A FAST BEAM-ION INSTABILITY *

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Abstract

Mutually driven transverse oscillations of an electron beam and residual gas ions may result in a fast transverse instability. This effect arises either during a single pass of a train of electron bunches or it is caused by ionization electrons oscillating within a single positron bunch. In both cases, the beam oscillations grow exponentially with an exponent proportional to the square root of time. In this report, instability rise times are calculated analytically and compared with computer simulations. The effect considered could be a significant limitation in many future designs.

I. INTRODUCTION

The effect we describe arises during the passage of a single electron bunch train or a single positron bunch; ions (or ionized electrons) created by the head of the train (bunch), via ionization of the residual gas, perturb the tail. Under certain conditions a fast transverse beam-ion instability can develop. The instability mechanism is the same in linacs and storage rings where we assume that the ions are not trapped from turn to turn. It differs from instabilities previously studied [4], where the ions, usually treated as being in equilibrium and trapped over many turns, interact with a circulating electron or antiproton beam. By contrast, the instability discussed in this report occurs in a transport line, linac, or a storage ring with a clearing gap to prevent ion trapping. In this paper we outline the basic ideas. For more details we refer to Refs. [1] and [2].

In Section II, instability rise times are calculated analytically. Section III compares the results of computer simulations with the analytical prediction. In Section IV rise times are evaluated for several operating or proposed storage rings and linear accelerators. Section V is devoted to a brief discussion of possible remedies. A summary is given in Section VI.

II. ANALYTICAL TREATMENT

The vertical motion of the beam and the ions or electrons that are generated during the beam passage via ionization may, in linear approximation, be described by two equations of motion. The first equation reads:

$$\left(\frac{d^2}{ds^2}+\omega_\beta^2\right) y_b(s,z) = K\Gamma(z)\,\left(y_i(s,s+z)-y_b(s,z)\right)\,. \quad (1)$$

The coordinate s denotes the longitudinal position along the beam line or storage ring. Equation (1) represents the vertical motion of the beam centroid $y_b(s,z)$ at a distance z from the bunch (or bunch train) center. In our convention positive values of z refer to trailing particles. The motion is a combination of: a betatron oscillation due to external focusing, represented by a harmonic oscillator of frequency $\omega_\beta \approx 1/\beta_y$; and a driving force that is proportional to the distance of beam and ion centroids, and also to the number of generated ions and thus to an integral over the beam density, $\Gamma(z) \equiv \int_{-\infty}^{z}\rho(z')dz'$, normalized such that $\Gamma(\infty)=1$. Here, and in the following, the term "ions" is understood as "ions or electrons, respectively." Finally, the coefficient K is

$$K \equiv \frac{2\lambda_{ion}(p_{gas})r_e}{\gamma\,\Sigma_y(\Sigma_y+\Sigma_x)} \approx \frac{4\lambda_{ion}(p_{gas})r_e}{\gamma 3\sigma_y(\sigma_x+\sigma_y)}\,, \quad (2)$$

where γ denotes the relativistic factor $\gamma = E/(mc^2)$ for the beam, r_e is the classical electron radius, and $\Sigma_{x,y}^2 \approx \frac{3}{2}\sigma_{x,y}^2$ is the sum of the squares of rms ion-cloud size and beam size $\sigma_{x,y}$. Assuming a cross section for collisional ionization of about 2 Mbarns (corresponding to carbon monoxide at 50 GeV) the density λ_{ion} of ions per meter at the end of the bunch (or bunch train) is $\lambda_{ion} \approx 6Np_{gas}$[torr], where N is the total number of particles in the beam and p_{gas} the residual gas pressure in torr. The second equation,

$$\frac{d^2\tilde{y}_t(s,t)}{dt^2} + \omega_i^2(t-s)\tilde{y}_t(s,t) = \omega_i^2(z)y_b(s,t-s)\,, \quad (3)$$

describes the oscillation of a transverse slice of ions inside the beam. It is here written as an equation in time t for a fixed position s. The variable $\tilde{y}_t(s,t)$ is the vertical centroid of the transverse slice of ions.. For convenience, here and in the following, the time t is quoted in units of length obtained from the actual time by multiplication with the velocity of light c. At a certain time t, beam particles at a distance $z=t-s$ from the bunch center reach the location s. Their centroid position is therefore $y_b(s,t-s)$. The oscillation frequency $\omega_i(t-s)=\omega_i(z)$ is proportional to the square root of the beam density ρ. In the case of electrons oscillating inside a single positron bunch, ω_i is given by $\left(4N\rho(z)r_e/(3\sigma_y(\sigma_x+\sigma_y))\right)^{\frac{1}{2}}$. For ions and an electron bunch train we have $\omega_i \equiv \left((4N_b r_p/(3L_{sep}\sigma_y(\sigma_x+\sigma_y)A)\right)^{\frac{1}{2}}$ where A designates the atomic mass number of the ions, N_b the number of particles per bunch, L_{sep} the bunch spacing, and r_p the classical proton radius.

The solution to Eq. (3) for a slice of ions generated at time $t'=s+z'$ is denoted as $\tilde{y}_t(s,t|s+z')$. The centroid of the ions $y_i(s,t)$ (or electrons) used in Eq. (1) is obtained by averaging $\tilde{y}_t(s,s+z')$ over all possible creation times:

$$y_i(s,t) = \frac{\int_{-\infty}^{z} dz'\rho(z')\tilde{y}_t(s,t|s+z')}{\int_{-\infty}^{z}\rho(z')dz'}\,. \quad (4)$$

Several approximations have been made so far. For instance, the force between beam and ions is assumed to be linear. Any Landau damping caused by the lattice is ignored. It is supposed that inside a bunch train the ions are not overfocused, but that they are lost between different trains. Ions generated by synchrotron

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radiation are ignored. To further simplify the calculations, we will now approximate the longitudinal bunch density $\rho(z)$ by a homogeneous rectangular distribution of length $2z_0$. The oscillation frequency ω_i is then constant inside the bunch (or along the bunch train). Equations (1), (3), and (4), can be combined into a single integral equation for the beam centroid $y_b(s,z)$ alone. The latter can be solved either as a perturbation series in K/ω_β [1] or by an averaging method [2]. The asymptotic solution for large distances s is

$$y_b(s,z) \approx \hat{y}\frac{1}{4\sqrt{\pi}}\frac{1}{\eta^{\frac{1}{4}}}e^{2\sqrt{\eta}}\sin(\omega_i z - \omega_\beta s + \theta)\ , \qquad (5)$$

where $\hat{y}$ is the initial Fourier component at frequency ω_i in the longitudinal beam distribution, and $\eta(s,z)$ denotes the dimensionless function $\eta(s,z) \equiv \left(K\omega_i(z+z_0)^2 s/(\omega_\beta 16 z_0)\right)$. Asymptotically, the oscillation amplitude grows roughly as $\exp(\sqrt{s/\tau_{asym}})$, where τ_{asym} is the time at which the exponent $2\sqrt{\eta}$ in Eq. (5) equals one. Note that τ_{asym} is not an e-folding time because the exponent is proportional to the square root of time. In the multi-bunch case, the asymptotic rise time τ_{asym} for trailing bunches can be expressed in terms of more basic parameters as [1]

$$\tau_{asym,e-}[\mathrm{s}] \approx \left[\frac{6p[\mathrm{torr}]\, N_b^{\frac{3}{2}} n_b^2 r_e r_p^{\frac{1}{2}} L_{sep}^{\frac{1}{2}} c}{\gamma \sigma_y^{\frac{3}{2}}(\sigma_x+\sigma_y)^{\frac{3}{2}} A^{\frac{1}{2}}\omega_\beta}\right]^{-1}\ , \qquad (6)$$

where N_b denotes the number of particles per bunch and n_b is the number of bunches. All quantities, except for the pressure, are given in SI units. A similar expression can be found for a single positron bunch. [1] In the asymptotic limit, ion and beam motion are of similar amplitude and in phase.

III. COMPUTER SIMULATIONS

To study this instability, we have written a computer simulation. The simulation treats the beam, the ions, and the ionized electrons as collections of macroparticles whose distributions are allowed to evolve self-consistently. Each bunch in the beam is divided into slices in z. Each slice is then represented by macroparticles whose number is chosen to reflect a Gaussian distribution between $\pm 3\sigma_z$. The initial macroparticle coordinates are random with Gaussian distributions. At four locations in each FODO cell, calculations are performed using a grid in x and y centered at the bunch train centroid. As each beam slice passes, macroparticles are created at the grid points representing the ions and ionized electrons generated by collisional ionization. The beam and ion fields are mapped onto the grid and then interpolated to the macroparticle positions. Ref. [1] presents details of the simulations.

Simulations have been performed for the PEP-II HER, the SLC Positron Arc and the NLC Damping Ring, typically using about 160 000 macroparticles. The results are consistent with the analytical calculation, and confirm the expected scaling of the amplitude growth with time, pressure, ion mass, and longitudinal position z. The absolute rise times found in the simulations agree with the analytical result to within a factor 2 or 3, which is smaller than the spread of values obtained for different random seeds. The analytical solution, Eq. (5), does not include the filamentation of ions: due, for instance, to the variation of the ion oscillation frequency with horizontal position. An approximative analytical solution which takes this ion-decoherence into account [2] predicts a rise time which is about a factor 2 or 3 larger than that of Eq. (6).

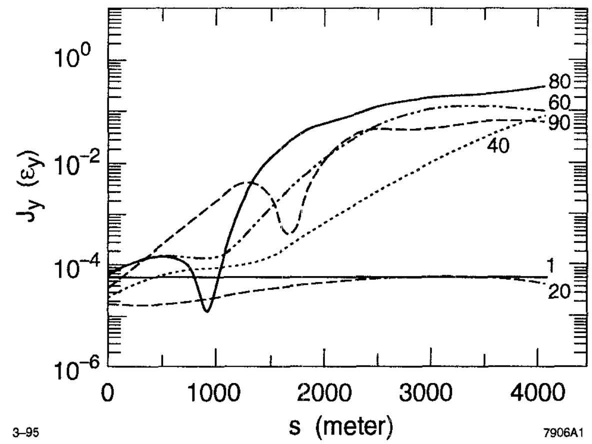

Figure. 1. Action of the vertical centroid as a function of distance for every twentieth bunch of a train of 90 bunches in the NLC-DR with a pressure of 10^{-8} torr.

Figure 1 shows a simulation result for the NLC Damping Ring (DR). The average action $< J_y(s,z) >$ is depicted as a function of the distance s for every twentieth bunch in the train of 90 bunches and a pressure of 10^{-8} torr. The initial amplitudes are due to the finite number of macroparticles. From this figure, the rise time for the trailing bunches is about 170 ns; within the uncertainty of the simulation this is close to the estimate of 47 ns obtained from Eq. (6). In the NLC-DR an average vacuum pressure of or below 10^{-9} torr has to be maintained, in order to sufficiently reduce the growth rate of the beam-ion instability; emittance dilutions due to other gas or ion effects do not require a pressure below 10^{-8} torr.

IV. RISE TIMES FOR SOME ACCELERATORS

Table I shows basic parameters and the asymptotic rise times for several accelerators proposed or under construction at SLAC and KEK: namely for the NLC Electron Damping Ring, the NLC main linac, the PEP-II HER, and for the ATF Damping Ring. Due to its much higher vacuum pressure, the smallest rise time is expected for the ATF Damping Ring. Values for the NLC systems vary between 40 ns and 1 μs. If the initial perturbation is purely due to Schottky noise, it takes about 200 rise times until the bunches oscillate at an amplitude comparable to the beam size. Even with the additional factor 200, the growth times are still very short.

A similar evaluation indicates that the beam-ion instability is not expected to occur in most of the existing accelerators [1]. For instance, the estimated rise time for the SLC e+ Damping Ring, is much larger than the synchrotron period, in which case the instability cannot develop, while the predicted rise time in the HERA electron ring at DESY is about a factor 1–2 larger than the damping time of the transverse multi-bunch feedback. From all the existing machines considered, only the ALS at LBL should show a significant fast beam-ion instability with a rise time of about 2 μs for an average pressure of 10^{-9} torr. Experience so far is unclear. Transverse instabilities are observed, but these are not necessarily caused by ions.

accelerator	NLC e− DR	NLC ML	HER	ATF
n_b	90	90	1658	60
N_b	$1.5 \cdot 10^{10}$	$1.5 \cdot 10^{10}$	$3 \cdot 10^{10}$	10^{10}
$\beta_{x,y}$ [m]	0.5, 5	8	15	0.5, 5
σ_x [μm]	62	35	1,060	22
σ_y [μm]	4	3.5	169	7
z_0	19 m	19 m	1000 m	25 m
E [GeV]	2	10	9	1.54
p [torr]	10^{-9}	10^{-8}	10^{-9}	$6 \cdot 10^{-8}$
τ_{asym}	465 ns	46 ns	6 μs	29 ns

Table I
Parameters and rise times for some future accelerators.

V. POSSIBLE CURES

If the oscillation amplitude of the trailing electron bunches, or positrons, saturates at about 1 σ_y due to the nonlinear character of the coupling force—not included in the analytical treatment—a reduction of the design vertical emittance by a factor of 2 results in about the desired projected final emittance after filamentation [8]. However, it is not yet known if the beam will continue to blow-up (though with decreasing growth rate) after partial filamentation. A second possibility is to use an optical lattice in which the product of the horizontal and vertical beta functions, and thus ω_i, vary substantially. Third, if additional gaps are introduced in the bunch train, the ions are over-focused between the shorter trains [9]. As an example, 10 additional bunch gaps in PEP-II increase the instability rise time from 5 μs to 0.5 ms, which is inside the bandwidth of the feedback system. Finally, in linear accelerators the trailing bunches might be realigned by use of fast kickers and feed-forward.

VI. SUMMARY AND ACKNOWLEDGMENT

The interaction of an electron bunch train or a single positron bunch with ions or ionization electrons can cause a fast transverse instability, which is characterized by an exponential growth of the vertical amplitude. The exponent is proportional to the position along the bunch train (or bunch) and to the square root of time, and is inversely proportional to the 3/4th power of the beam sizes.

The expected rise time of the instability is exceedingly short. For instance, for the various NLC rings and linacs, it varies between 40 ns and 800 ns, while, for the PEP-II HER, it is estimated at 5 μs.

The analytical model used is a linearized approximation and does not include nonlinearities of the ion-beam force or the lattice. However, these nonlinearities are included in the simulations which, for the parameter regimes compared, yield rise times that are in good agreement with the analytical model. In Ref. [2] the linear model is extended to include Landau damping due to the nonlinearity of the beam-ion force; this decreases the growth rate by a factor of two. A large number of questions remain to be answered; among them are the emittance growth due to filamentation and detuning as the oscillation saturates, the effect of synchrotron motion on the growth rate, the rise time in the presence of different ion species, the possible damping due to the nonlinearity of the beam-beam interaction in circular colliders, and the study of coherent oscillation modes of higher order.

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