Multipoles

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February 13, 2009

If there are no longitudinal fields, the magnetic field can be expressed as

$$B_y(x,y) + iB_x(x,y) = \frac{1}{n!}B_n e{-}i(n+1)\theta_n e^{in\theta} r^n$$

where (R,θ) are polar coordinates corresponding to (x,y). The n^{th} order multipole is

$$\frac{qL}{P_0}(B_y + iB_x) = (b_n + ia_n)(x+iy)^n$$

where P_0 is the momentum. $(P_0/q = B\rho)$. Note that $b_n x^n$ is the kick (in radians) due to the n^{th} order. Sasha likes to expand the vertical field component of a quadrupole, measured in the midplane, in the series

$$\frac{\Delta B}{B} = m_1 x + m_2 x^2 + ...$$

where B is the nominal field at x. For the quadrupole

$$B(x) = \frac{B_0}{r_0}x + c_2 x^2 + c_3 x^3 + ...$$

where B_0 and r_0 are the field at the pole tip and the pole tip radius. Therefore

$$\frac{\Delta B}{B} = \frac{c_2 x^2 + c_3 x^3 + ...}{(B_0/r_0)x} = m_1 x + m_2 x^2 + ..$$

So $m_1 = \frac{r_0 c_2}{B_0}x$ and in general

$$m_n = \frac{r_0 c_{n+1}}{B_0}x^n$$

BMAD

The BMAD manual says that in tracking it uses

$$[a_n(\text{actual}), b_n(\text{actual})] = [a_n(\text{input}), b_n(\text{input})] \cdot \frac{\theta_{nom}(r_0)}{r_0^n}$$

where $\theta_{nom}(r_0)$ is the nominal kick at radius r_0 if there are no multipole errors. For a quadrupole we have that

$$[a_n(\text{actual}), b_n(\text{actual})] = [a_n(\text{input}), b_n(\text{input})] \cdot kL\frac{r_0}{r_0^n}$$

In tracking, the kick due to the n^{th} multipole is $\theta_n \sim b_n^{act} r^n$.

$$\begin{aligned} \theta_n^{act} &= b_n^{act} r^n = b_n^{inp} \theta_0(r_0) \left(\frac{r}{r_0}\right)^n \\ \rightarrow \frac{\theta_n(r_0)}{\theta_0(r_0)} &= b_n^{inp} \end{aligned}$$

Temnykh notation

Now to relate [a_n(input),b_n(input)] to Sasha's m_i. We see that the fractional contribution

$$\begin{aligned} m_i r^i &= \frac{\Delta B_i}{B_{nom}} = \frac{\theta_{i+n_{ref}}(r)}{\theta_0(r)} \\ &= b_n^{inp} \frac{\theta_0(r_0)}{\theta_0(r)} \left(\frac{r}{r_0}\right)^{i+n_{ref}} \end{aligned}$$

Define b_n^{inp} at $r = r_0$ and

$$\begin{aligned} m_i r_0^i &= b_n^{inp} \frac{\theta_0(r_0)}{\theta_0(r_0)} \left(\frac{r_0}{r_0}\right)^{i+n_{ref}} \\ &\rightarrow b_n^{inp} = m_i r_0^i \end{aligned}$$

Mark II quad

The table includes data from Sasha's memo of February 26, 2008. The coefficients are defined according to

$$\frac{\Delta B}{B} = m_1 x + m_4 x^4 + m_{12} x^{12}$$

If $r_0 = 4.0$cm, then

	$m_n[\text{m}^{-n}]$	$b_{n+1} = m_n \cdot r_0^n$
m_1	3.5471×10^{-5}	1.41884×10^{-6}
m_4	9.12×10^2	2.33×10^{-3}
m_{12}	-2.078×10^{16}	-0.5

60 cm IR quad (Q2)

From CBN 96-19 becomes

$m[\mathrm{m}^{-m}]$	Q2E	Q2W
1	-2.55×10^{-3}	7.35×10^{-3}
4	$2.41{\times}10^{1}$	$-1.73{\times}10^{1}$
8	5.65×10^{5}	2.68×10^{6}
12	-7.18×10^{10}	$-1.06{\times}10^{11}$

$b(r_{ref} = 0.04)$	Q2E	Q2W
2	-1.02×10^{-4}	2.94×10^{-4}
5	$6.16{\times}10^{-5}$	$-4.44{\times}10^{-5}$
9	3.70×10^{-6}	1.74×10^{-5}
13	-1.2×10^{-6}	$-1.8{\times}10^{-6}$

0.1 Q00

Multipoles measured versus excitation. $I = 50$A is typical.

Table 1: Q00

b_1	$0.13689 + 6.1347 \cdot I$
b_2	$-0.011053 + 0.00063431 \cdot I$
b_3	$0.00088409 - 0.0013833 \cdot I$
b_5	$-3.2331{\times}10^{-5} + 0.00010689 \cdot I$
b_9	$2.1285e - 8 - 1.701{\times}10^{-7} \cdot I$
a_2	$0.00080125 - 0.00055671 \cdot I$
a_3	$0.00022806 + 1.5123{\times}10^{-5} \cdot I$

PEM quad

Q1E

Multipoles depend on excitation. $I = 100$A is typical for the PEM quads in the low energy CTA optics. Note that in the language of BMAD Qxxx[$\mathrm{b_m}$] $= b_m/b_1$

Table 2: PEM Q1E

b_1	$-0.034232 + 2.8677 \cdot I$
b_2	$-0.025623 + 0.00061253 \cdot I$
b_3	$0.0007069 + 0.00097505 \cdot I$
b_4	$0.00028722 + 2.5505{\times}10^{-5} \cdot I$
b_5	$-6.5954e - 05 - 0.0001738 \cdot I$
a_2	$0.0047057 - 0.001142 \cdot I$
a_3	$0.003029 - 7.1252{\times}10^{-5} \cdot I$
a_4	$-3.2331e - 05 + 1.4096{\times}10^{-5} \cdot I$
a_5	$-8.9376e - 05 + 1.6747{\times}10^{-7} \cdot I$

Q1W

Table 3: PEM Q1W

b_1	$0.28368 + 2.7106 \cdot I$
b_2	$0.0028307 - 0.024744 \cdot I$
b_3	$-0.0016348 - 0.001014 \cdot I$
b_4	$0.00019477 - 0.000222 \cdot I$
b_5	$1.6563e - 5 - 0.00017878 \cdot I$
a_2	$0.01033 - 0.0034731 \cdot I$
a_3	$-0.0021764 - 2.0905{\times}10^{-5} \cdot I$
a_4	$-3.1489{\times}10^{-5} + 0.00027539 \cdot I$
a_5	$0.00013888 + 2.5687{\times}10^{-5} \cdot I$

Bending magnet

Sextupole

Vertical steering

According to CON 96-5, (A. Mikhailichenko), for 1 kA-turns, the horizontal field dependence as a function of vertical displacement is

$$B_x(y, x = 0) \approx 234. - 0.16 \cdot y^2 - 0.19 \cdot y^4 + 1.53 \cdot 10^{-3} y^6 \ [Gauss]$$

Table 4: Bend magnet

n	$m_n[\mathrm{m}^{-n}]$	n	$b_n = m_n r_0^n$
1	-2.46×10^{-3}	1	-9.85×10^{-5}
2	-4.85×10^{-2}	2	-7.76×10^{-5}
12	-5.78×10^{11}	12	-9.70×10^{-6}