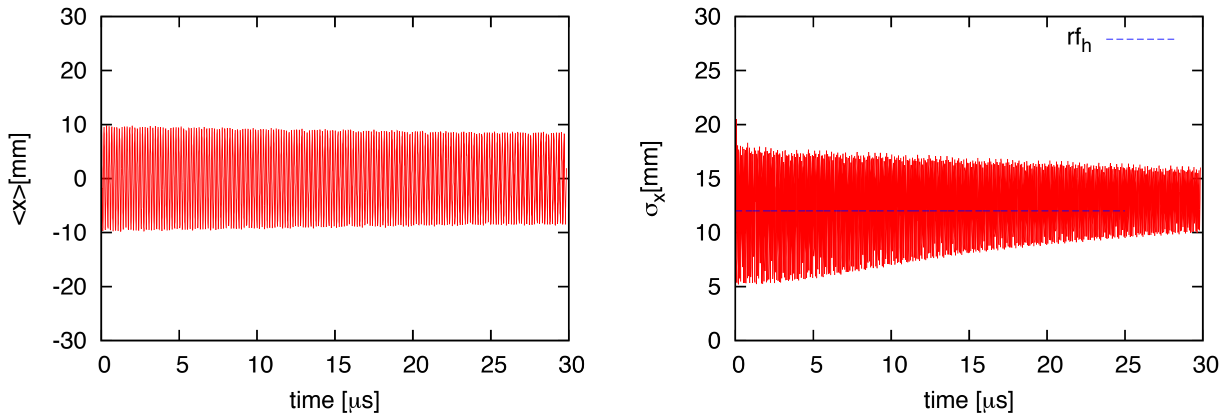

Figure 7: The centroid is horizontally displaced by 10mm. Quad nonlinearities are included but energy and temporal spread are set near zero. RF is off. Displacement of the centroid is shown at left and beam width at right.

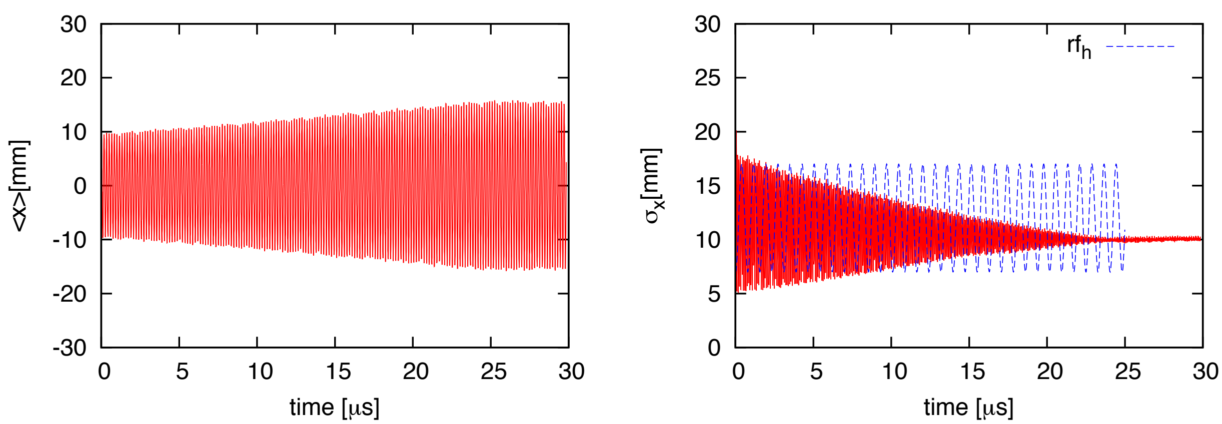

Figure 8: The centroid is horizontally displaced by 10mm. Quad nonlinearities are included but energy and temporal spread are set near zero. At left, the amplitude of the coherent centroid motion increases due the RF drive while at right, the effect of the RF to decrease the width is evidently independent of the amplitude of the centroid motion.

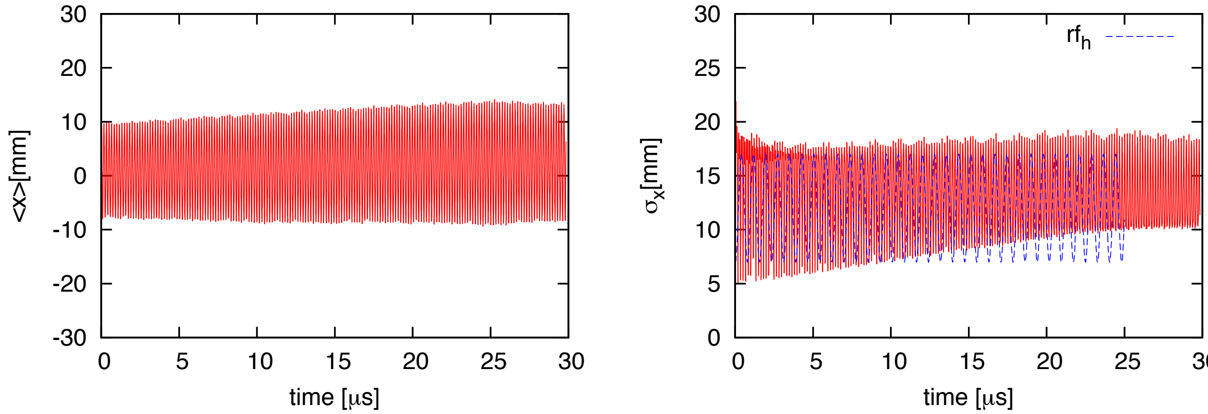

Figure 9: The centroid is horizontally displaced by 10mm. Quad nonlinearities are included. $\sigma_E/E = 0.112\%$, and bunch length is 120 ns. At left, the amplitude of the coherent centroid motion increases with the RF drive, while at right, the effect of the RF is to increase the width of the distribution.

4 Vertical

The RF frequency that matches modulation of the vertical beam size is $f_{RF} = 2(1 - Q_y)f_{rev}$ where $Q_y = 0.43271$. The amplitude ($V_{RF} = 217\ V$) and phase ($\phi_v = 0.16$ rad) are chosen to match vertical phase space in the first $\sim 25\ \mu$s of the fill as shown in Figure 10.

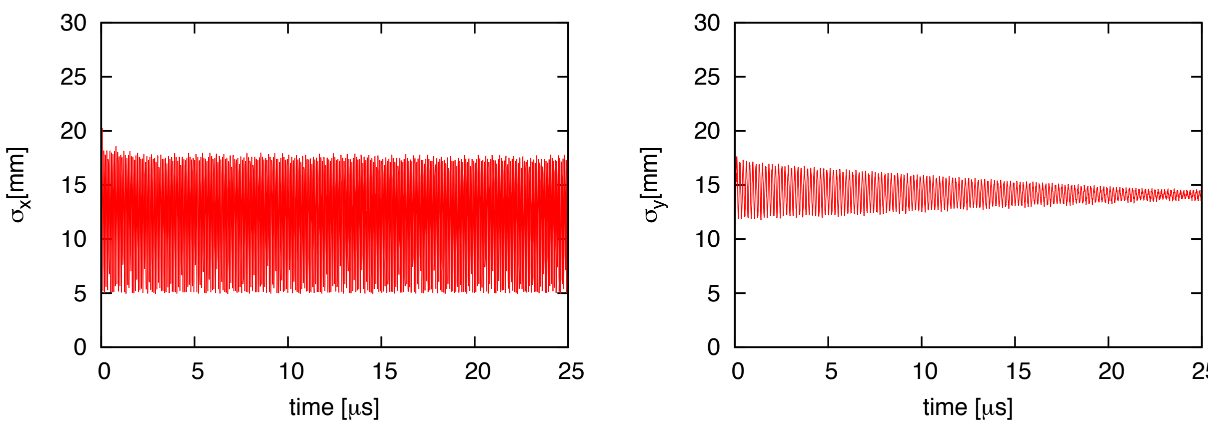

Figure 10: Bunch length and energy spread are near zero. Quad multipoles are turned off. At left, the width (σ_x) of the distribution and at right the height σ_y. The RF frequency is $f_{RF} = 2(1 - Q_y)f_{rev}$.

Next horizontal and vertical RF voltage are superimposed according to

$$V = V_x \cos(2\omega_x t + \phi_x) + V_y \cos(2\omega_y t + \phi_y) \tag{3}$$

In Figure 11 we see that beam width and height are "damped" simultaneously. The vertical size is not altogether unaffected by the signal at the horizontal tune, but that effect is small.

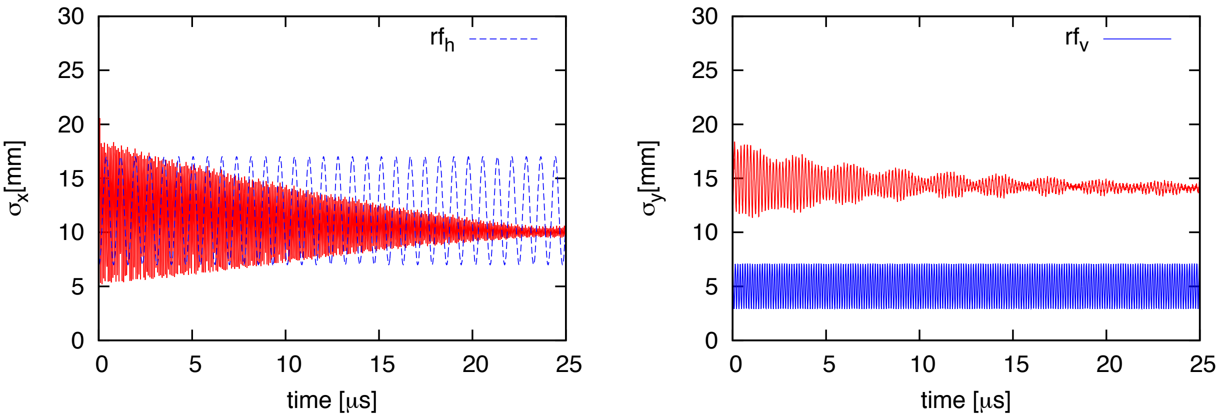

Figure 11: Bunch length and energy spread are near zero. Quad multipoles are turned off. At left, the width (σ_x) of the distribution and at right the height σ_y. The RF frequencies are $f_{RF}^{(x,y)} = 2(1 - Q_{(x,y)})f_{rev}$.

Finally, put all of the pieces together, including quad multipoles, 0.112% energy spread and 120ns bunch length, and an RF signal with components at both twice the horizontal and vertical betatron frequencies. The centroid of the beam is horizontally displaced 10mm. The effect on CBO amplitude, horizontal and vertical beam size is shown in Figure 12.

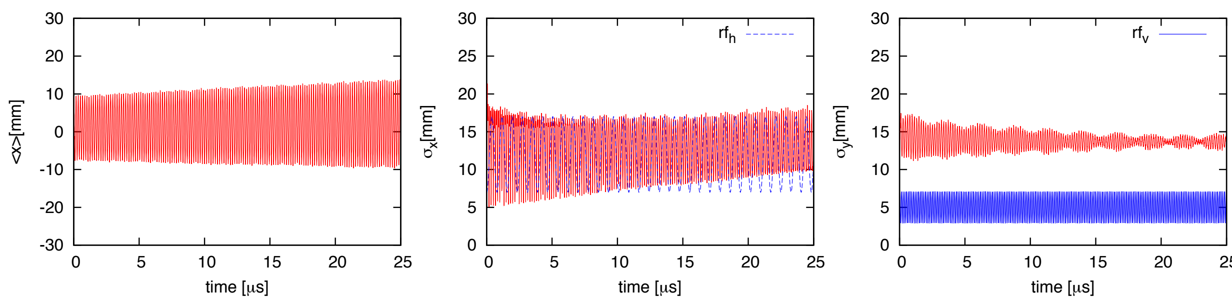

Figure 12: Bunch length and energy spread are near zero. Quad multipoles are turned on At left, the amplitude of the centroid motion (CBO) increases with time due to the RF as does the width (σ_x) of the distribution (center). At right the height σ_y is reduced. The RF frequencies are $f_{RF}^{(x,y)} = 2(1 - Q_{(x,y)})f_{rev}$.

5 Conclusion

The RF "damping" is effective as long as there is no energy spread and no coherent oscillation of the centroid of the distribution. The benefit vanishes with the anticipated energy spread, and can increase the amplitude of centroid motion.