2-D Fourier Transforms of Wiggler Fields and Adjustment of End Pole Lowerings

1. Two-Dimensional Fourier Transforms of Wiggler Field

- FT in each dimension decoupled ⇒ linear problem

  ⇒ Easy if you can figure out the indices (≃ three weeks)

  ⇒ First systematic way of obtaining k_x

- Get all 3 components in all 3 dimensions simply from knowledge of B_Y in one plane (Really? YES.)

  ⇒ Number of terms for given accuracy now known. For example, satisfying DCS criterion (Taylor-series tracking à la Étienne Forest) on maximum k_x, k_y, k_z implies particular accuracy.

- Will it suffice to vary the fit coefficients alone?
  Nope. Fit needs freedom to vary k_x and k_z.

- Does use of FT as input to fit help?
  YES. Got best fit to date last night with many fewer iterations. Limit of improvement not yet known.

2. Effect of Changing End Pole Lowering from 3.5 to 3.6 mm (Present models mistakenly use 3.5 mm)

- Transverse field profile

  ⇒ B_Y changes by less than 0.5 per mil

- Tracking result

  ⇒ Transverse kick changes by 20 μrad

Analytic Form for B_x, B_y, B_Z

$$
\begin{aligned}
B_X &= C \ \sin x k_x \ \sinh y k_y \ \cos z k_z \\
B_Y &= C \ \cos x k_x \ \cosh y k_y \ \cos z k_z \\
B_Z &= C \ \cos x k_x \ \sinh y k_y \ \sin z k_z
\end{aligned}
$$

$$k_y^2 = {k_x}^2 + {k_z}^2$$

2-D Fourier Transform for B_Y Table

$$B_Y(i_x, i_z) =$$

$$\frac{1}{N_X\,N_Z} \Sigma_{j=0}^{N_X-1} \ C_j \ \exp\left(2\pi i \ j \, i_x / N_X \ \ \Sigma_{k=0}^{N_Z-1}\right) C_k \ \exp\left(2\pi i \ k \, i_z / N_Z\right)$$

where the C_j and C_k are the 2-D Fourier coefficients ($16 \times 512 = 8192$ of them)