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September 8, 2008

BPM Tilt and Crunch vs Shear

The BPM buttons are mounted as a top block and a bottom block, each with two buttons. So the distance between the two buttons in each block is well defined. The two blocks are very nearly parallel. But there may be a horizontal displacement of the top block compared to the bottom block. Then

$$\begin{aligned} x_4 &= -x_1 = x_0 - \Delta x \\ x_2 &= -x_3 = x_0 + \Delta x \\ -y_1 &= -y_2 = y_3 = y_4 = y_0 \end{aligned}$$

Assuming that all buttons have the same gain, we can wite for $\Delta x/x_0 \ll 1$,

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & \frac{3\Delta x}{r_0^2} y_0 \\ (-1 + \frac{3x_0^2}{r_0^2})\frac{\Delta x}{y_0} & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

In terms of g_x, g_y and ϕ,

$$\begin{aligned} 1 &= g_x \cos(\theta + \phi) \\ \frac{3\Delta x}{r_0^2} y_0 &= g_x \sin(\theta + \phi) \\ (-1 + \frac{3x_0^2}{r_0^2})\frac{\Delta x}{y_0} &= -g_y \sin(\theta - \phi) \\ 1 &= g_y \cos(\theta - \phi) \end{aligned}$$

Then if we assume that θ and ϕ are both small

$$\begin{aligned} 2\theta &\approx \left(\frac{1}{y_0} + \frac{3}{r_0^2}(-\frac{x_0^2}{y_0} + y_0)\right)\Delta x = \left(1 + \frac{3}{r_0^2}(-x_0^2 + y_0^2)\right)\frac{\Delta x}{y_0} \\ \rightarrow \theta &\approx \frac{1}{2}(1 - 3\cos 2\beta)\frac{\Delta x}{y_0} \qquad (1) \end{aligned}$$

where $\beta = \tan -1(y_0/x_0)$ and

$$\begin{aligned} 2\phi &\approx \left(\frac{-1}{y_0} + \frac{3}{r_0^2}(\frac{x_0^2}{y_0} + y_0)\right)\Delta x = \left(-1 + 3\frac{x_0^2 + y_0^2}{r_0^2}\right)\frac{\Delta x}{y_0} \\ \rightarrow \phi &\approx \frac{\Delta x}{y_0} \qquad (2) \end{aligned}$$

Conclusion

In the limit where $x_0 = y_0$, the tilt angle $\theta \rightarrow \frac{1}{2}\phi$. In the limit of a very wide spacing, $x \gg y_0$, then $\beta \rightarrow 0$ and $\theta \rightarrow -\frac{1}{2}\phi$. Finally, if the chamber is tall, $\beta \rightarrow \frac{\pi}{2}$ and $\theta \rightarrow 2\phi$.

Example

For the CESR vacuum chamber $x_0 = 12.98$mm, $y_0 = 24.92$mm. Consider the shear $\Delta x =$ 1mm. Then

$$\begin{aligned} \phi &= 0.04\text{rad} \\ \theta &= 0.054\text{rad} \end{aligned}$$

and

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0.094 \\ -0.011 & 1 \end{pmatrix}$$