Optimization of Dispersion Matching

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We consider the dispersion matching (or the degree of mismatch) into the ring that will maximize capture and minimize losses in the ring itself. Consider 3 possibilities

1. Choose dispersion in the inflector so that it will match ring dispersion
2. Set dispersion in inflector to be zero to maximize transmission
3. Partial mismatch: Set dispersion through inflector at 1/2 the value required for matching into the ring

In all three cases we will assume that $\beta_x = 2$m and $\beta_y = 19$m are at respective minima halfway through the existing inflector in which case we know that all of the 40mm-mrad phase space volume of the beam will clear apertures (See GM2-doc-1110-v1). There is a mismatch into the ring and as a result the beam size will be modulated in the ring at twice the betatron tunes. The depth of the modulation in the horizontal dimension is $\sqrt{\frac{\beta_{max}}{\beta_{min}}} \sim 4$. The depth of modulation of the vertical size is small, of order ~ 1.1.

Transport through the magnet iron, cryostat and inflector includes the effect of all fringe fields and the inflector fields based on field maps provided by Wuzeng. (See GM2-doc-1109-v2)

1 Matched dispersion

The dispersion function for the closed ring is very close to $\eta_0 = 8$m. Since the aperture of the storage volume is $A \sim \pm 4.0$cm, we find that the maximum energy offset that will store is $\frac{\Delta E}{E} = \frac{A}{\eta} = 0.05$. That is the absolute maximum energy acceptance. This assumes that the dispersion is matched through the inflector to the ring value, because if there is a mismatch, the maximum dispersion in the ring will necessarily be greater than η_0. But therefore, if the dispersion is matched into the ring then the energy acceptance will be determined by the aperture of the inflector ($A_x = \pm 9$mm) and we find $\frac{\Delta E}{E} = \frac{0.009}{8} = 0.011$. No particles outside of the ring energy acceptance (0.05) will make it into the ring. All off energy particles outside of the ring acceptance will have been scraped off in the inflector, or upstream. Indeed we could easily arrange an even larger dispersion in the upstream quadrupole so that we could scrape the off energy particles well before they enter the storage ring. The β and η-functions through the injection channel are shown in Figure 1(left). The corresponding beam sizes where $\sigma = \sqrt{\epsilon\beta + (\eta\delta)^2}$ and $\delta = \Delta E/E = 0.011$ are shown in Figure 1(right). We see that everything that gets through the inflector will comfortably fit into the ring aperture. (Of course we are neglecting scattering in inflector end coils and quad plates.)

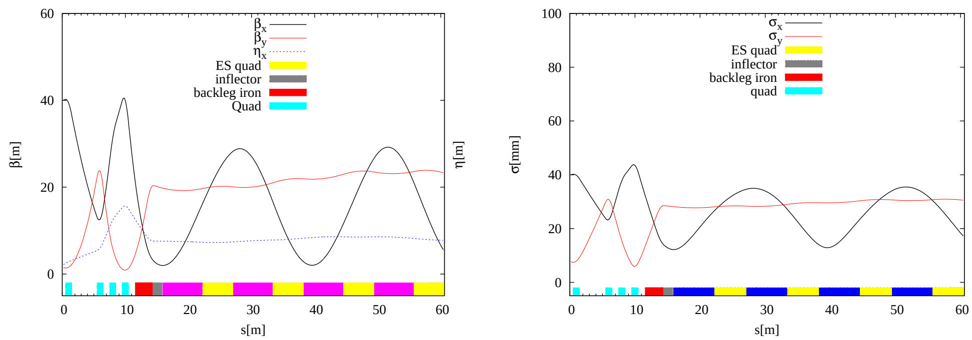

Figure 1: The horizontal axis of the plots begins with the four final focus quadrupoles upstream of the ring, and extends through the backlog iron and inflector and then for two complete revolutions in the ring. (Left)Values for , α and β at the upstream end of the injection channel (s=0), are chosen so that $\beta_x = 2.1$m and $\beta_y = 19.6$m at the midpoint of the inflector. η is chosen to match value of closed ring. (Right) Beam size $\sigma = \sqrt{\epsilon\beta + (\eta\delta)^2}$ and $\delta = \Delta E/E = 0.011$

2 Zero dispersion in inflector

If the dispersion is zero through the injection channel and into the inflector than there is a substantial mismatch into the ring. The result is that there will be dispersion modulation that will advance at the betatron tune and with a depth $\frac{\eta_{max}}{\eta_{min}}$. $\eta_{min} = 0$ since that is the value at the exit of the inflector by design, making for a large modulation depth. The maximum dispersion will be twice the matched value, namely $\eta_{max} = 2\eta_0 = 16$m. All energies will clear the inflector. But the ring acceptance will be only $\frac{\Delta E}{E} = 0.025$. All energies outside of ± 0.025 (with zero betatron amplitude) will be scraped off in the ring. The β and η-functions through the injection channel and the corresponding beam sizes with $\Delta E/E = 0.05$ are shown in Figure 2. From Figure 2(right) we see that many muons our outside of the ± 4cm aperture and will be lost in the ring.

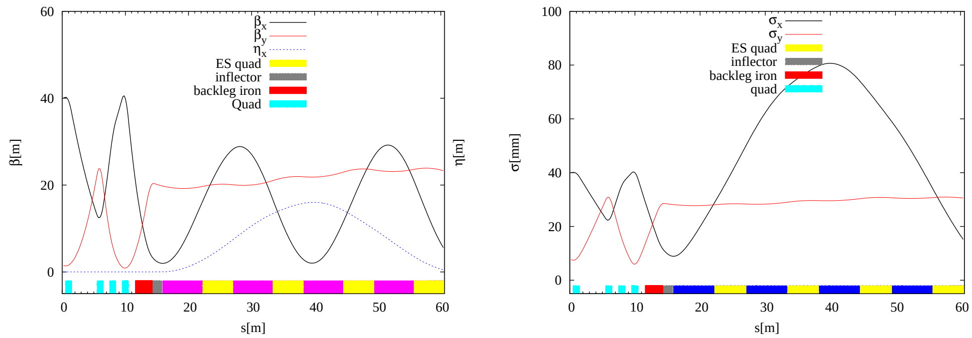

Figure 2: (Left)Values for , α and β at the upstream end of the injection channel (s=0), are chosen so that $\beta_x = 2.1$m and $\beta_y = 19.6$m at the midpoint of the inflector. η at $s = 0$ is chosen so that $\eta = 4$m in the middle of the inflector. (Right) Beam size $\sigma = \sqrt{\epsilon\beta + (\eta\delta)^2}$ and $\delta = \Delta E/E = 0.05$

3 Partial mismatch, $\eta_{inflector} = \frac{1}{2}\eta_0$

If $\eta_{inf} = \frac{1}{2}\eta_0$, then the maximum and minimum dispersion in the ring are $\eta_{max} = 12$m and $\eta_{min} = 4$m respectively. The depth of the energy modulation is 3. Muons with energy within the range $\frac{\Delta E}{E} = \frac{0.009}{4} = 0.023$ will clear the inflector (assuming zero betatron amplitude) and all of those energies will be within the acceptance of the ring. It would seem that this middle ground is optimal as a maximum energy spread is captured and off energy particles that will not fit within the ring aperture are scraped away before they enter the ring. The twiss functions and beam sizes with $\Delta E/E = 0.025$ are shown in Figure 3.

4 Conclusion

We find that while the nominal energy acceptance of the ring is $\Delta E/E = 0.05$, the effective energy acceptance of the combination of the existing inflector and ring is only $\frac{\Delta E}{E} \sim 0.025$. If the optics of the injection line are arranged so that there is zero dispersion in the inflector, all particles with energies outside the effective acceptance are lost in the ring (and into the detectors) and the off energy particles remaining contribute a large beam size modulation at the betatron tune. If the injection line optics are arranged so that there is large dispersion in the final horizontally focusing quadrupole, then the off energy particles outside the effective acceptance can be scraped away before entering the ring. Such an optimized optical configuration is closest to the *partial* dispersion mismatch shown in Figure 3.

In the event that the E821 inflector is replaced with a device with larger horizontal aperture, then the balance between optimum dispersion . If for example, the horizontal

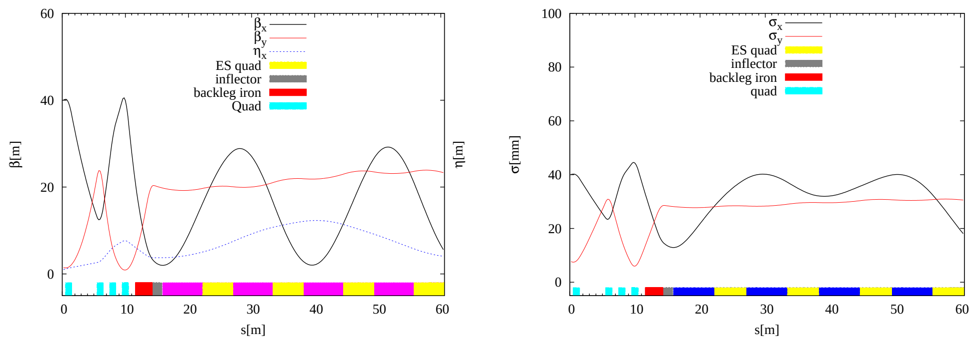

Figure 3: (Left)Values for $,\alpha$ and β at the upstream end of the injection channel (s=0), are chosen so that $\beta_x = 2.1$m and $\beta_y = 19.6$m at the midpoint of the inflector. η at $s = 0$ is chosen so that $\eta = 4$m in the middle of the inflector. (Right) Beam size $\sigma = \sqrt{\epsilon\beta + (\eta\delta)^2}$ and $\delta = \Delta E/E = 0.025$

aperture of the inflector is increased to ± 20mm from the E821 aperture ± 9mm, then we capture more beam if the dispersion in the inflector is increased to 6 m. Then the energy aperture of the inflector would be $\frac{\Delta E}{E} = 0.02/5.5 = 0.36\%$. The maximum and minimum dispersion in the ring would be $\eta_{max} = 10.5$m and $\eta_{min} = 5.5$m. The ring energy acceptance becomes $\frac{\Delta E}{E} = \frac{0.04}{10} = 0.38\%$, so that everything that cleared the inflector would be accepted into the ring.

It would be best if the upstream optics were arranged so that the dispersion in the inflector can be tuned over the range $4\text{m} < \eta_{inf} < 8\text{m}$ in order that we have the flexibility to optimize the matching.