Study of Wakefields and Methods for Their Reduction in an Energy Recovery Linac

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The defining characteristic of the Energy Recovery Linac (ERL) is the high efficiency derived from recycling particle beam energy. The efficacy of the ERL is greatly enhanced when the particles in the beam are close in energy, a criteria that minimizes particle loss during beam transit. A pipe structure comprised of a thin dielectric layer with coupling devices attached at the ends may fulfill this criteria for the bunch specifications of the ERL by using extracting wakefields in the deceleration phase and injecting them to correct wakefields in the acceleration phase. Simulations were used to pinpoint usable design specifications for this novel approach and demonstrate its plausibility for use in the implementation of an ERL.

I. INTRODUCTION

Energy Recovery Linacs have been heralded for their efficiency; beam energy is recycled by deceleration and reacceleration phases of operation, resulting in a relatively inexpensive, yet bright, x-ray source. The first major ERL x-ray source is being designed at Cornell to supersede the present light source, the Cornell High Energy Synchrotron Source.[4] The deviation from perfect efficiency can be attributed in part to energy losses in the form of wakefields. Wakefields are excited by the beam due to the shunt impedance of the structures the beam traverses, which is a function of structural geometric discontinuities, pipe resistance, and dielectric properties of the materials used in the construction of the pipe structure. Parasitic losses due to wakefields spread the energy of the constituents of the beam.[1]

The focus of this project is the optimization of the design of a structure that reduces the energy spread of the particle beam without sacrificing efficiency.[2] The basic schematic is based on the "dielectric power extractor" presented in a paper published for the Argonne National Laboratory for the Argonne Wakefield Accelerator along with the details governing optimal RF power extraction.[3] Figure 1 depicts a cross-sectional view of the dielectric power extractor as conceived at ANL. In the present study, the structure was placed under close scrutiny and many parameters such as the structure length and endpoint taper lengths were varied in an attempt to optimize the structure's characteristics. The goal was to design a structure for a which a mode could be found in which to excite a wakefield with a high quality factor and R/Q value using the two-beam wakefield acceleration scheme described in the aforementioned ANL paper.

The high quality factor of a particular mode in the structure implies that little power is dissipated in the structure. This characteristic insures that the majority of the power generated by the excited wakefield can be coupled out of the structure locally and that the power excited would interfere with the wakefields of neighboring bunches in a well-behaved manner.

R/Q is a measurement of the "shunt impedance" of the structure that is independent of the quality factor.[1] A structure with a large R/Q is advantageous because more power can be

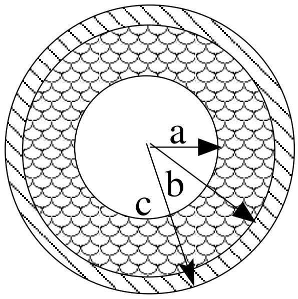

FIG. 1: Basic schematic for the cross section of a dielectric power extractor. The material in the region $a < r < b$ represents a thin film of dielectric material. The material in the region for $b < r < c$ represents the conducting pipe wall. The hollow region represents vacuum through which the bunch traverses.[3]

extracted from the wakefield with fewer structures. A structure with a small R/Q would have to be coupled with many identical structures to produce the same effect as a structure with a large R/Q.

II. DEFINITIONS

$\omega_{RF} \equiv$ Resonance frequency, $\omega \equiv$ bunch frequency, $U_{\text{stored}} \equiv$ Energy stored, $P \equiv$ Power dissipated in c

$$Q \equiv \text{Quality Factor} \equiv \frac{\omega_{RF} U_{\text{stored}}}{P}$$

$$W_{||}(s) \equiv \text{Logitudinal wake as a function of position } s$$

$$\lambda(s) \equiv \text{Bunch charge density as a function of position } s$$

$$k \equiv \text{Loss Factor} \equiv \int_{-\infty}^{\infty} ds \lambda(s) W_{||}(s)$$

$$R/Q \equiv \text{Shunt impedance divided by Q} \equiv \frac{4k(\omega_{RF})}{\omega_{RF}}$$

[1]

III. IDEAL CHARACTERISTICS

Prior to running any simulation, the undesired wakefield produced by a 77 pC 0.6 mm bunch[2] was analyzed to determine the ideal traits of the dielectric power extractor. The wakefield measurements as a function of distance along the pipe can be used to express the wakefield of any bunch length. Given $W_{||}(s)$ for a certain bunch length, $W_{||}(s)$ for other bunch lengths can be calculated as follows:

$$W'_{||}(s) = W_{||}(s) \bigotimes \mathbf{F}^{-1} \left[\frac{\mathbf{F}[\lambda'(s)]}{\mathbf{F}[\lambda(s)]} \right] \quad (1)$$

where $W'_{||}(s)$ represents the longitudinal wake of a different charge distribution, $\lambda'(s)$ and $W^{\delta}_{||}(s)$ represents the Green's response-function wake for the structure. $W^{\delta}_{||}(s)$ does not need to be solved for explicitly. Using a trapezoidal approximation scheme, a program was used to numerically solve eqn. 1 and generate the plot shown in fig. 2. The derivation of eqn. 1 is given in section VIII.

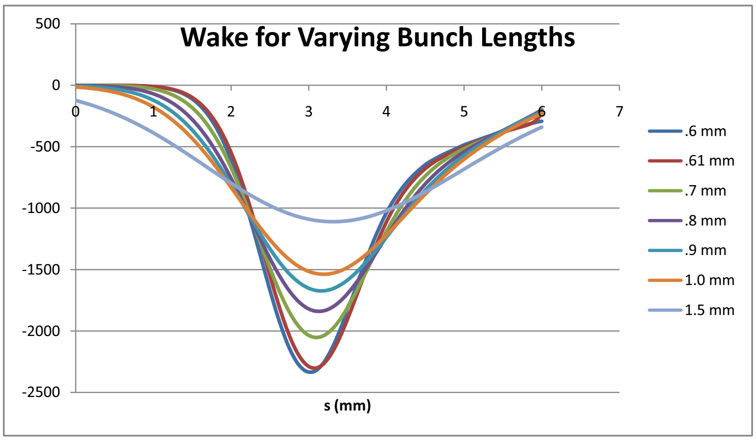

FIG. 2: As the bunch is lengthened, the total wake potential decreases and spreads along the transverse direction.

Choosing the wake of the 1 mm bunch, a manually varied RF voltage was added to the wake to produce the plot in fig. 3. Evidently, a well defined sinusoidal RF wave followed by

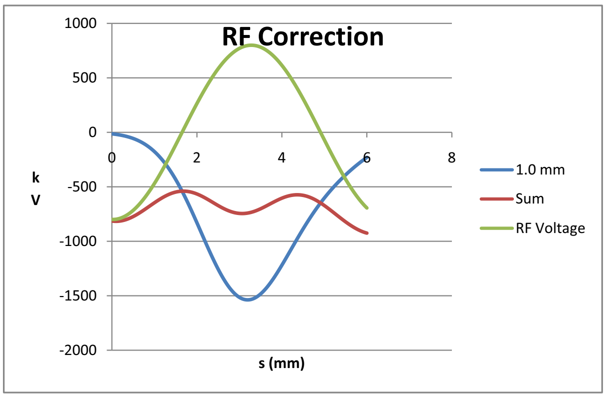

FIG. 3: In this figure, the wake potential of a 1 mm bunch is superimposed with a sinusoidal RF wave.

an accelerating voltage can effectively mitigate the wakefield. The RF wave shown in fig. 3 had a frequency of approximately 45 GHz and an amplitude of 1600 kV. Thus, a usable dielectric power extractor must be able to extract voltages on the order of 2 megawatts to account for additional energy losses in the structure at a frequency in the tens of GHz range. Undershooting the frequency target can be alleviated by further lengthening of the bunch because, as fig. 2 suggests, longer bunches can be corrected with lower frequency RF waves.

IV. POWER EXTRACTION

This section outlines the basic storyboard by which power is extracted from the bunch. A more detailed treatment is given in the paper from the Argonne National Laboratory.[3] The first bunch in a series of bunches traversing through the pipe at frequency ω enters the dielectric power extractor. The bunch then excites a wakefield at a specific mode based on the design of the structure. The voltage it generates is given by eqn. 2.

$$V_{RF} = 2k(\omega'_{RF})q cos(\omega'_{RF}t)e^{-\frac{\omega'_{RF}t}{2Q_{\text{loaded}}}} \tag{2}$$

V_{RF} is a function of the adjusted resonance frequency, the loaded quality factor (assume matched impedance condition), and the loss factor given in eqn. 3. The RF wake in the dielectric propagates at a group velocity slower than the bunch velocity (assumed ideally relativistic in vacuum).

$$k(\omega_{RF}) = \frac{1}{2}\omega'_{RF}\frac{R}{Q}e^{-\frac{{\omega'_{RF}}^2\sigma_z^2}{c^2}} \tag{3}$$

The RF pulse is terminated because the bunch leaves the dielectric power extractor. The total duration of the RF pulse is given by eqn. 4.

$$\tau(\omega'_{RF}) \equiv \text{RF pulse duration for a particular mode with frequency } \omega'_{RF} = \frac{2Q_{\text{loaded}}}{\omega'_{RF}} \tag{4}$$

While the RF pulse is still propagating within the dielectric power extractor, a subsequent bunch enters the structure and excites its own RF wake that superimposes itself with the tail of the existing RF pulse. This superposition continues until the total RF voltage produced peaks at V_o; V_o is expressed in eqn. 8. V_o is a function of the RF pulse length, $\tau(\omega'_{RF})$, and the length between each bunch, given in eqn. 7.

$$n \equiv \text{the \# of bunches that fit on the RF trail of a previous bunch} = \left\lfloor \frac{Q\omega}{\omega_{RF}} \right\rfloor \tag{5}$$

$$\omega'_{RF} = \frac{2Q_{\text{loaded}}\omega}{n} \tag{6}$$

$$t_b \equiv \text{time between bunches} = \frac{2\pi}{\omega} \tag{7}$$

$$V_o = 2kq(\frac{\tau(\omega'_{RF})}{t_b} - 1) \tag{8}$$

Note that for the dielectric power extractor to function properly, the structure must be tuned to an adjusted frequency given by eqn. 6. This ensures that the numerous RF pulses generated by each bunch add (the bunch frequency is a harmonic of the structure frequency). The frequency of the structure, as will be shown, can be tuned by adjusting the lengths or other geometric dimensions of the power couplers attached to the ends of the dielectric layer, or the dielectric itself. A simulation was written to implement this flowchart and automatically tune the resonance frequency and determine the output characteristics of a given mode.

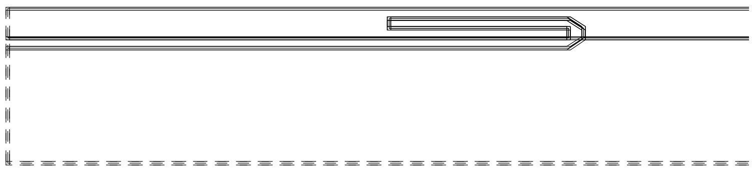

FIG. 4: A transverse view of the dielectric power extractor. The horizontal dashed line on the bottom indicates a rotational transformation about the radial axis of the pipe. The vertical dashed line on the left indicates a reflection so the structure is symmetric.

V. DESIGN PRINCIPLES

The general shape of the dielectric power extractor is depicted in fig 4. As mentioned before, the end objective was to use URMEL-T to discover a mode for a given structure that has a high $\frac{R}{Q}$, high ω_{RF}, and high quality factor. These all manifest themselves by rendering the structure adept at extracting power at a controllable frequency without leaking power outside the structure into the rest of the beam pipe. Batch files were written to process a high volume of files varying the following parameters: the length of the dielectric layer, the thickness of the dielectric layer, the taper angle at the endpoints of the dielectric layer, the size of the beam pipe, and the length of the coupler attached to the endpoints. These parameters were varied independently and the simulation queue constantly changed as a function of the results of the prior simulations. It was discovered that without the couplers attached to the dielectric layer, it was very difficult to identify a structure for which a mode was truly trapped within the dielectric. An example of such a mode is shown in fig. 5. The

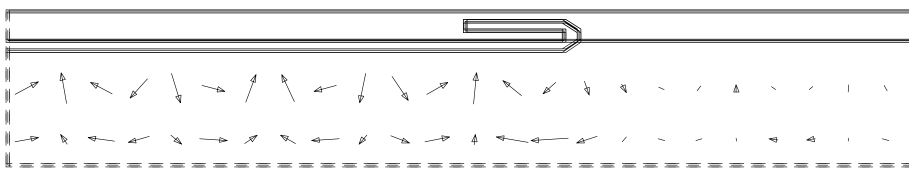

FIG. 5: A structure found to possess a "trapped" mode. The dimensions are: length = 0.05700 m, taper length = 0.00150 m, coupler length = 0.00149 m

corresponding data for this mode is given in table I.

Note that by changing the coupler length, one can "tune" the structure according to the specification of eqn. 6 as long as ω'_{RF} does not deviate from ω_{RF} by much more than 200 MHz. For the mode pictured in fig. 5, simulations alluded to in section IV were used to determine whether this mode met the qualifications outlined in section III. The results of the analysis will be presented in section VI. Several important trends were discovered or verified. First, the frequency of the mode is inversely related to the total length of the dielectric layer and the couplers. Second, As the length is varied such that the quantity $\frac{R}{Q}$ passes through a maximum, the modes neighboring the mode for which $\frac{R}{Q}$ is maximized become more spread out in frequency. Third, $\frac{R}{Q}$ varies sinusoidally as the length of the couplers are varied. This observation is shown in fig. 6 and was instrumental in demonstrating that the tuning range of the dielectric power extractor was comfortably greater than 100 MHz. Fourth, quality factors, Q, continually decrease as the resonance frequencies increase as evident in fig. 7.

TABLE I: Results from a variation in the coupler length for a dielectric length of 0.00150 m.

Coupler Length	FREQUENCY / MHZ	(R/Q)/OHM AT R0	Q FACTOR
0.0177	15505.9	20.808	6036
0.0178	15495.2	21.476	5528
0.0179	15482.6	21.902	5039
0.018	15465.6	21.276	4531
0.0181	15446.8	21.255	4080
0.0182	15424.3	20.9	3680
0.0183	15396	18.862	3329
0.0184	15370.4	21.24	3082
0.0185	15334	18.477	2880

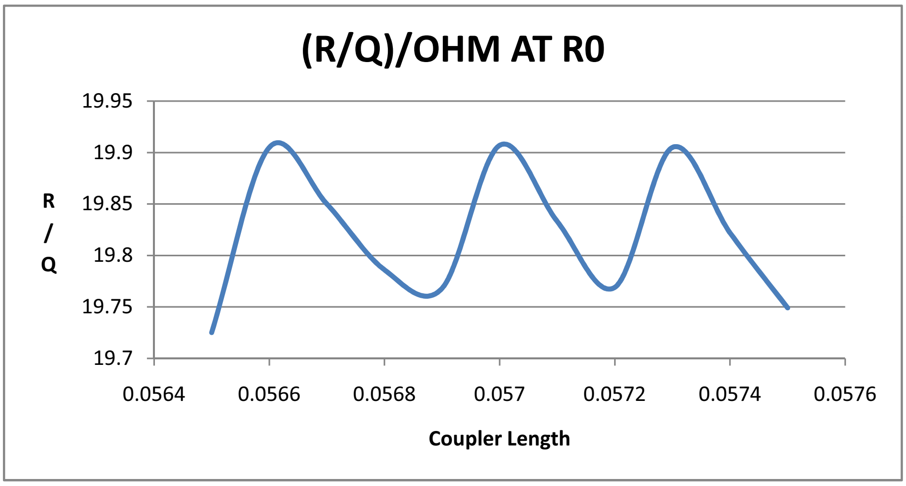

FIG. 6: Plot demonstrating that $\frac{R}{Q}$ varies sinusoidally with the length of the dielectric plus the length of the coupler.

Again, these trends were established by series simulations of URMEL-T input files with minute, independently varied, changes.

VI. RESULTS

After many optimizations, the mode and structure used in attempt to demonstrate the plausibility of this design was mode TM0-EE-16 in a structure with the specifications given in table II.

The specifics regarding mode TM0-EE-16 are as follows: $\omega_{RF} = 15074.2$ MHz, $\frac{R}{Q} = 21.016\Omega$, and $Q = 3848$. Processing this data with the simulation discussed in section IV, the structure was found to have the output characteristics in table III, given a bunch frequency $\frac{\omega}{2\pi}$ of 1.3 GHz and charge of 77 pC.

First, note that the tuned frequency ω'_{RF} falls well within the tuning range previously mentioned. In addition, to extract power on the order of 2 MV, approximately 250 of these dielectric power extractors are needed in phase for the deceleration portion of the

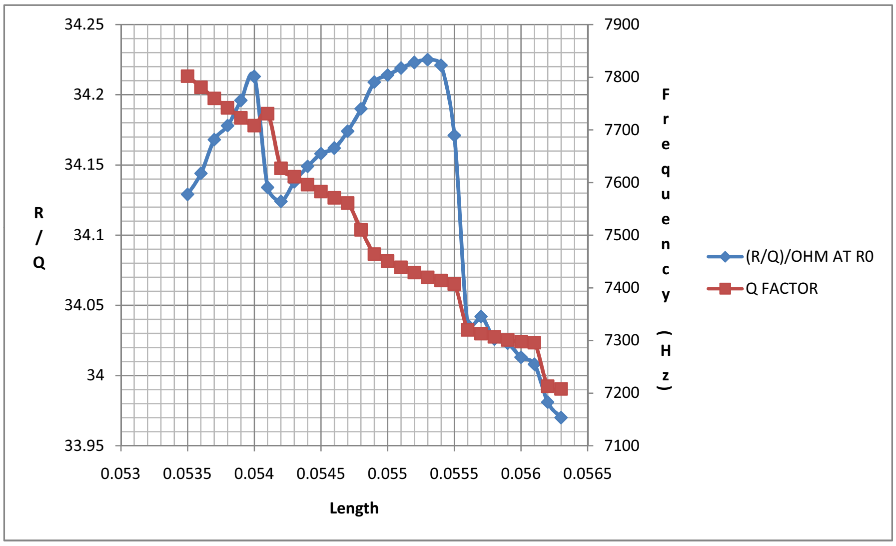

FIG. 7: Plot demonstrating that $\frac{R}{Q}$ undergoes a peak while Q consistently decreases. The aberration located at roughly 0.0541 m is due to a mesh deformity and can be ignored.

TABLE II: Specifications of an optimized dielectric power extractor

Dielectric Constant	10
Dielectric Length	0.114 m
Dielectric Thickness	0.001 m
Taper Length	0.0015 m
Coupler Length	0.012 m
Coupler Thickness	0.001 m
Pipe Radius	0.0127 m

ERL. Additional structures must be budgeted to compensate for energy attenuation within the waveguides used in construction. The total length of each individual dielectric power extractor is on the order of 0.1 meters. Leaving room to account for phase changes, this amounts to a spacial requirement of approximately 60 meters of structure. This is not an unreasonable requirement, especially if it can eliminate the wake generated in a cavity as was seen in fig. 3.

TABLE III: Results of a two beam acceleration simulation for an optimized dielectric power extractor

Rise/Fall Time:	39.23 ns
Tuned Frequency:	15.311 GHz
Peak Voltage:	7.164 kV
RF Pulse Length:	40 ns
Time between bunches:	0.769 ns

VII. CONCLUSIONS

The use of a dielectric power extractor to accelerate and decelerate the beam has much potential to lessen the energy spread of the particle beam. It has been demonstrated that the TBA scheme, implemented by a structure consisting of a dielectric and two endpoint couplers, can adequately serve as a framework for further research and development in the full realization of the ERL. Future studies should give consideration to the calculation of higher order modes, which may prove to have similar or greater $\frac{R}{Q}$ or Q parameters at even higher frequencies. Additional analysis will need to be executed to characterize the effects present when several dielectric power extractors are connected in series. Such a study may provide insight into the advantages and disadvantages associated with utilizing many short dielectric power extractors versus a few long dielectric power extractors.

VIII. APPENDIX:EXPRESSING $W'_{||}(s)$ IN TERMS OF $W_{||}(s)$ AND CHARGE DISTRIBUTION DERIVATION

$$W_{||}(s) = \lambda(s) \bigotimes W^{\delta}_{||}(s)$$

$$\mathbf{F}[W_{||}(s)] = \mathbf{F}[\lambda(s)] \cdot \mathbf{F}[W^{\delta}_{||}(s)]$$

$$\mathbf{F}[W'_{||}(s)] = \mathbf{F}[\lambda'(s)] \cdot \mathbf{F}[W^{\delta}_{||}(s)]$$

$$\mathbf{F}[W'_{||}(s)] = \frac{\mathbf{F}[\lambda'(s)]}{\mathbf{F}[\lambda(s)]} \cdot [W_{||}(s)]$$

$$W'_{||}(s) = W_{||}(s) \bigotimes \mathbf{F}^{-1}\left[\frac{\mathbf{F}[\lambda'(s)]}{\mathbf{F}[\lambda(s)]}\right]$$

IX. ACKNOWLEDGMENTS

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[1] For a fuller treatise on wakefields, refer to Weiland and Wanzenberg's *Wake Fields and Impedances*, a 1990 Joint US-Cern Accelerator Course.

[2] Problem is addressed and a treatment suggestion is given by G.H. Hoffstaetter, M.G. Billing, and Y.H. Lau in a presentation at EPAC08 in Genoa, Italy entitled *Wake-Field Compensation in Energy Recovery Linacs*

[3] Design of 7.8 GHz power extractor proposed by F. Gao et al. in Phys. Rev. Special Topics **11**, 041301 (2008).

[4] To learn the general operating principles of the Cornell ERL and read a discussion of its planned implementation, refer to http://www.lepp.cornell.edu/Research/AP/ERL/.