0.1 Touschek parameter

Suppose that we parameterize loss rate (inverse lifetime) as

$$\Gamma = -\frac{1}{I}\frac{dI}{dt} = bI + R$$

where R is the loss rate in the absence of current dependent effects, (scattering from residual gas) and bI characterize the current dependence of loss rate. Then

$$\begin{aligned}
-\int \frac{dI}{I(bI+R)} &= \int dt \\
\frac{\log(I) - \log(bI+R)}{-R} \,\Big|_{I_0}^{I} &= t \,\Big|_{t_0}^{t} \\
\log\frac{I}{I_0} - \log\frac{bI+R}{bI_0+R} &= -R(t-t_0) \\
\log\frac{I(bI_0+R)}{I_0(bI+R)} &= -R(t-t_0) \\
I(bI_0+R) &= I_0(bI+R)Ce^{-Rt} \\
\rightarrow I(bI_0+R-I_0bCe^{-Rt}) &= I_0RCe^{-Rt} \\
\rightarrow I &= \frac{I_0RCe^{-Rt}}{(bI_0+R-I_0bCe^{-Rt})}
\end{aligned}$$

Then since $I(t=0) = I_0$, $C = 1$ and we have

$$I = \frac{I_0Re^{-Rt}}{(bI_0+R-I_0be^{-Rt})}$$

Let's check to see if this is right.

$$\begin{aligned}
\frac{dI}{dt} &= \frac{I_0R(-R)e^{-Rt}}{((bI_0+R)e^{Rt}-I_0b)} - \frac{I_0^2R^2be^{-2Rt}}{(bI_0+R-I_0be^{-Rt})^2} \\
&= -RI - bI^2 \\
\rightarrow -\frac{1}{I}\frac{dI}{dt} &= bI + R
\end{aligned}$$

In the limit where $Rt \ll 1$ we have that

$$I \sim \frac{I_0R}{(bI_0+R-I_0b(1-Rt)}$$

$$\sim \frac{I_0 R}{(I_0 b R t + R)}$$
$$\sim \frac{I_0}{b I_0 t + 1}$$

We define $\tau_B = 1/b$ as the Touschek parameter. It has dimensions of current-time.