

## Multiple scattering in injection channel

There is multiple scattering of injected beam in the transfer line due to the titanium windows at each end of the pulsed septum, and the helium gas that fills that septum aperture.

The root-mean-square deflection is given in Perkins for an ultra-relativistic particle as

$$\phi_{rms} = \langle \phi^2 \rangle^{1/2} = \frac{21MeV}{E} \sqrt{\frac{t}{X_0}}$$

where  $E$  is the beam energy,  $t$  the thickness of the material and  $X_0$  the radiation length in the material. The titanium window that isolates CESR from the septum has a thickness of  $0.025\text{mm}^1$ . We assume that there is an identical window isolating septum from synchrotron. The meter long septum magnet aperture is filled with helium at atmospheric pressure.

$$X_0(\text{titanium}) = 35.6\text{mm}$$

$$X_0(\text{helium}) = 4.8 \times 10^3\text{m}$$

Then

$$\phi_{rms} = \frac{21MeV}{E} \left[ \left( \sqrt{\frac{2(0.025)}{35.6}} \right)^2 + \left( \sqrt{\frac{1}{4.8 \times 10^3}} \right)^2 \right]^{1/2} = 4.7 \times 10^{-4}$$

at  $E = 1800\text{MeV}$ . Along one axis in a plane it will be

$$\frac{1}{\sqrt{2}} \phi_{rms} = 3.3 \times 10^{-4}$$

The contribution from the helium gas to the multiple scattering is about 8%.

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## Emittance and beam size of injected bunch

Since the angular divergence of the beam

$$\sigma' = \sqrt{\frac{\epsilon}{\beta}}$$

We estimate the contribution to its emittance as

$$\epsilon_x = (3.3 \times 10^{-4})^2 \beta = 4.6 \times 10^{-6} (m - rad)$$

and

$$\epsilon_y = (3.3 \times 10^{-4})^2 \beta = 1.9 \times 10^{-6} (m - rad)$$

using  $\beta_x = 42m$  and  $\beta_y = 17m$ , namely the value at the injection point in CESR in the BMAD\_1843MEV.LAT optics. It is assumed that the twiss parameters in the transfer line match CESR. Then the rms beam size of the injected bunch in CESR at peak  $\beta = 40m$  is  $\sigma_x = 13.5mm$ .

## Energy dependence

The emittance scales as the square of the energy. At  $E = 5.3GeV$ ,  $\epsilon = 5.3 \times 10^{-7}m-rad$  and the size of the injected bunch is  $\sigma_x = 4.6mm$

## Beryllium window and evacuated septum

The radiation length in beryllium is  $X_0 = 353mm$  If we replace each of the 0.025mm thick titanium windows with 0.05mm thick beryllium, then the multiple scattering in the helium is nearly as much as in the two windows and the emittance of the injected bunch at 1800MeV is  $\epsilon_x = 1.4 \times 10^{-6}m-rad$ . If the septum magnet aperture is evacuated, the emittance is reduced to  $\epsilon_x = 8.1 \times 10^{-7}m-rad$ . The corresponding beams size in CESR at  $\beta = 40m$  is  $\sigma_x = 5.7mm$ .

Table 1: Material parameters

Material	Thickness	$X_0$	$t/X_0$
Ti	0.025mm	35.6mm	$7 \times 10^{-4}$
He	1m	$4.8 \times 10^3$ m	$2.1 \times 10^{-4}$
Be	0.05mm	353mm	$1.4 \times 10^{-4}$

Table 2: Emittance and beam size

Energy	window	gas/pressure/length	emittance	$\sigma_x$
5.3	Ti (2X0.025)mm	He(1A-1m)	0.53mm-mrad	4.6mm
1.8	Ti (2X0.025)mm	He(1A-1m)	4.6mm-mrad	13.5mm
1.8	Be(2X0.05)mm	He(1A-1m)	1.4mm-mrad	7.5mm
1.8	Be(2X0.05)mm	0	0.81mm-mrad	5.7mm
1.8	Be(2X0.05)mm	He(0.5A-1m)	1.1mm-mrad	6.6mm
1.8	Be(2X0.05)mm	He(1A-0.5m)	1.1mm-mrad	6.6mm