

High-Precision Predictions for Higgs and Top-Quark Pair Production at Hadron Colliders

Effective Field Theories for LHC Processes

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Theory Seminar

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Based on:

- ♦ IR singularities of scattering amplitudes in non-abelian gauge theories

Thomas Becher, MN: 0901.0722 (PRL), 0903.1126 (JHEP), 0904.1021 (PRD)

Andrea Ferroglia, MN, Ben Pecjak, Li Lin Yang: 0907.4791 (PRL), 0908.3676 (JHEP)

- ♦ Threshold resummation for Higgs production

Valentin Ahrens, Thomas Becher, MN, Li Lin Yang: 0808.3008 (PRD), 0809.4283 (EPJC) & 1008.3162 (PLB)

- ♦ Threshold resummation for top-pair production

Valentin Ahrens, Andrea Ferroglia, MN, Ben Pecjak, Li Lin Yang: 0912.3375 (PLB), 1003.5827 (JHEP), 1103.0550 (NEW!)

A tale of many scales

- ◆ Collider processes characterized by many scales: s , s_{ij} , M_i , Λ_{QCD} , ...
- ◆ Large Sudakov logarithms arise, which need to be resummed (e.g. parton showers, mass effects, aspects of underlying event)
- ◆ Effective field theories provide modern, elegant approach to this problem based on **scale separation (factorization theorems)** and **RG evolution (resummation)**

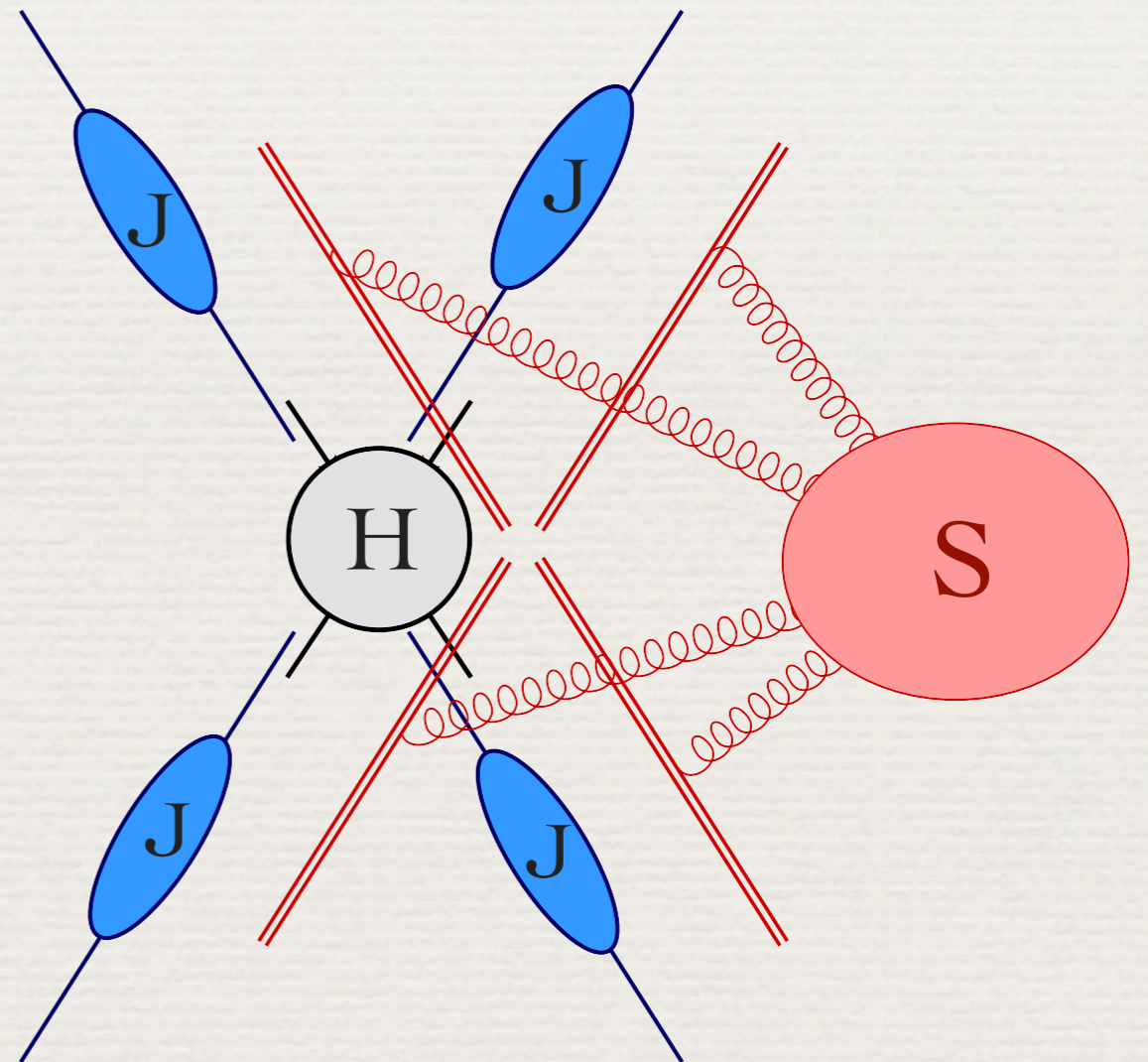
Soft-collinear factorization

Sen 1983; Kidonakis, Oderda, Sterman 1998

- ◆ Factorize cross section:

$$d\sigma \sim H(\{s_{ij}\}, \mu) \prod_i J_i(M_i^2, \mu) \otimes S(\{\Lambda_{ij}^2\}, \mu)$$

- ◆ Define components in terms of field theory objects in SCET
- ◆ Resum large Sudakov logarithms directly in momentum space using RG equations



Soft-collinear effective theory (SCET)

Bauer, Pirjol, Stewart et al. 2001 & 2002; Beneke et al. 2002; ...

- Two-step matching procedure:



- Integrate out hard modes, describe collinear and soft modes by fields in SCET

$$S_{ij} \frac{\text{hard}}{\text{---}}$$
$$M_i^2 \frac{\text{collinear}}{\text{---}}$$

- Integrate out collinear modes (if perturbative) and match onto a theory of Wilson lines

$$\Lambda_{ij}^2 = \frac{M_i^4}{S_{ij}} \frac{\text{soft}}{\text{---}}$$

NLO+NNLL resummation

in few cases (Drell-Yan, Higgs production) NNLO+N³LL resummation

- ◆ Necessary ingredients:
 - ◆ **Hard functions:** from fixed-order results for on-shell amplitudes (but need amplitudes!)
 - ◆ **Jet functions:** from imaginary parts of two-point functions (depend on cuts, jet definitions)
 - ◆ **Soft functions:** from matrix elements of Wilson-line operators
 - ◆ **Anomalous dimensions:** known!
- ◆ Yields **jet cross sections**, not parton rates
- ◆ Goes beyond **parton showers**, which are accurate only at LL order even after matching

Anomalous dimension to two loops

- General result for arbitrary processes: Becher, MN 2009

$$\Gamma(\{\underline{p}\}, \{\underline{m}\}, \mu) = \sum_{(i,j)} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{2} \gamma_{\text{cusp}}(\alpha_s) \ln \frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)$$

massless partons

$$- \sum_{(I,J)} \frac{\mathbf{T}_I \cdot \mathbf{T}_J}{2} \gamma_{\text{cusp}}(\beta_{IJ}, \alpha_s) + \sum_I \gamma^I(\alpha_s) + \sum_{I,j} \mathbf{T}_I \cdot \mathbf{T}_j \gamma_{\text{cusp}}(\alpha_s) \ln \frac{m_I \mu}{-s_{Ij}}$$

massive partons

$$+ \sum_{(I,J,K)} i f^{abc} \mathbf{T}_I^a \mathbf{T}_J^b \mathbf{T}_K^c F_1(\beta_{IJ}, \beta_{JK}, \beta_{KI})$$

new!

$$+ \sum_{(I,J)} \sum_k i f^{abc} \mathbf{T}_I^a \mathbf{T}_J^b \mathbf{T}_k^c f_2\left(\beta_{IJ}, \ln \frac{-\sigma_{Jk} v_J \cdot p_k}{-\sigma_{Ik} v_I \cdot p_k}\right) + \mathcal{O}(\alpha_s^3).$$

- Generalizes structure found for massless case
- Novel three-parton terms appear at two loops

Mitov, Sterman, Sung 2009; Becher, MN 2009
 Ferroglia, MN, Pecjak, Yang 2009



EFT-based predictions for Higgs production at Tevatron and LHC

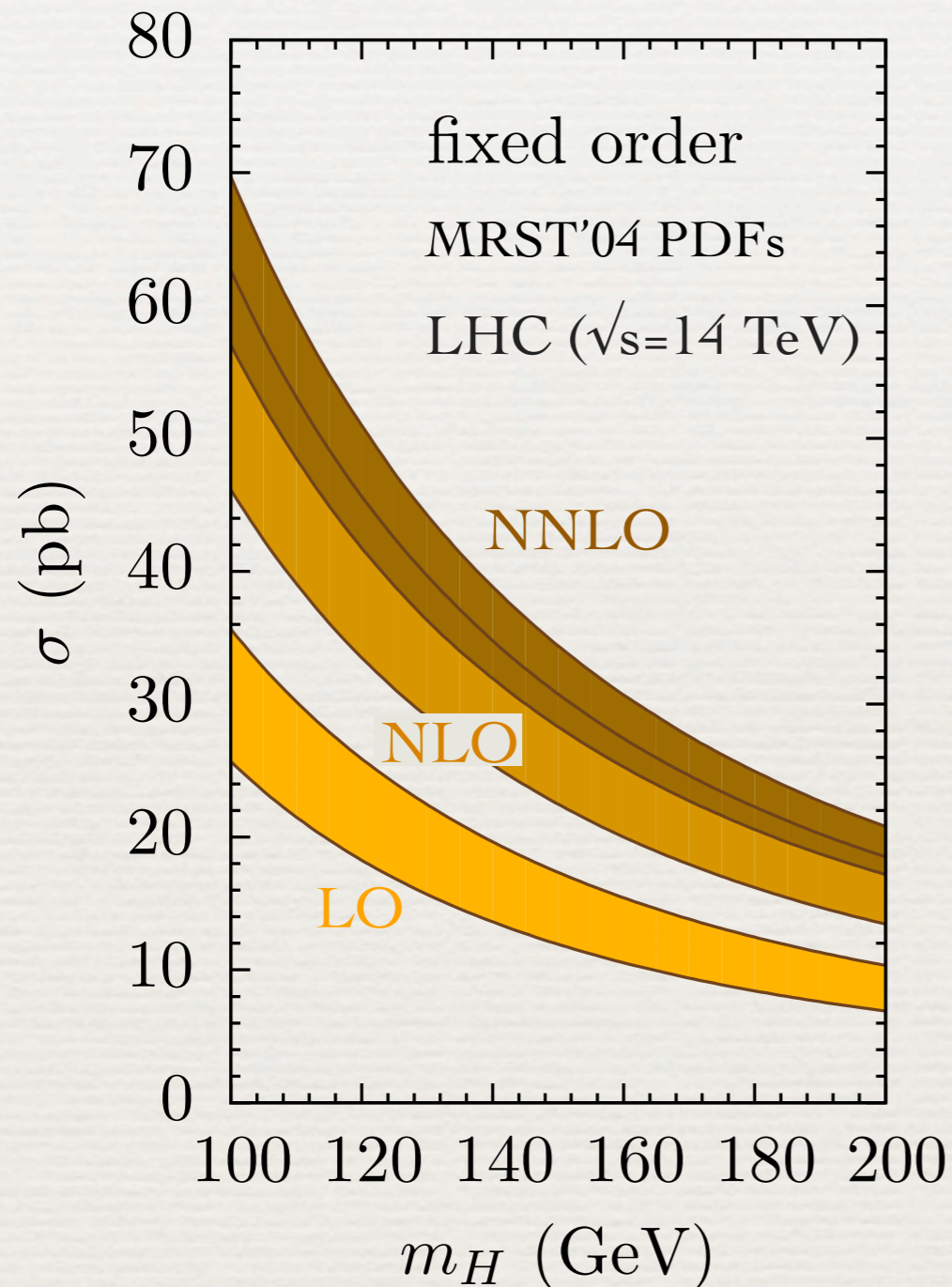
Ahrens, Becher, MN, Yang 2008 & update for ICHEP 2010



<http://projects.hepforge.org/rghiggs/>

arXiv:0803.0988 [hep-th]

Large higher-order corrections



- ♦ **Corrections are large:**
70% at NLO + 30% at NNLO
[130% and 80% if PDFs and α_s are held fixed]
- ♦ Only gg channel contains leading singular terms, which give 90% of NLO and 94% of NNLO correction
- ♦ Contributions of qg and qq channels are small: -1% and -8% of the NLO correction

Effective theory analysis

- ◆ Separate contributions associated with different scales, turning a multi-scale problems into a series of single-scale problems
- ◆ Evaluate each contribution at its natural scale, leading to improved perturbative behavior
- ◆ Use renormalization group to evolve contributions to a common factorization scale, thereby exponentiating (resumming) large corrections

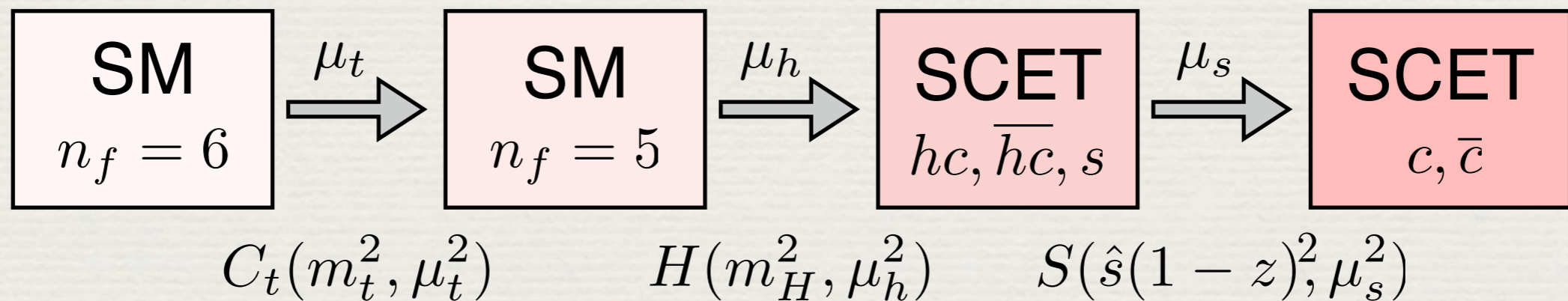
When this is done consistently, large K-factors should not arise, since no large perturbative corrections are left unexponentiated!

Scale hierarchy

- ♦ Will analyze the Higgs cross section assuming the scale hierarchy ($z = M_H^2 / \hat{s}$)

$$2m_t \gg m_H \sim \sqrt{\hat{s}} \gg \sqrt{\hat{s}}(1 - z) \gg \Lambda_{\text{QCD}}$$

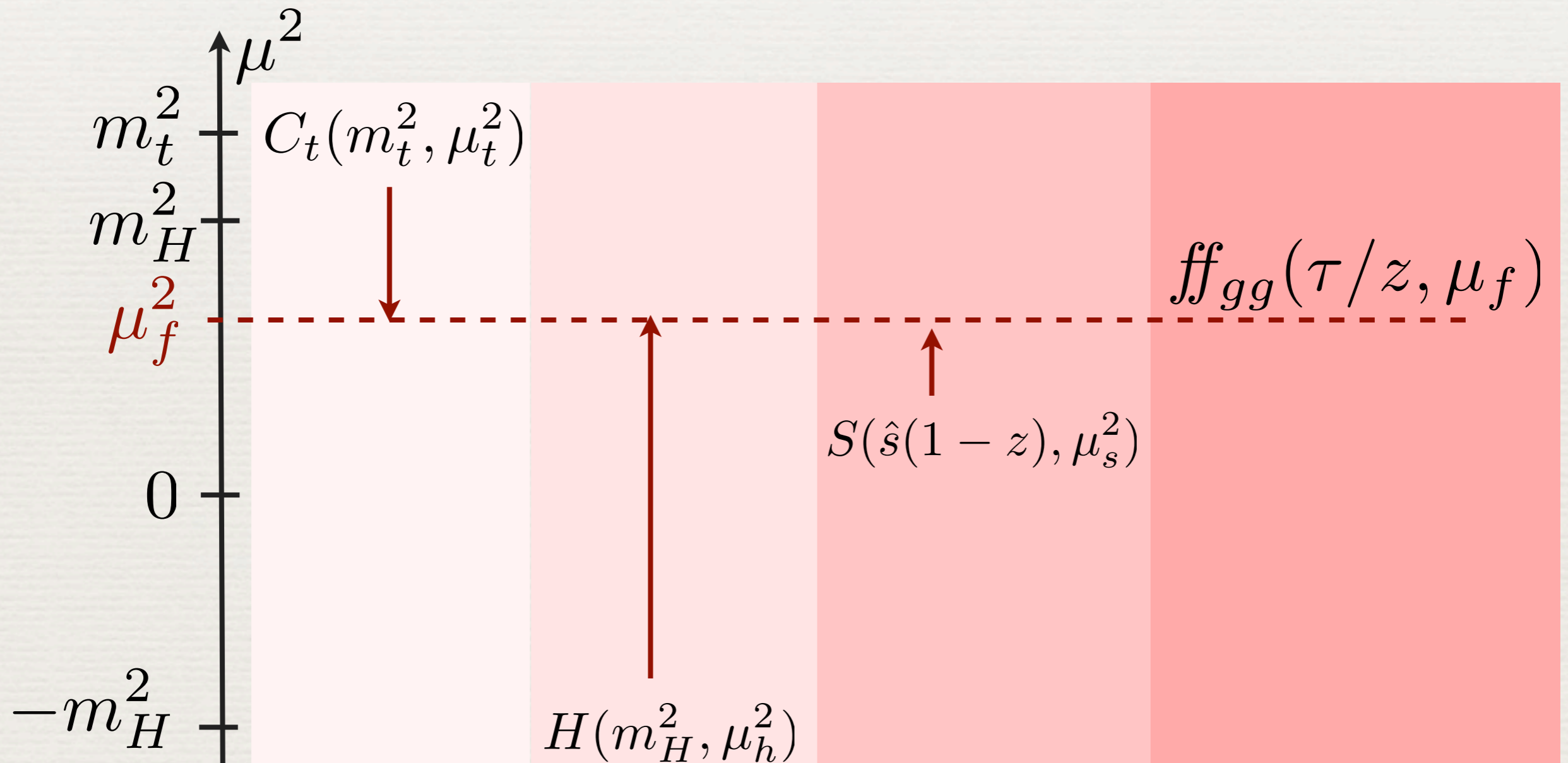
- ♦ Treating one scale at a time leads to a sequence of effective theories:



- ♦ Effects associated with each scale absorbed into matching coefficients

Scale hierarchy

- ♦ Evaluate each part at its characteristic scale and evolve to a common scale using RGEs:



RG evolution equations

- ◆ Top function:

$$\frac{d}{d \ln \mu} C_t(m_t^2, \mu^2) = \gamma^t(\alpha_s) C_t(m_t^2, \mu^2)$$

- ◆ Hard function $H(m_H^2, \mu^2) = |C_S(-m_H^2 - i\epsilon, \mu^2)|^2$:

$$\frac{d}{d \ln \mu} C_S(-m_H^2 - i\epsilon, \mu^2) = \left[\Gamma_{\text{cusp}}^A(\alpha_s) \ln \frac{-m_H^2 - i\epsilon}{\mu^2} + \gamma^S(\alpha_s) \right] C_S(-m_H^2 - i\epsilon, \mu^2)$$

- ◆ Soft function:

$$\begin{aligned} \frac{dS(\omega^2, \mu^2)}{d \ln \mu} = & - \left[2\Gamma_{\text{cusp}}(\alpha_s) \ln \frac{\omega^2}{\mu^2} + 2\gamma^W(\alpha_s) \right] S(\omega^2, \mu^2) \\ & - 4\Gamma_{\text{cusp}}(\alpha_s) \int_0^\omega d\omega' \frac{S(\omega'^2, \mu^2) - S(\omega^2, \mu^2)}{\omega - \omega'} \end{aligned}$$

Sudakov (cusp) logarithms



RG evolution equations

- ◆ Closed analytic solutions (Laplace transform):

Becher, MN 2006

$$C(z, m_t, m_H, \mu_f) = [C_t(m_t^2, \mu_t^2)]^2 |C_S(-m_H^2 - i\epsilon, \mu_h^2)|^2 U(m_H, \mu_t, \mu_h, \mu_s, \mu_f) \\ \times \frac{z^{-\eta}}{(1-z)^{1-2\eta}} \tilde{\mathcal{S}}_{\text{Higgs}} \left(\ln \frac{m_H^2(1-z)^2}{\mu_s^2 z} + \partial_{\eta, \mu_s^2} \right) \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)}$$

with:

$$U(m_H, \mu_t, \mu_h, \mu_s, \mu_f) = \frac{\alpha_s^2(\mu_s^2)}{\alpha_s^2(\mu_f^2)} \left[\frac{\beta(\alpha_s(\mu_s^2))/\alpha_s^2(\mu_s^2)}{\beta(\alpha_s(\mu_t^2))/\alpha_s^2(\mu_t^2)} \right]^2 \left| \left(\frac{-m_H^2 - i\epsilon}{\mu_h^2} \right)^{-2a_{\Gamma}(\mu_h^2, \mu_s^2)} \right| \\ \times \left| \exp \left[4S(\mu_h^2, \mu_s^2) - 2a_{\gamma_S}(\mu_h^2, \mu_s^2) + 4a_{\gamma_B}(\mu_s^2, \mu_f^2) \right] \right|.$$

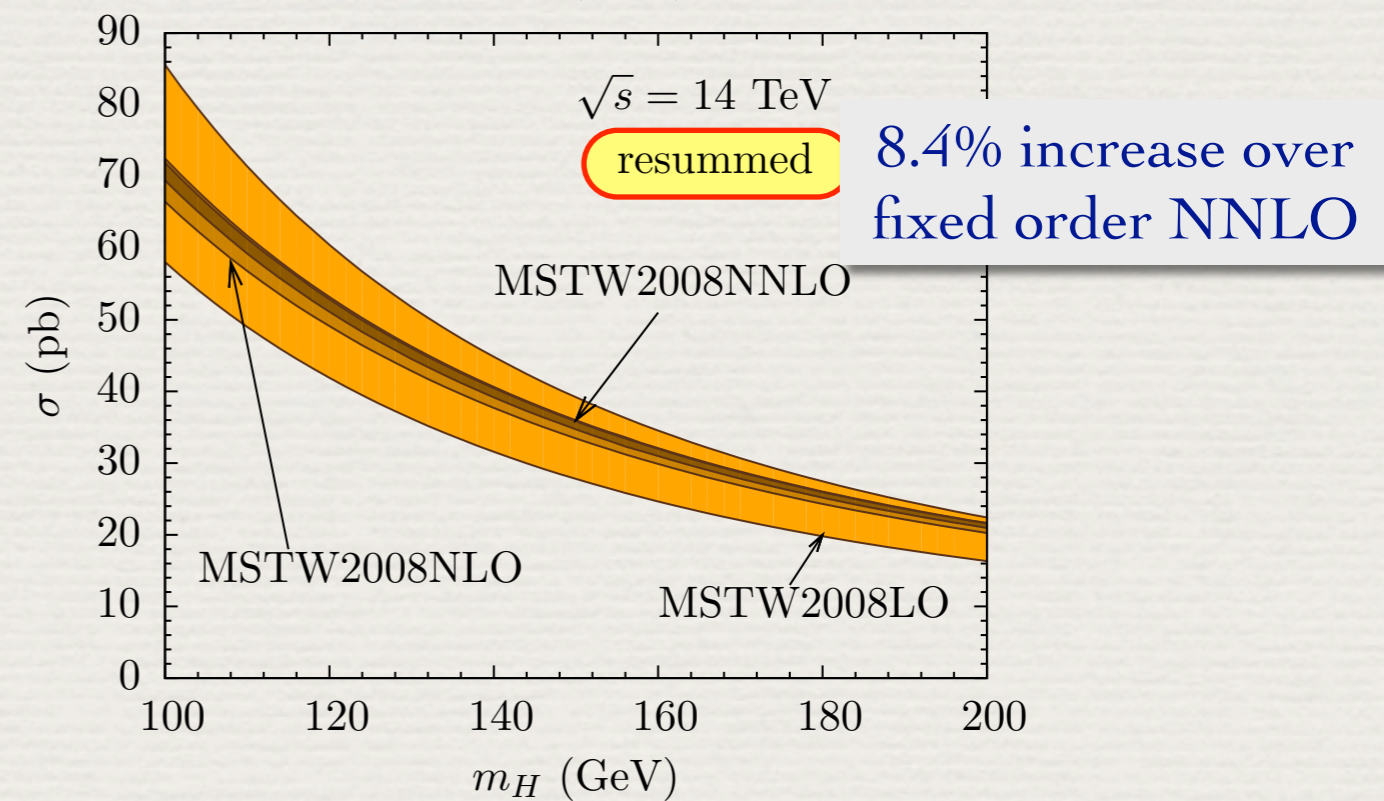
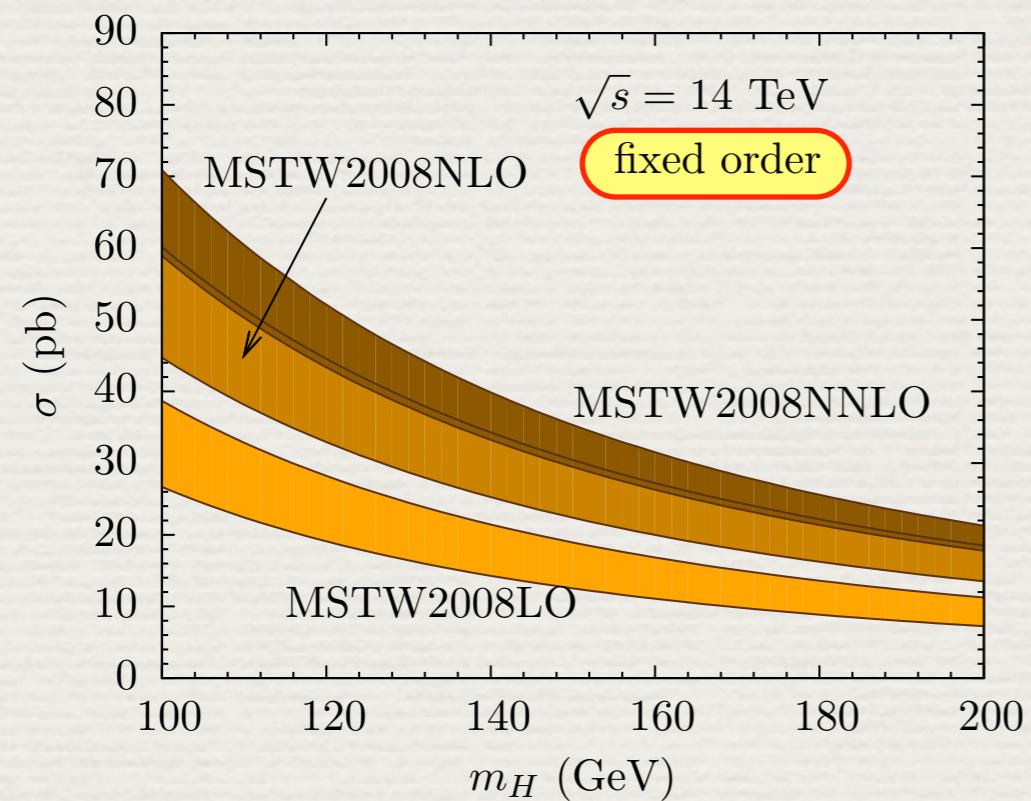
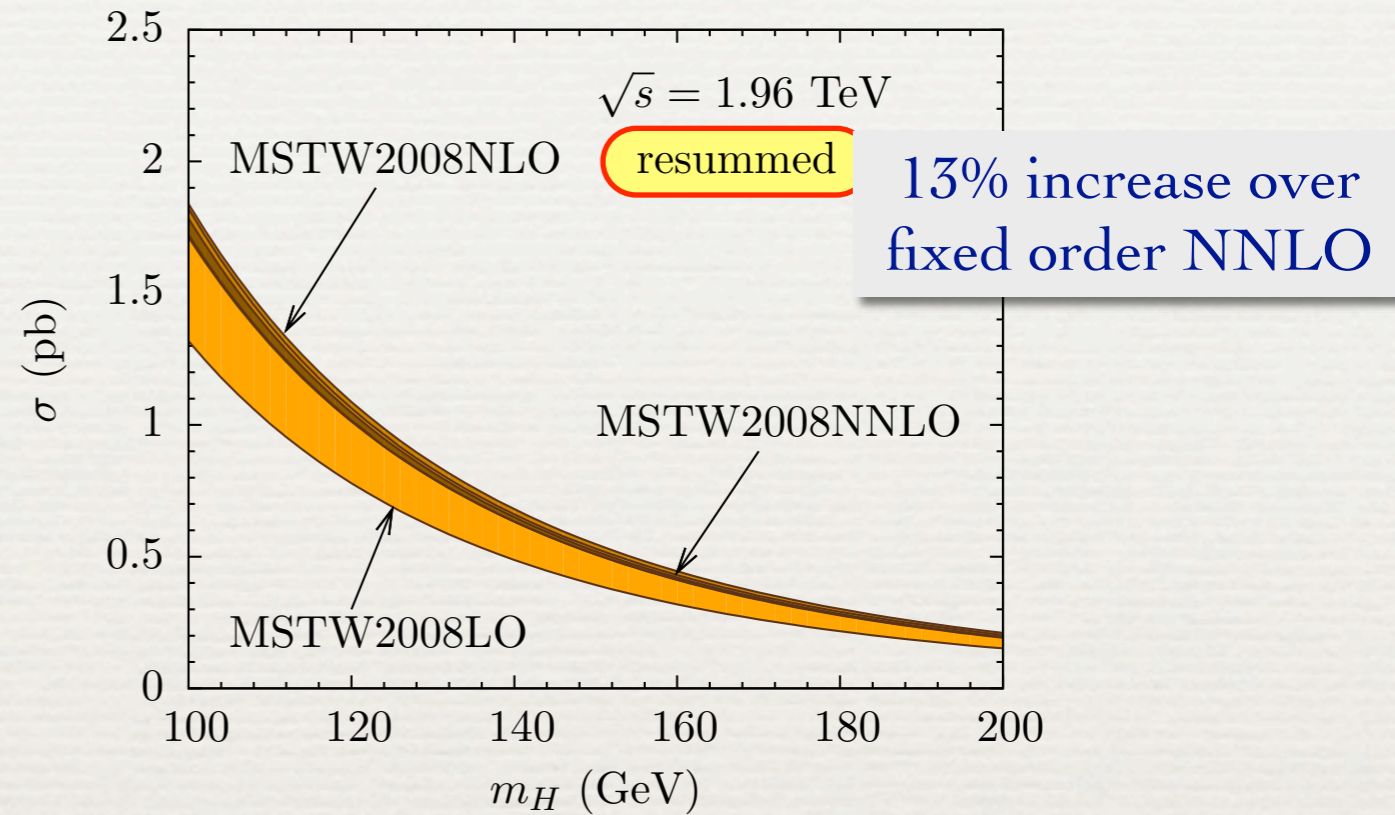
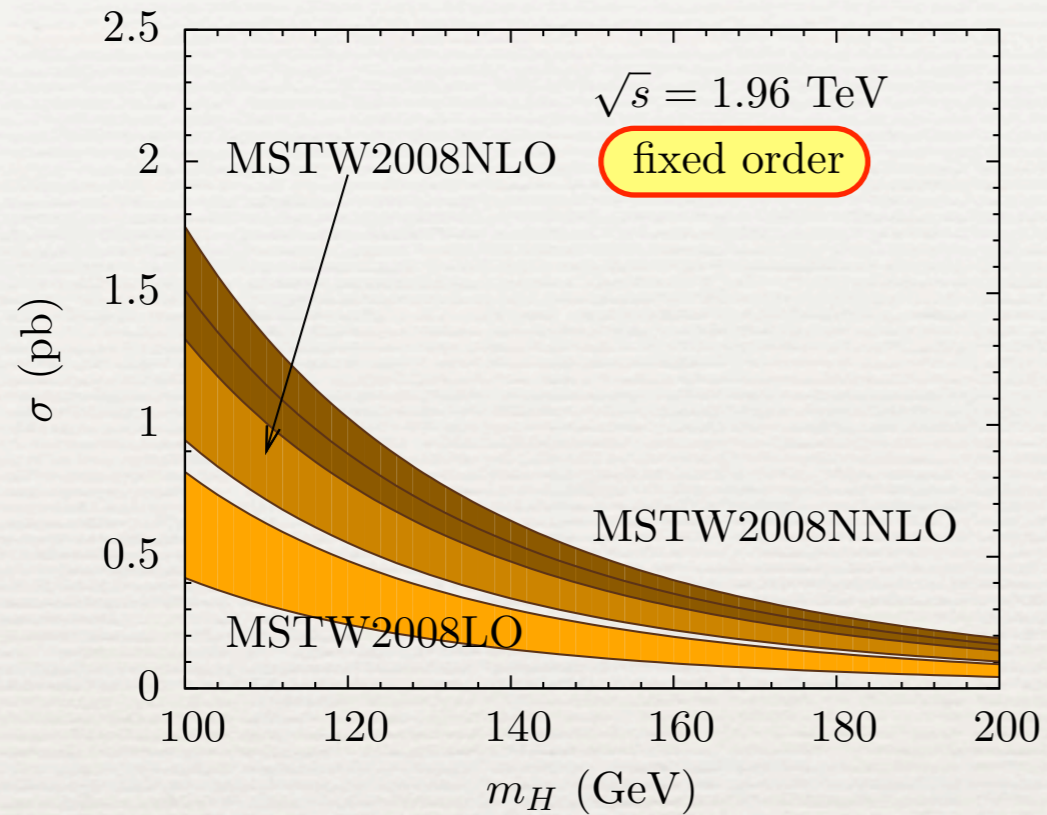
and:

$$\mu_t \approx m_t, \quad \mu_h^2 \approx -m_H^2, \quad \mu_s \text{ set dynamically}$$

Advantages over standard approach

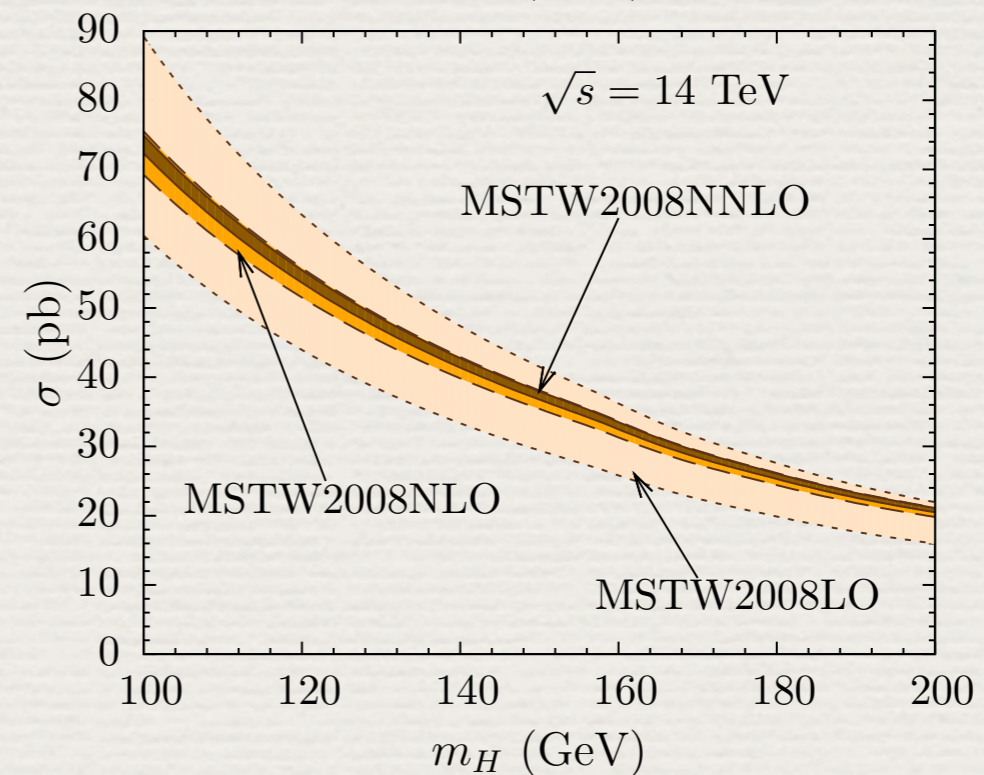
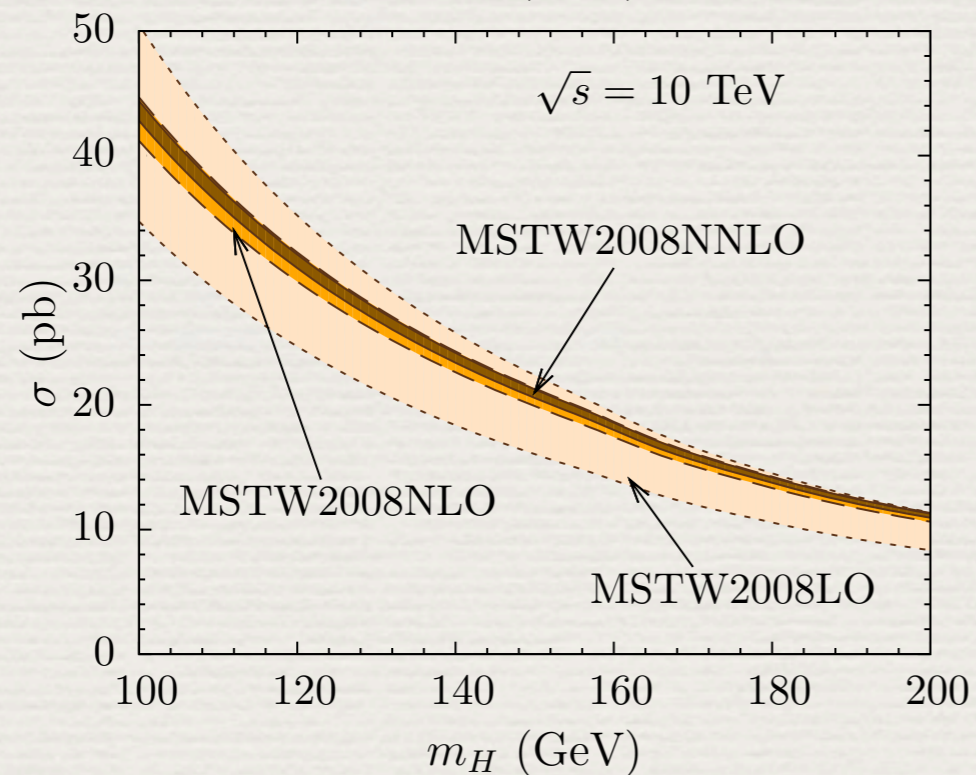
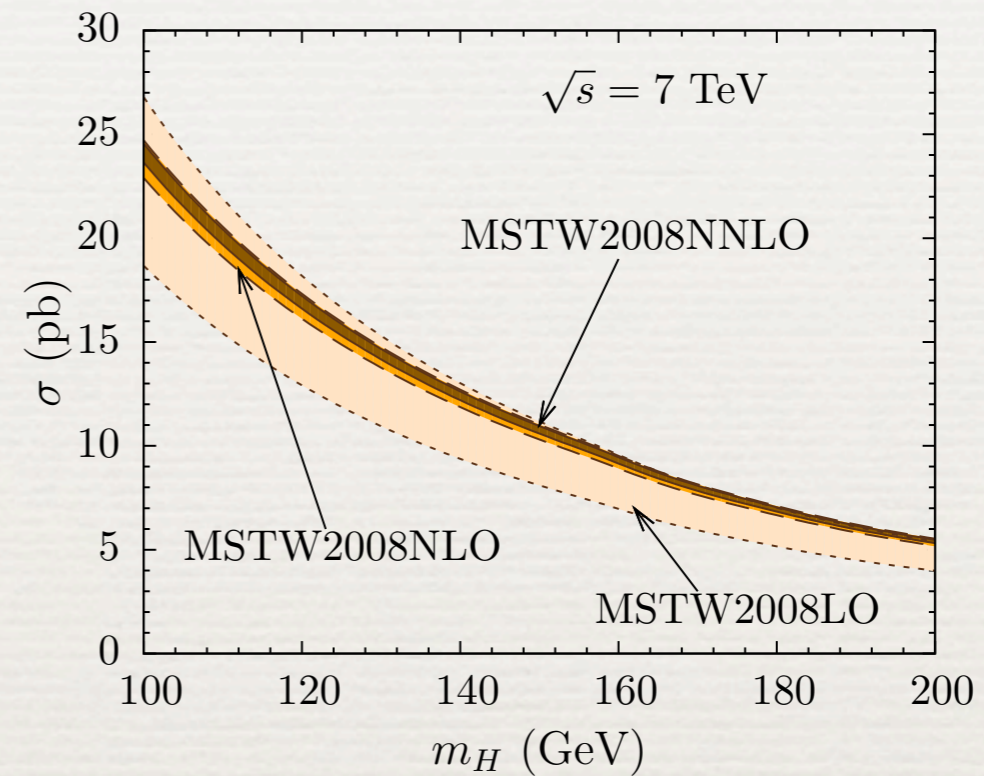
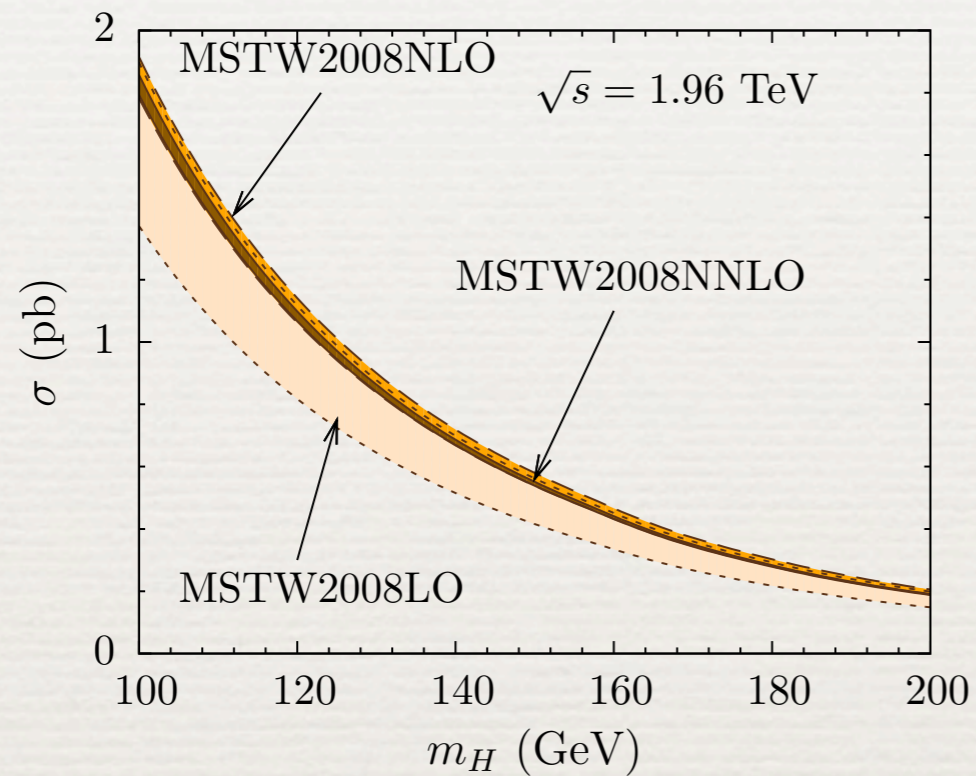
- ◆ Traditionally, resummation is performed in Mellin-moment space e.g.: Catani, de Florian, Grazzini, Nason 2003
- ◆ While equivalent at any fixed order in α_s , our approach offers several advantages:
 - ◆ Large corrections $\sim (C_A \pi \alpha_s)^n$ from analytic continuation of gluon form factor resummed
 - ◆ No integrals over Landau pole of running coupling $\alpha_s(\mu^2)$, hence no regularization prescription
 - ◆ No need for numerical Mellin inversion
 - ◆ Trivial matching onto fixed-order results

Cross section predictions



Predictions including EW corrections

Ahrens, Becher, MN, Yang 2010

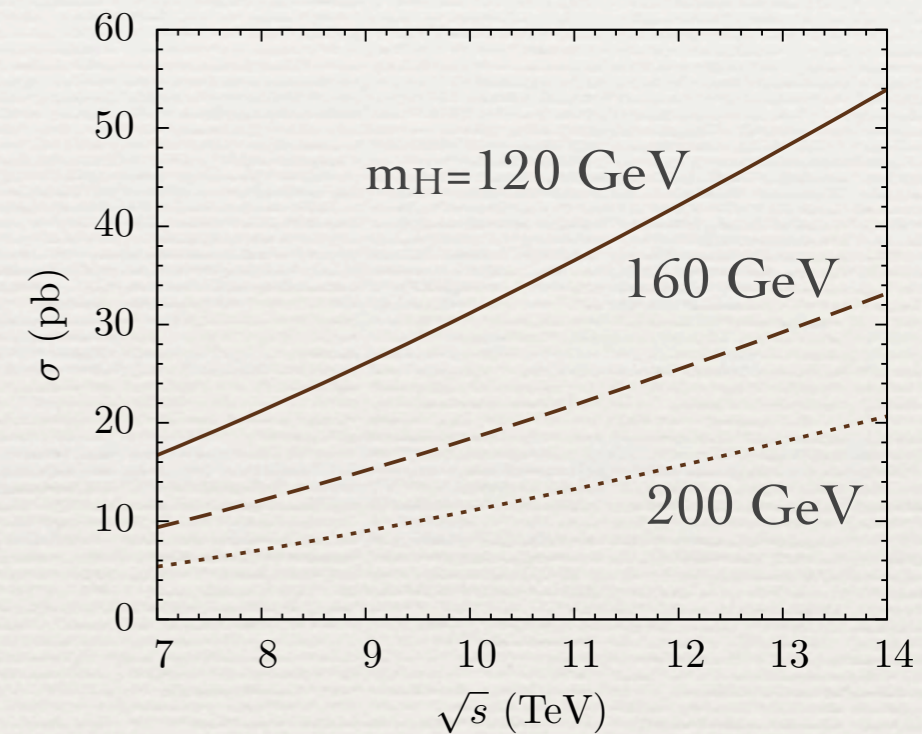


Predictions including EW corrections

Ahrens, Becher, MN, Yang 2010

- State-of-the-art results (most complete to date), using MSTW2008NNLO PDFs:

m_H [GeV]	Tevatron	LHC (7 TeV)	LHC (10 TeV)	LHC (14 TeV)
115	$1.215^{+0.031+0.141}_{-0.007-0.135}$	$18.19^{+0.53+1.46}_{-0.14-1.39}$	$33.7^{+1.0+2.6}_{-0.2-2.5}$	$57.9^{+1.6+4.4}_{-0.3-4.2}$
120	$1.073^{+0.026+0.126}_{-0.006-0.121}$	$16.73^{+0.48+1.34}_{-0.13-1.28}$	$31.2^{+0.9+2.4}_{-0.2-2.3}$	$54.0^{+1.5+4.1}_{-0.3-3.9}$
125	$0.950^{+0.022+0.113}_{-0.005-0.108}$	$15.43^{+0.44+1.23}_{-0.12-1.18}$	$29.0^{+0.8+2.2}_{-0.2-2.1}$	$50.4^{+1.4+3.8}_{-0.3-3.6}$
130	$0.844^{+0.019+0.102}_{-0.004-0.098}$	$14.27^{+0.40+1.14}_{-0.11-1.09}$	$27.0^{+0.7+2.1}_{-0.2-2.0}$	$47.2^{+1.3+3.5}_{-0.3-3.4}$
135	$0.753^{+0.016+0.093}_{-0.004-0.088}$	$13.23^{+0.36+1.06}_{-0.10-1.01}$	$25.2^{+0.7+1.9}_{-0.2-1.8}$	$44.3^{+1.2+3.3}_{-0.3-3.2}$
140	$0.672^{+0.014+0.084}_{-0.003-0.080}$	$12.29^{+0.33+0.98}_{-0.09-0.94}$	$23.5^{+0.6+1.8}_{-0.2-1.7}$	$41.6^{+1.1+3.1}_{-0.3-3.0}$
145	$0.602^{+0.012+0.076}_{-0.003-0.072}$	$11.44^{+0.31+0.91}_{-0.08-0.88}$	$22.1^{+0.6+1.7}_{-0.1-1.6}$	$39.2^{+1.0+2.9}_{-0.2-2.8}$
150	$0.541^{+0.010+0.070}_{-0.002-0.066}$	$10.67^{+0.28+0.85}_{-0.08-0.82}$	$20.7^{+0.5+1.6}_{-0.1-1.5}$	$37.0^{+1.0+2.7}_{-0.2-2.6}$
155	$0.486^{+0.009+0.064}_{-0.002-0.060}$	$9.95^{+0.26+0.80}_{-0.07-0.77}$	$19.4^{+0.5+1.5}_{-0.1-1.4}$	$34.9^{+0.9+2.6}_{-0.2-2.5}$
160	$0.433^{+0.008+0.058}_{-0.002-0.054}$	$9.21^{+0.24+0.74}_{-0.07-0.71}$	$18.1^{+0.5+1.4}_{-0.1-1.3}$	$32.7^{+0.8+2.4}_{-0.2-2.3}$
165	$0.385^{+0.006+0.052}_{-0.002-0.049}$	$8.50^{+0.22+0.68}_{-0.06-0.66}$	$16.8^{+0.4+1.3}_{-0.1-1.2}$	$30.5^{+0.8+2.2}_{-0.2-2.1}$
170	$0.345^{+0.005+0.047}_{-0.002-0.044}$	$7.89^{+0.20+0.63}_{-0.06-0.61}$	$15.7^{+0.4+1.2}_{-0.1-1.1}$	$28.6^{+0.7+2.1}_{-0.2-2.0}$
175	$0.310^{+0.005+0.043}_{-0.001-0.040}$	$7.36^{+0.18+0.59}_{-0.05-0.57}$	$14.7^{+0.4+1.1}_{-0.1-1.1}$	$27.0^{+0.7+1.9}_{-0.2-1.9}$
180	$0.280^{+0.004+0.040}_{-0.001-0.037}$	$6.88^{+0.17+0.56}_{-0.05-0.54}$	$13.8^{+0.3+1.0}_{-0.1-1.0}$	$25.5^{+0.6+1.8}_{-0.2-1.8}$
185	$0.252^{+0.003+0.036}_{-0.001-0.033}$	$6.42^{+0.15+0.52}_{-0.04-0.50}$	$13.0^{+0.3+1.0}_{-0.1-0.9}$	$24.0^{+0.6+1.7}_{-0.1-1.7}$
190	$0.228^{+0.003+0.033}_{-0.001-0.031}$	$6.02^{+0.14+0.49}_{-0.04-0.47}$	$12.2^{+0.3+0.9}_{-0.1-0.9}$	$22.7^{+0.5+1.6}_{-0.1-1.6}$
195	$0.207^{+0.002+0.031}_{-0.001-0.028}$	$5.67^{+0.13+0.46}_{-0.04-0.45}$	$11.6^{+0.3+0.9}_{-0.1-0.8}$	$21.6^{+0.5+1.6}_{-0.1-1.5}$
200	$0.189^{+0.002+0.028}_{-0.001-0.026}$	$5.35^{+0.12+0.44}_{-0.03-0.42}$	$11.0^{+0.3+0.8}_{-0.1-0.8}$	$20.6^{+0.5+1.5}_{-0.1-1.4}$



scale uncertainty

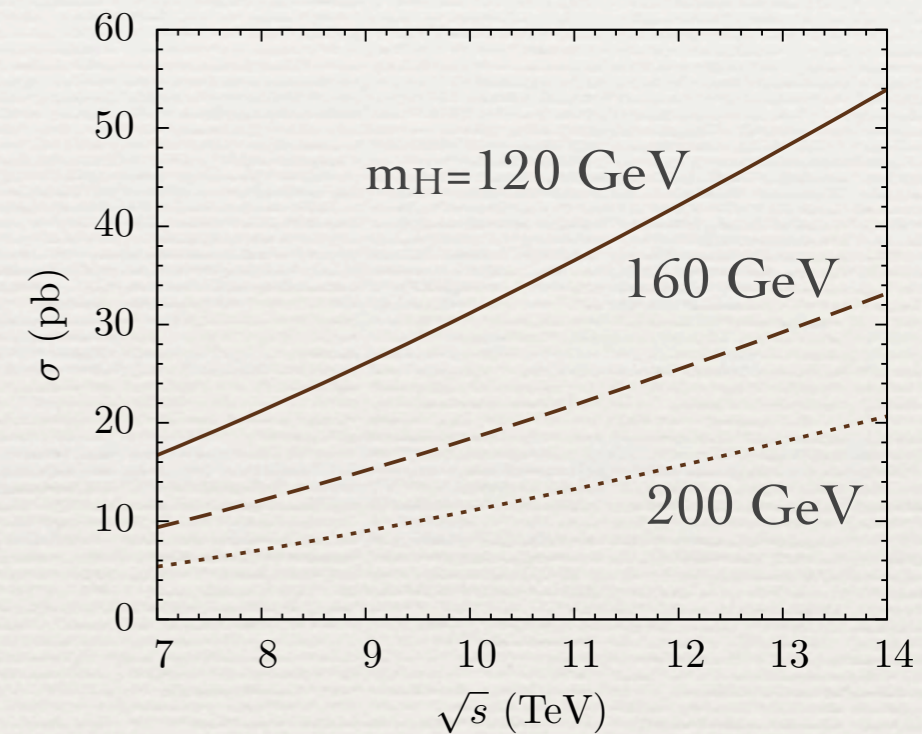
PDF & α_s uncertainty

Predictions including EW corrections

Ahrens, Becher, MN, Yang 2010

- State-of-the-art results (most complete to date) using CT10 PDFs:

m_H [GeV]	Tevatron	LHC (7 TeV)	LHC (10 TeV)	LHC (14 TeV)
115	$1.215^{+0.031+0.105}_{-0.007-0.095}$	$18.34^{+0.54+0.95}_{-0.14-1.00}$	$34.1^{+1.0+1.8}_{-0.2-1.9}$	$58.8^{+1.7+3.1}_{-0.4-3.5}$
120	$1.073^{+0.026+0.096}_{-0.005-0.087}$	$16.86^{+0.49+0.87}_{-0.13-0.91}$	$31.5^{+0.9+1.6}_{-0.2-1.8}$	$54.7^{+1.6+2.9}_{-0.3-3.2}$
125	$0.950^{+0.022+0.088}_{-0.005-0.079}$	$15.54^{+0.45+0.80}_{-0.12-0.83}$	$29.3^{+0.8+1.5}_{-0.2-1.6}$	$51.1^{+1.4+2.6}_{-0.3-3.0}$
130	$0.845^{+0.019+0.081}_{-0.004-0.072}$	$14.36^{+0.41+0.74}_{-0.11-0.76}$	$27.2^{+0.8+1.4}_{-0.2-1.5}$	$47.8^{+1.3+2.5}_{-0.3-2.7}$
135	$0.753^{+0.016+0.075}_{-0.004-0.067}$	$13.31^{+0.37+0.68}_{-0.10-0.70}$	$25.4^{+0.7+1.3}_{-0.2-1.4}$	$44.8^{+1.2+2.3}_{-0.3-2.5}$
140	$0.673^{+0.014+0.069}_{-0.003-0.061}$	$12.35^{+0.34+0.63}_{-0.09-0.65}$	$23.7^{+0.7+1.2}_{-0.2-1.3}$	$42.1^{+1.1+2.1}_{-0.3-2.3}$
145	$0.604^{+0.012+0.064}_{-0.003-0.057}$	$11.50^{+0.31+0.59}_{-0.08-0.60}$	$22.2^{+0.6+1.1}_{-0.2-1.2}$	$39.7^{+1.1+2.0}_{-0.2-2.2}$
150	$0.542^{+0.010+0.059}_{-0.002-0.052}$	$10.71^{+0.29+0.55}_{-0.08-0.56}$	$20.9^{+0.6+1.0}_{-0.1-1.1}$	$37.4^{+1.0+1.9}_{-0.2-2.0}$
155	$0.487^{+0.009+0.055}_{-0.002-0.049}$	$9.99^{+0.26+0.51}_{-0.07-0.52}$	$19.6^{+0.5+1.0}_{-0.1-1.0}$	$35.2^{+0.9+1.7}_{-0.2-1.9}$
160	$0.435^{+0.008+0.050}_{-0.002-0.045}$	$9.24^{+0.24+0.48}_{-0.07-0.48}$	$18.2^{+0.5+0.9}_{-0.1-0.9}$	$33.0^{+0.9+1.6}_{-0.2-1.7}$
165	$0.387^{+0.007+0.046}_{-0.002-0.041}$	$8.52^{+0.22+0.44}_{-0.06-0.44}$	$16.9^{+0.4+0.8}_{-0.1-0.9}$	$30.7^{+0.8+1.5}_{-0.2-1.6}$
170	$0.347^{+0.006+0.043}_{-0.002-0.038}$	$7.91^{+0.20+0.41}_{-0.05-0.41}$	$15.8^{+0.4+0.8}_{-0.1-0.8}$	$28.8^{+0.7+1.4}_{-0.2-1.5}$
175	$0.313^{+0.005+0.039}_{-0.001-0.035}$	$7.38^{+0.19+0.38}_{-0.05-0.38}$	$14.8^{+0.4+0.7}_{-0.1-0.7}$	$27.2^{+0.7+1.3}_{-0.2-1.4}$
180	$0.282^{+0.004+0.037}_{-0.001-0.032}$	$6.89^{+0.17+0.36}_{-0.05-0.36}$	$13.9^{+0.3+0.7}_{-0.1-0.7}$	$25.7^{+0.6+1.2}_{-0.2-1.3}$
185	$0.254^{+0.004+0.034}_{-0.001-0.030}$	$6.43^{+0.16+0.34}_{-0.04-0.33}$	$13.1^{+0.3+0.6}_{-0.1-0.7}$	$24.2^{+0.6+1.1}_{-0.1-1.2}$
190	$0.230^{+0.003+0.032}_{-0.001-0.028}$	$6.02^{+0.15+0.32}_{-0.04-0.31}$	$12.3^{+0.3+0.6}_{-0.1-0.6}$	$22.9^{+0.6+1.1}_{-0.1-1.2}$
195	$0.210^{+0.003+0.030}_{-0.001-0.026}$	$5.67^{+0.14+0.30}_{-0.04-0.30}$	$11.6^{+0.3+0.6}_{-0.1-0.6}$	$21.8^{+0.5+1.0}_{-0.1-1.1}$
200	$0.191^{+0.002+0.028}_{-0.001-0.024}$	$5.35^{+0.13+0.29}_{-0.03-0.28}$	$11.1^{+0.3+0.5}_{-0.1-0.5}$	$20.8^{+0.5+1.0}_{-0.1-1.0}$



scale uncertainty

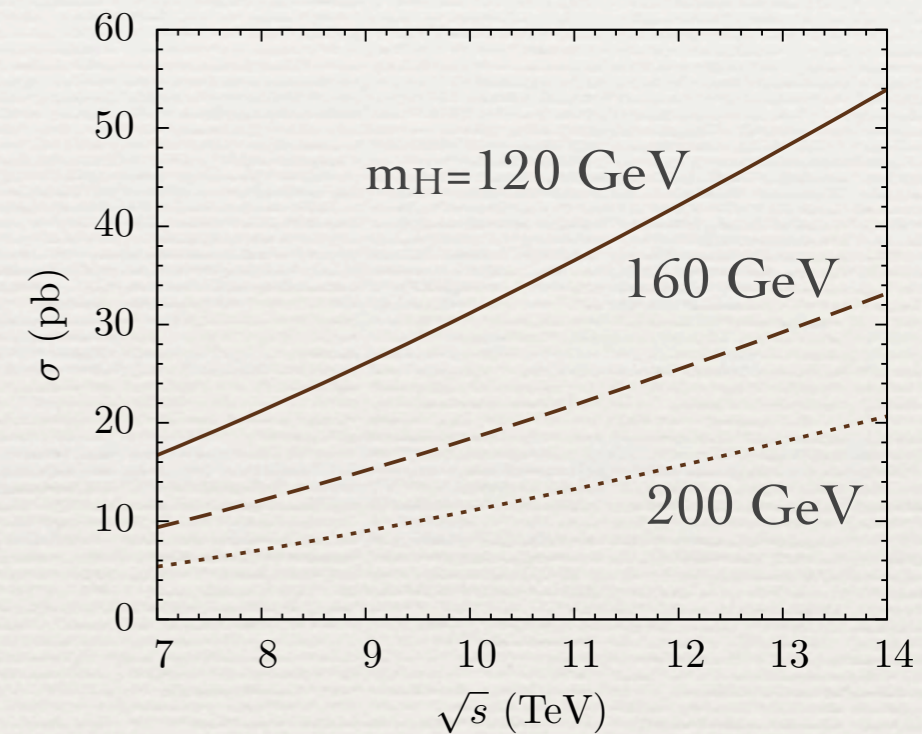
PDF & α_s uncertainty

Predictions including EW corrections

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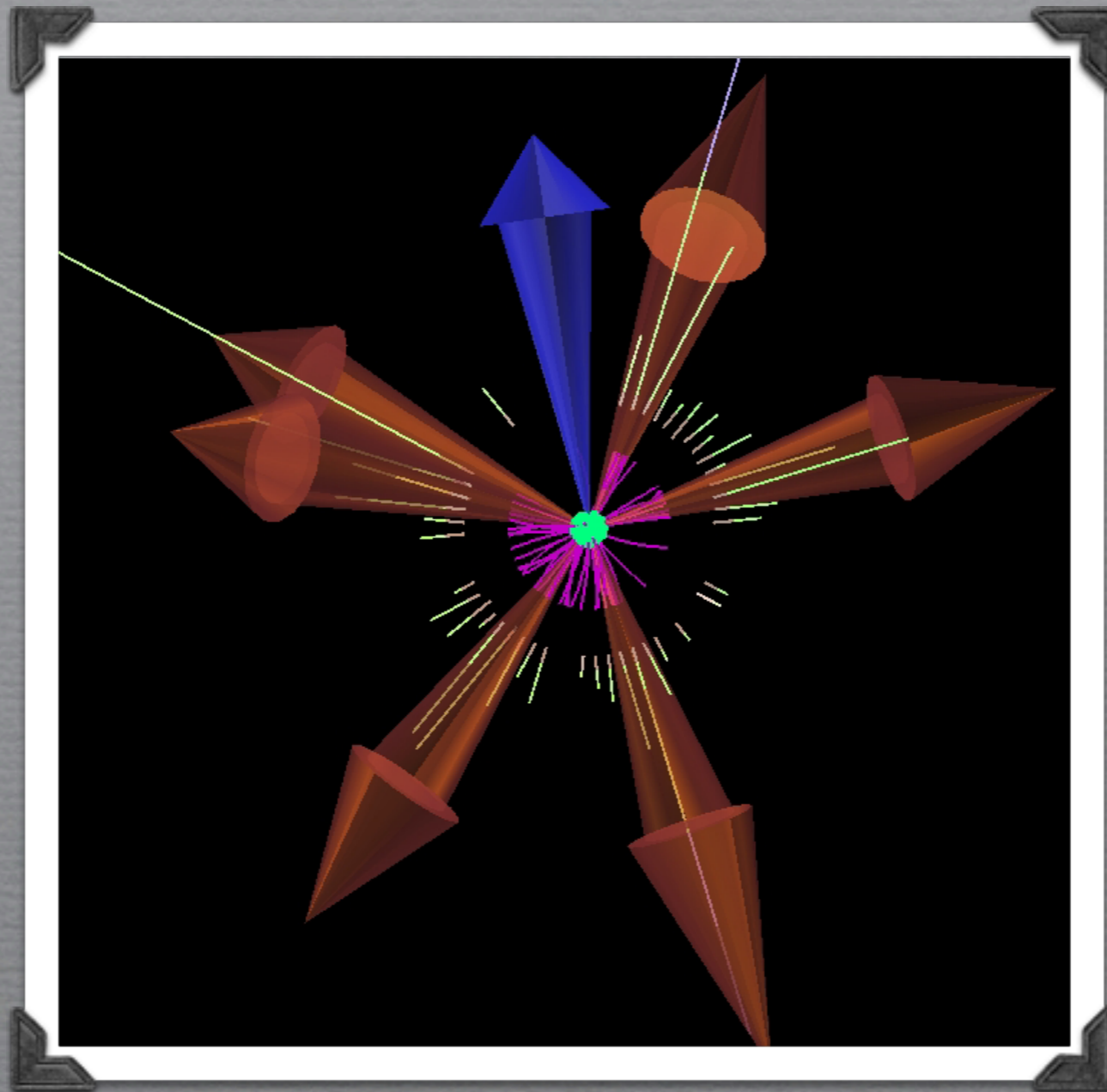
- State-of-the-art results (most complete to date) using NNPDF2.0 PDFs:

m_H [GeV]	Tevatron	LHC (7 TeV)	LHC (10 TeV)	LHC (14 TeV)
115	$1.341^{+0.037+0.143}_{-0.018-0.143}$	$19.35^{+0.60+1.36}_{-0.29-1.36}$	$35.4^{+1.1+2.4}_{-0.5-2.4}$	$60.3^{+1.8+3.9}_{-0.7-3.9}$
120	$1.184^{+0.032+0.129}_{-0.016-0.129}$	$17.82^{+0.54+1.25}_{-0.29-1.25}$	$32.8^{+1.0+2.2}_{-0.5-2.2}$	$56.3^{+1.7+3.7}_{-0.7-3.7}$
125	$1.049^{+0.027+0.116}_{-0.014-0.116}$	$16.45^{+0.50+1.15}_{-0.28-1.15}$	$30.5^{+0.9+2.0}_{-0.5-2.0}$	$52.6^{+1.5+3.4}_{-0.8-3.4}$
130	$0.932^{+0.023+0.105}_{-0.013-0.105}$	$15.23^{+0.45+1.07}_{-0.28-1.07}$	$28.5^{+0.8+1.9}_{-0.5-1.9}$	$49.3^{+1.4+3.2}_{-0.8-3.2}$
135	$0.831^{+0.020+0.096}_{-0.011-0.096}$	$14.13^{+0.41+0.99}_{-0.27-0.99}$	$26.6^{+0.8+1.8}_{-0.5-1.8}$	$46.3^{+1.3+3.0}_{-0.8-3.0}$
140	$0.742^{+0.017+0.087}_{-0.010-0.087}$	$13.14^{+0.38+0.93}_{-0.26-0.93}$	$24.9^{+0.7+1.7}_{-0.5-1.7}$	$43.6^{+1.2+2.8}_{-0.8-2.8}$
145	$0.665^{+0.015+0.080}_{-0.009-0.080}$	$12.24^{+0.35+0.86}_{-0.25-0.86}$	$23.3^{+0.7+1.5}_{-0.5-1.5}$	$41.1^{+1.1+2.6}_{-0.8-2.6}$
150	$0.597^{+0.013+0.073}_{-0.008-0.073}$	$11.42^{+0.32+0.81}_{-0.24-0.81}$	$21.9^{+0.6+1.5}_{-0.4-1.5}$	$38.8^{+1.1+2.5}_{-0.7-2.5}$
155	$0.536^{+0.011+0.067}_{-0.007-0.067}$	$10.66^{+0.30+0.76}_{-0.23-0.76}$	$20.6^{+0.6+1.4}_{-0.4-1.4}$	$36.6^{+1.0+2.3}_{-0.7-2.3}$
160	$0.478^{+0.010+0.061}_{-0.006-0.061}$	$9.88^{+0.27+0.70}_{-0.22-0.70}$	$19.2^{+0.5+1.3}_{-0.4-1.3}$	$34.3^{+0.9+2.2}_{-0.7-2.2}$
165	$0.425^{+0.008+0.055}_{-0.005-0.055}$	$9.11^{+0.25+0.65}_{-0.21-0.65}$	$17.8^{+0.5+1.2}_{-0.4-1.2}$	$32.0^{+0.9+2.0}_{-0.7-2.0}$
170	$0.380^{+0.007+0.050}_{-0.005-0.050}$	$8.46^{+0.24+0.61}_{-0.19-0.61}$	$16.6^{+0.5+1.1}_{-0.4-1.1}$	$30.0^{+0.8+1.9}_{-0.6-1.9}$
175	$0.342^{+0.006+0.046}_{-0.004-0.046}$	$7.90^{+0.22+0.57}_{-0.18-0.57}$	$15.6^{+0.4+1.0}_{-0.4-1.0}$	$28.4^{+0.8+1.8}_{-0.6-1.8}$
180	$0.308^{+0.005+0.042}_{-0.003-0.042}$	$7.38^{+0.20+0.53}_{-0.17-0.53}$	$14.7^{+0.4+1.0}_{-0.3-1.0}$	$26.8^{+0.7+1.7}_{-0.6-1.7}$
185	$0.277^{+0.005+0.039}_{-0.003-0.039}$	$6.90^{+0.19+0.50}_{-0.16-0.50}$	$13.8^{+0.4+0.9}_{-0.3-0.9}$	$25.3^{+0.7+1.6}_{-0.6-1.6}$
190	$0.250^{+0.004+0.036}_{-0.002-0.036}$	$6.46^{+0.18+0.47}_{-0.15-0.47}$	$13.0^{+0.4+0.9}_{-0.3-0.9}$	$23.9^{+0.7+1.5}_{-0.5-1.5}$
195	$0.227^{+0.004+0.033}_{-0.002-0.033}$	$6.08^{+0.17+0.44}_{-0.14-0.44}$	$12.3^{+0.4+0.8}_{-0.3-0.8}$	$22.8^{+0.6+1.4}_{-0.5-1.4}$
200	$0.207^{+0.003+0.031}_{-0.002-0.031}$	$5.74^{+0.17+0.42}_{-0.13-0.42}$	$11.7^{+0.3+0.8}_{-0.3-0.8}$	$21.7^{+0.6+1.4}_{-0.5-1.4}$



scale uncertainty

PDF & α_s uncertainty



EFT-based predictions for top-pair production
at Tevatron and LHC:

First NNLL+NLO results for distributions

Ahrens, Ferroglia, MN, Pecjak, Yang 2009, 2010 & 2011

State of the art

- ◆ Fixed-order NLO calculations:

- ◆ total cross section

Nason, Dawson, Ellis 1988
Beenakker et al. 1989

- ◆ differential

Nason, Dawson, Ellis 1989
Mangano, Nason, Ridolfi 1992
Frixione, Mangano, Nason, Ridolfi 1995

- ◆ A_{FB}^t

Kühn, Rodrigo 1998

- ◆ Fixed-order NNLO calculations:

- ◆ **none exist!** (but several pieces available)

- ◆ “leading terms” (enhanced near threshold)
for total cross section

Beneke, Falgari, Schwinn 2009
Czakon, Mitov, Sterman 2009
Ahrens, Ferroglia, MN, Pecjak, Yang 2010

- ◆ “leading terms” for distributions

Ahrens, Ferroglia, MN, Pecjak, Yang 2009

State of the art

- ◆ Threshold resummation at NLL:

- ◆ total cross section

Bonciani, Catani, Mangano, Nason 1998

Berger, Contopanagos 1995

Kidonakis, Laenen, Moch, Vogt 2001

- ◆ distributions

Kidonakis, Vogt 2003; Banfi, Laenen 2005

- ◆ A_{FB}^t

Almeida, Sterman, Vogelsang 2008

- ◆ Resummation at NNLL+NLO matching:

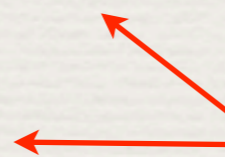
- ◆ total cross section

Beneke, Falgari, Schwinn 2009

Czakon, Mitov, Sterman 2009

- ◆ distributions

Ahrens, Ferroglia, MN, Pecjak, Yang 2010



Different kinematics

- ◆ Pair-invariant mass (PIM) kinematics:

Valentin Ahrens, Andrea Ferroglia, MN, Ben Pecjak, Li Lin Yang: 0912.3375 (PLB), 1003.5827 (JHEP)

- ◆ observe both top quarks

- ◆ define threshold limit as $z = M_{t\bar{t}}^2/\hat{s} \rightarrow 1$

- ◆ One-particle inclusive (1PI) kinematics:

Valentin Ahrens, Andrea Ferroglia, MN, Ben Pecjak, Li Lin Yang: 1103.0550

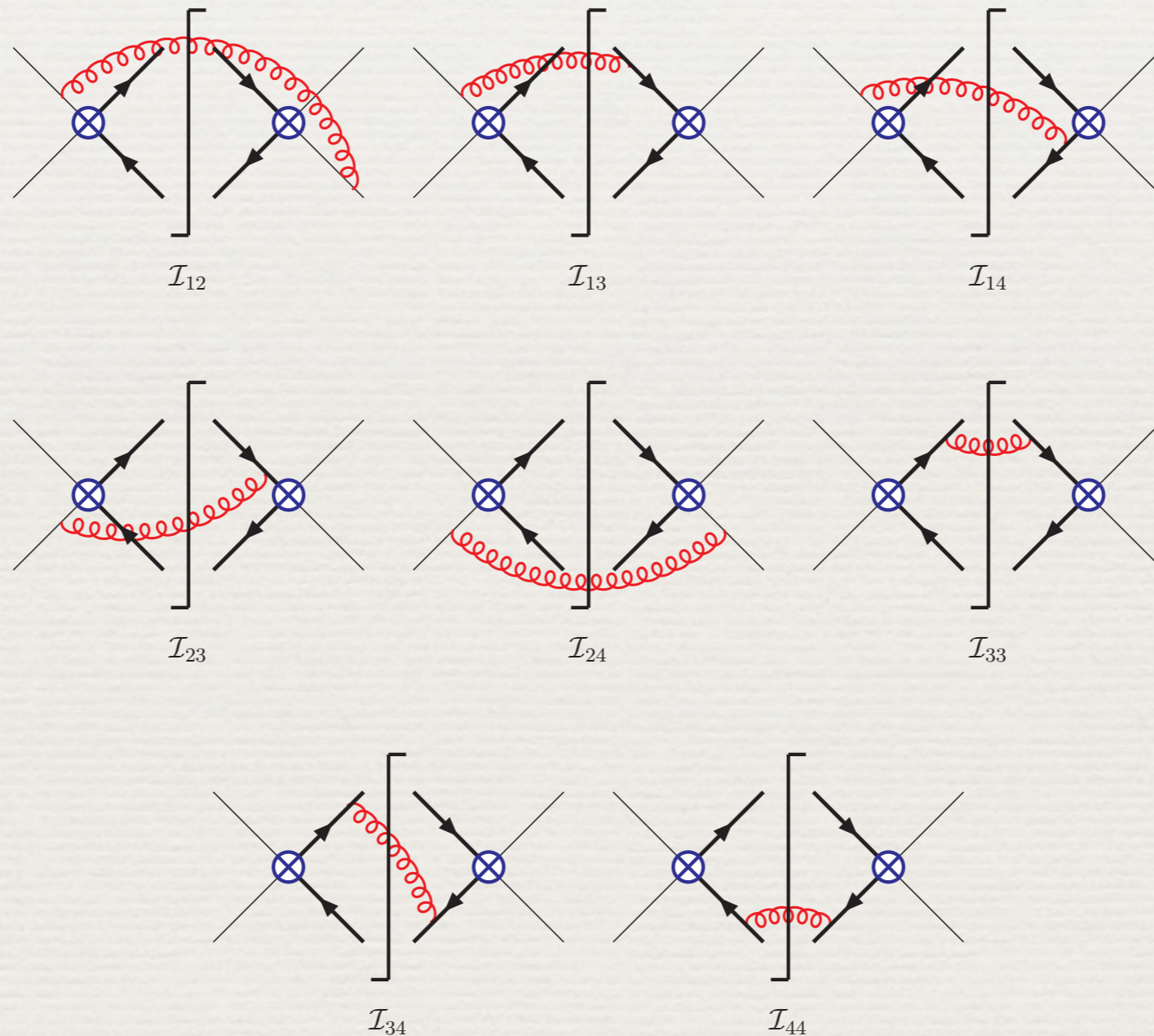
- ◆ observe only one top quark

- ◆ define threshold limit as $s_4 = p_{t+X}^2 - m_t^2 \rightarrow 0$

- ◆ Which kinematics to use is a matter of choice of the observables (none is intrinsically better than the other!)

Top-pair production at NLO+NNLL

- Soft functions from time-like Wilson-line correlation function:



Top-pair production at NLO+NNLL

Ferrogia, MN, Pecjak, Yang 2009

- ♦ Anomalous-dimension matrices in s-channel singlet-octet basis for $q\bar{q}, gg \rightarrow t\bar{t}$ channels:

$$\begin{aligned}
 \mathbf{\Gamma}_{q\bar{q}} &= \left[C_F \gamma_{\text{cusp}}(\alpha_s) \ln \frac{-s}{\mu^2} + C_F \gamma_{\text{cusp}}(\beta_{34}, \alpha_s) + 2\gamma^q(\alpha_s) + 2\gamma^Q(\alpha_s) \right] \mathbf{1} \\
 &+ \frac{N}{2} \left[\gamma_{\text{cusp}}(\alpha_s) \ln \frac{(-s_{13})(-s_{24})}{(-s) m_t^2} - \gamma_{\text{cusp}}(\beta_{34}, \alpha_s) \right] \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \\
 &+ \gamma_{\text{cusp}}(\alpha_s) \ln \frac{(-s_{13})(-s_{24})}{(-s_{14})(-s_{23})} \left[\begin{pmatrix} 0 & \frac{C_F}{2N} \\ 1 & -\frac{1}{N} \end{pmatrix} + \frac{\alpha_s}{4\pi} g(\beta_{34}) \begin{pmatrix} 0 & \frac{C_F}{2} \\ -N & 0 \end{pmatrix} \right] + \mathcal{O}(\alpha_s^3) \\
 \\
 \mathbf{\Gamma}_{gg} &= \left[N \gamma_{\text{cusp}}(\alpha_s) \ln \frac{-s}{\mu^2} + C_F \gamma_{\text{cusp}}(\beta_{34}, \alpha_s) + 2\gamma^g(\alpha_s) + 2\gamma^Q(\alpha_s) \right] \mathbf{1} \\
 &+ \frac{N}{2} \left[\gamma_{\text{cusp}}(\alpha_s) \ln \frac{(-s_{13})(-s_{24})}{(-s) m_t^2} - \gamma_{\text{cusp}}(\beta_{34}, \alpha_s) \right] \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
 &+ \gamma_{\text{cusp}}(\alpha_s) \ln \frac{(-s_{13})(-s_{24})}{(-s_{14})(-s_{23})} \left[\begin{pmatrix} 0 & \frac{1}{2} & 0 \\ 1 & -\frac{N}{4} & \frac{N^2-4}{4N} \\ 0 & \frac{N}{4} & -\frac{N}{4} \end{pmatrix} + \frac{\alpha_s}{4\pi} g(\beta_{34}) \begin{pmatrix} 0 & \frac{N}{2} & 0 \\ -N & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] + \mathcal{O}(\alpha_s^3).
 \end{aligned} \tag{55}$$

Top-pair production at NLO+NNLL

- ♦ Can use these results to predict leading singular terms near partonic threshold $z = M^2/\hat{s} \rightarrow 1$

- ♦ Obtain NNLO coefficients of distributions

$$P'_n(z) = \left[\frac{1}{1-z} \ln^n \left(\frac{M^2(1-z)^2}{\mu^2 z} \right) \right]_+$$

and (partially) of $\delta(1-z)$

- ♦ Yields **presently best estimate** of NNLO terms

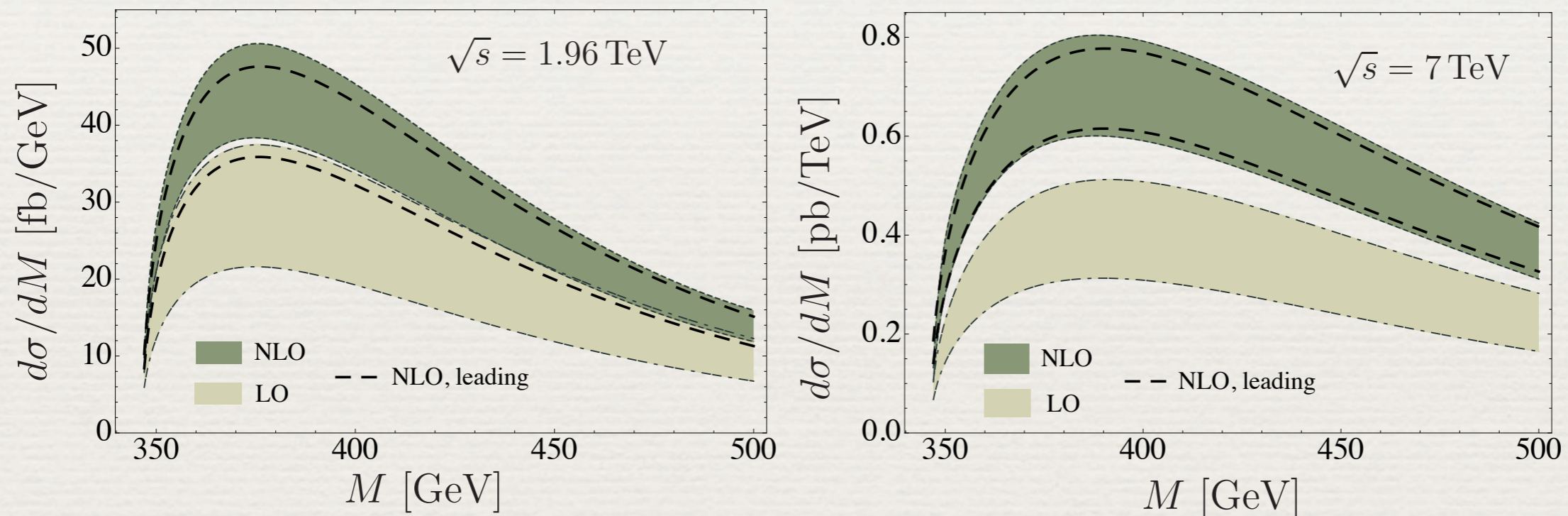
- ♦ **Note:** includes some subleading terms $\sim \ln(z)$

beyond distributions

$$P_n(z) = \left[\frac{\ln^n(1-z)}{1-z} \right]_+$$

Dominance of threshold terms

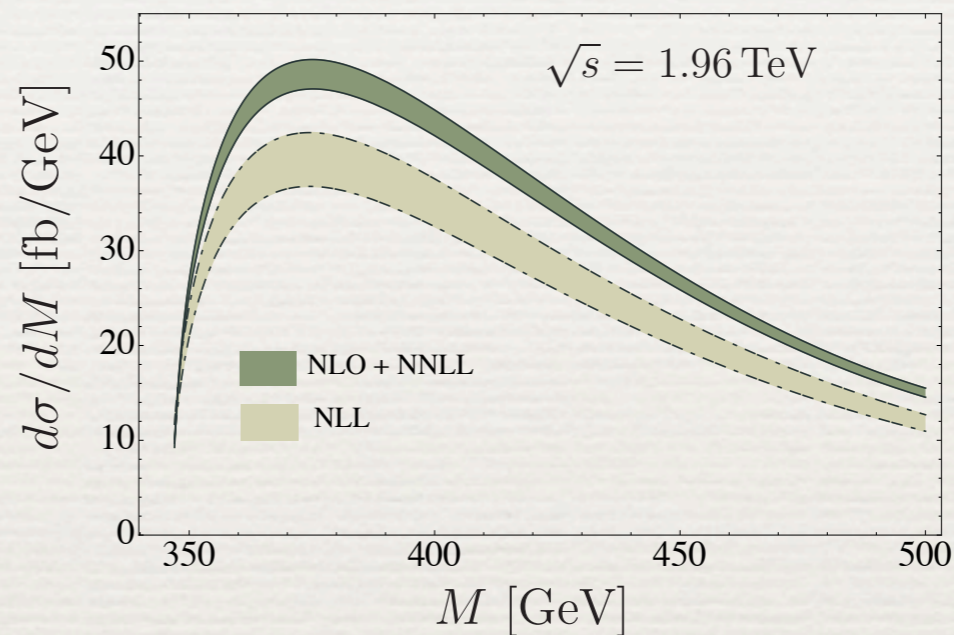
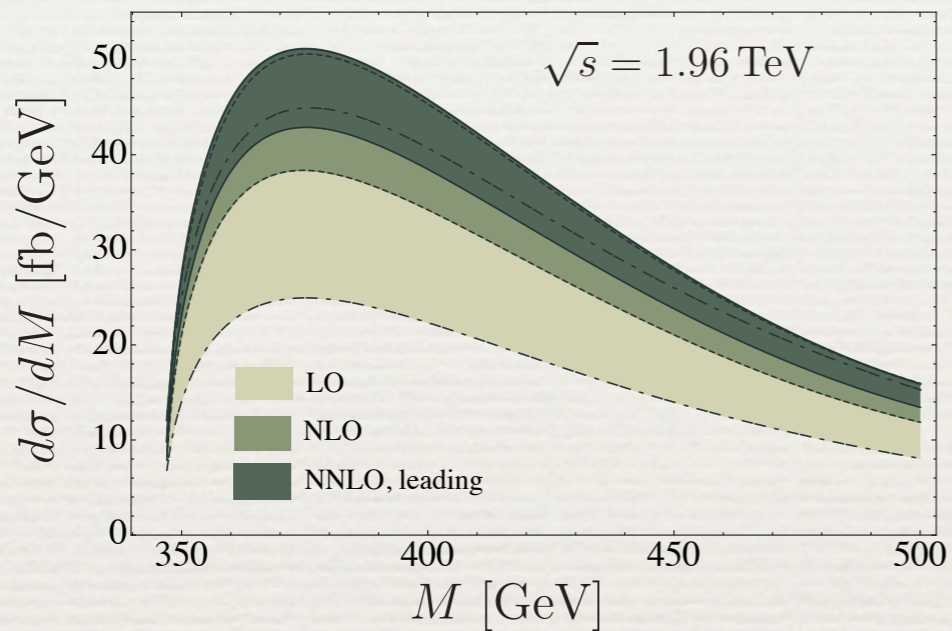
- ◆ Fixed-order results for invariant mass distribution at Tevatron and LHC:



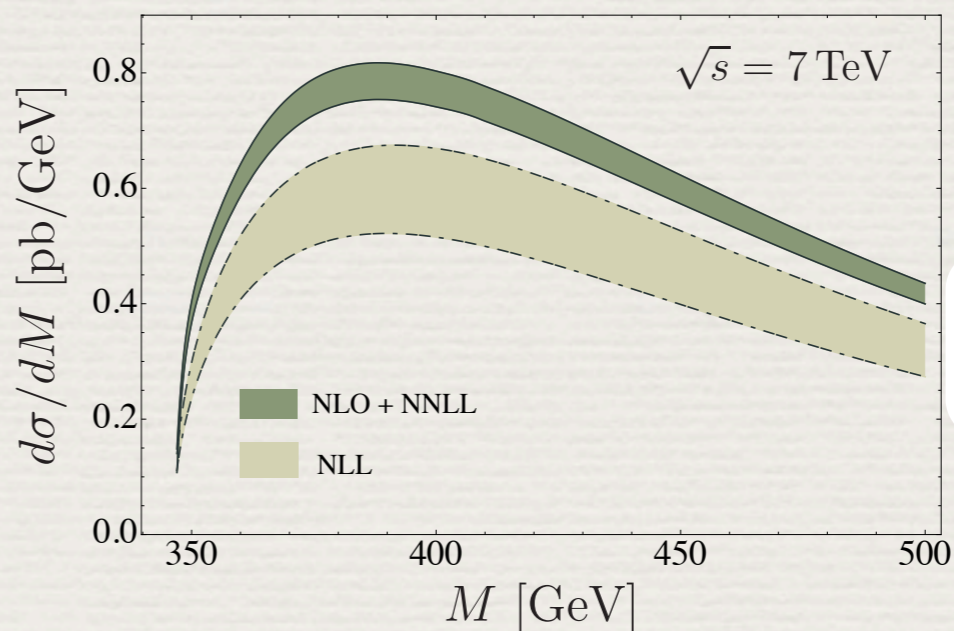
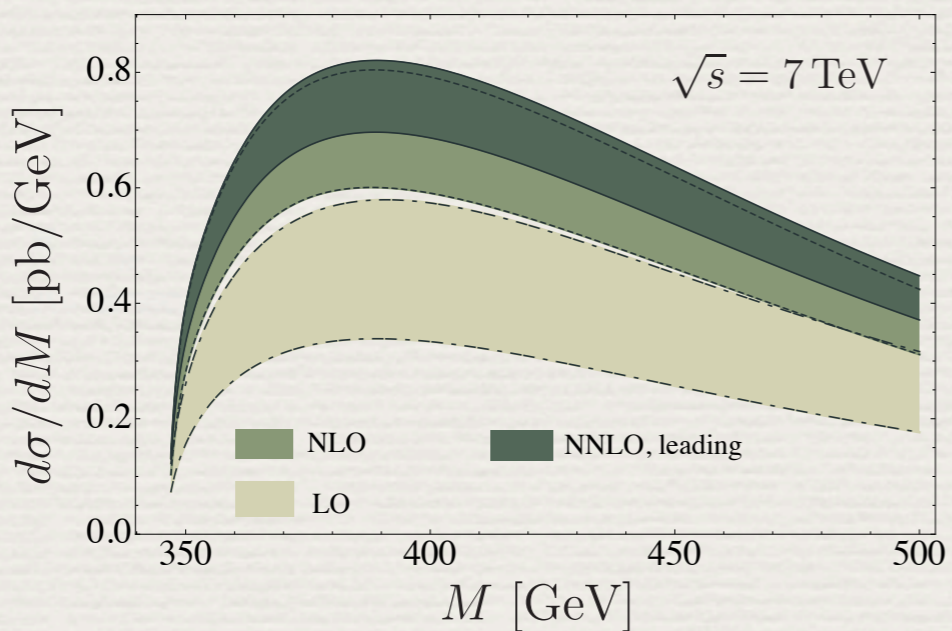
- ◆ Leading singular terms near partonic threshold $z = M^2/\hat{s} \rightarrow 1$ give dominant contributions even at low and moderate M values

Invariant mass distributions (PIM)

- Fixed-order vs. resummed PT (matched to NLO):



Tevatron



LHC

NNLO
(partial)

NLO

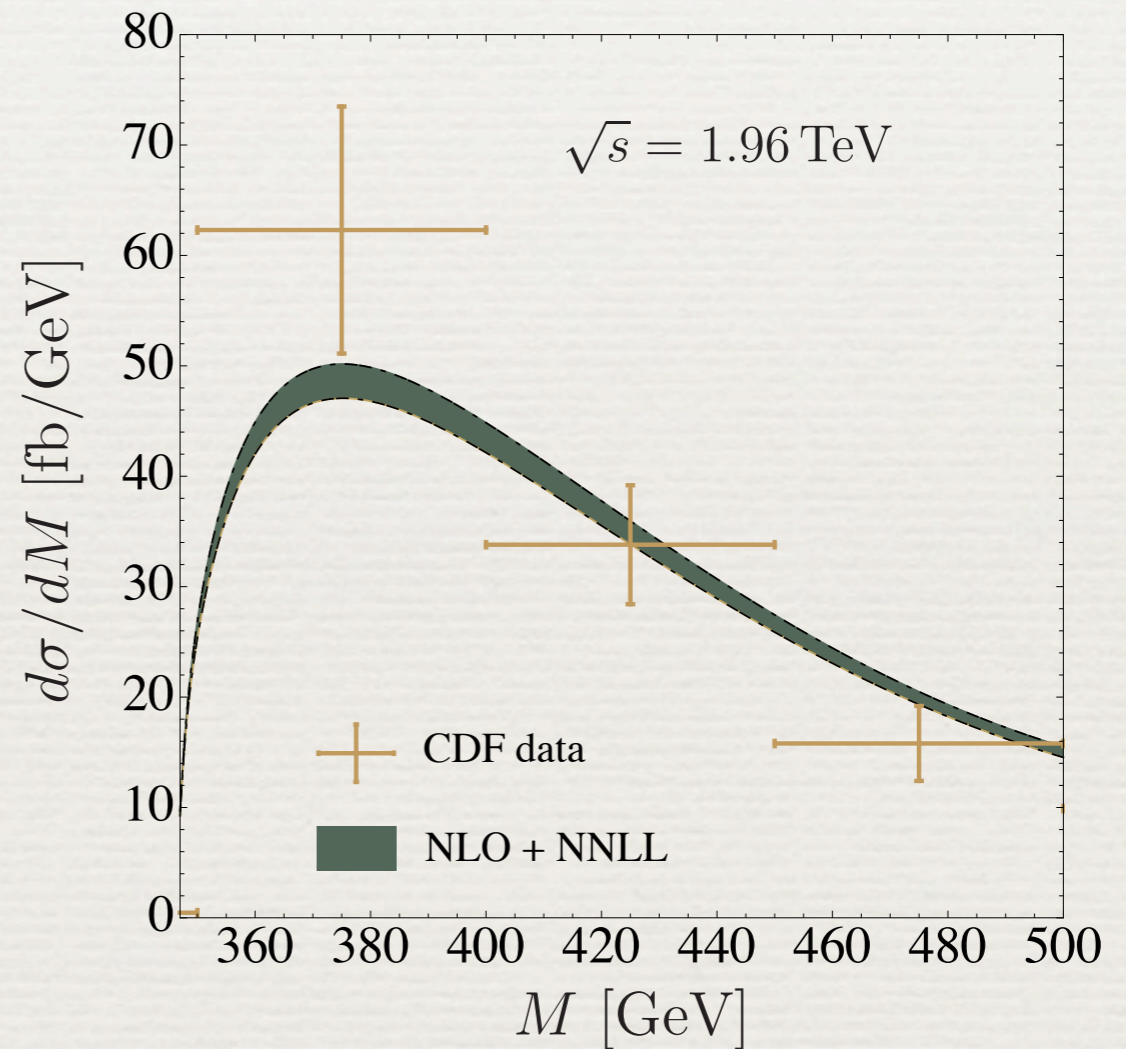
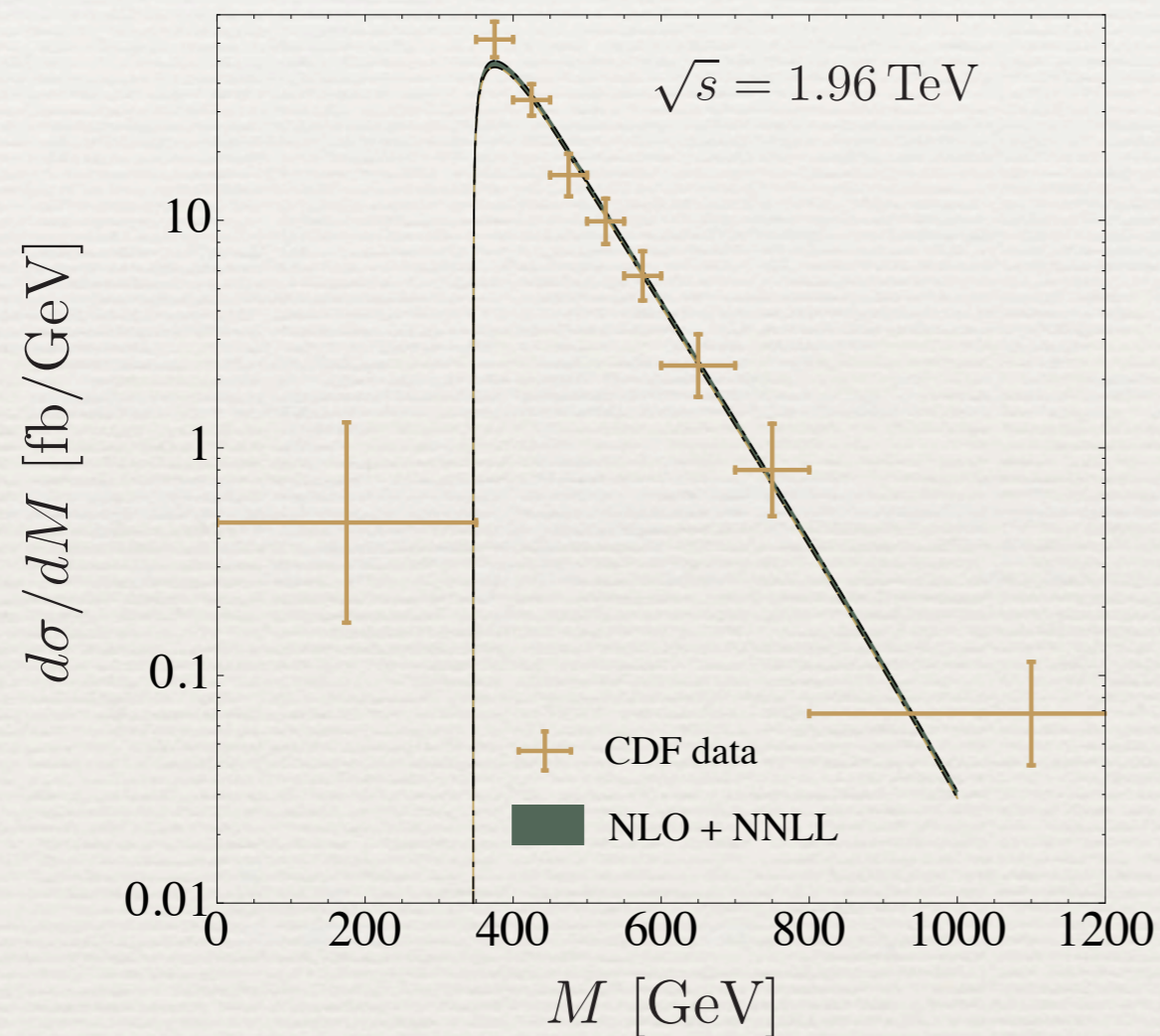
LO

NNLL+NLO

NLL+LO

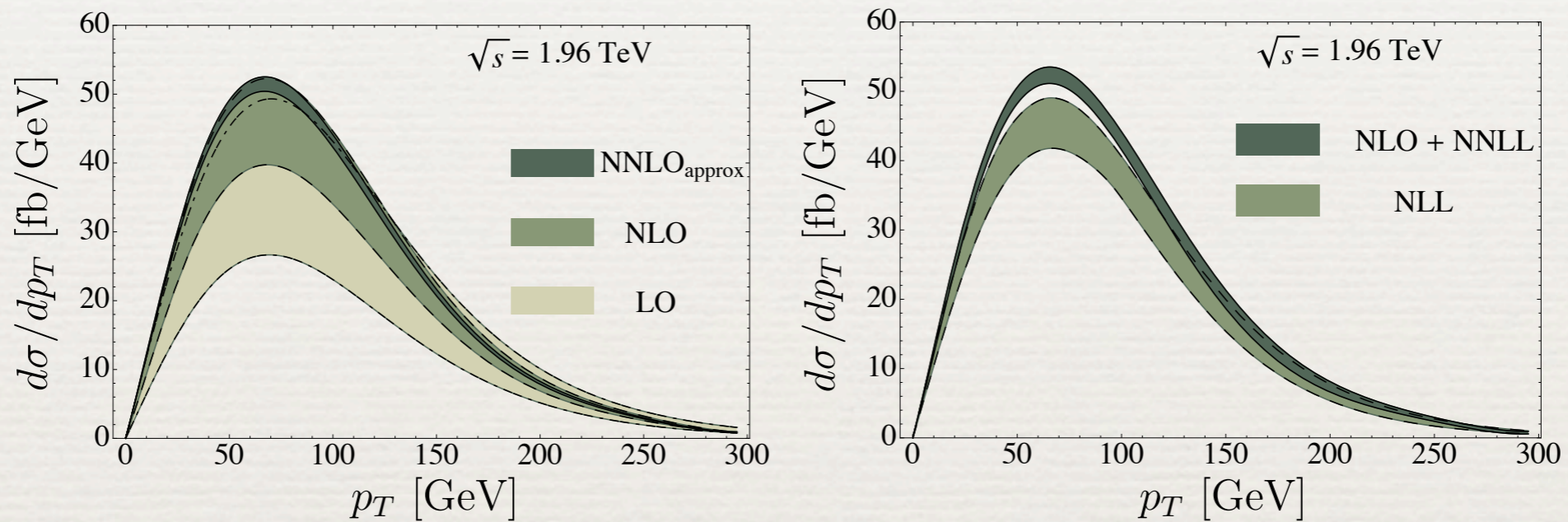
Comparison with CDF data

- ◆ Overlay (not a fit!) for $m_t=173.1$ GeV:

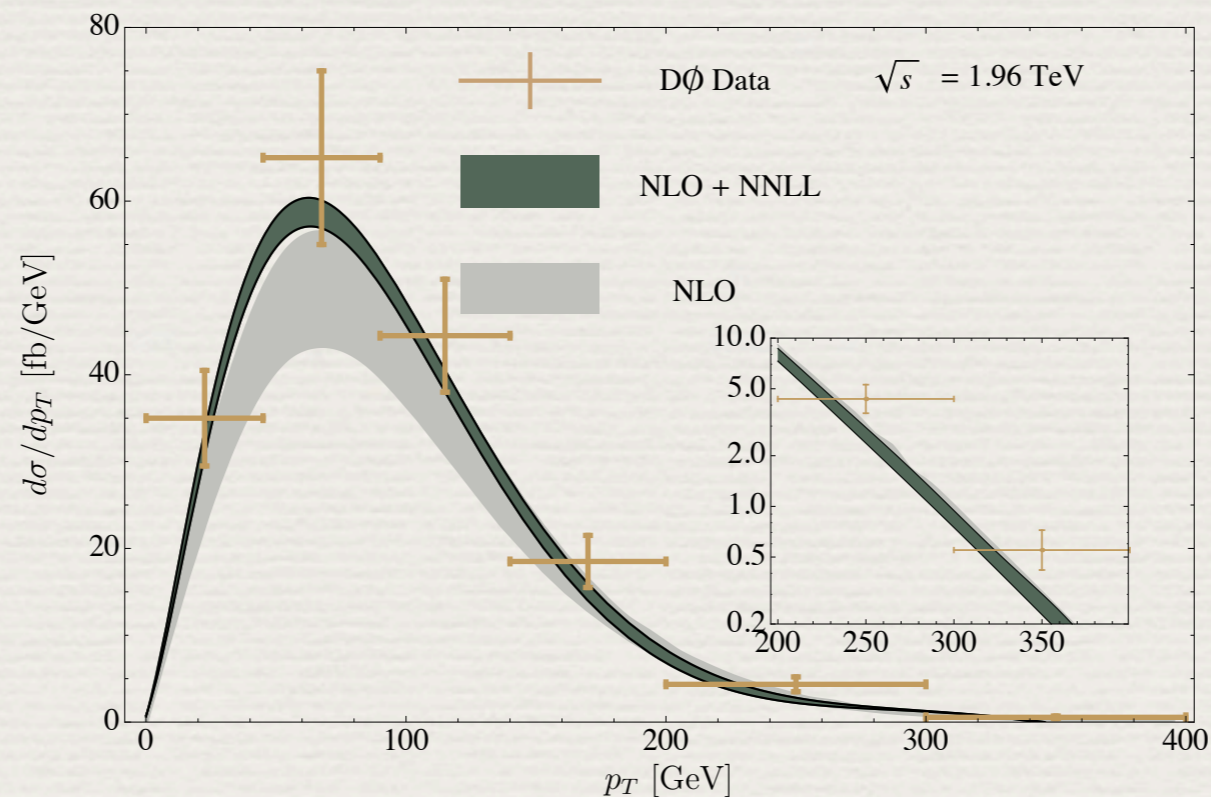


Top-quark p_T distributions (1PI)

- ◆ Fixed-order vs. resummed PT (matched to NLO):

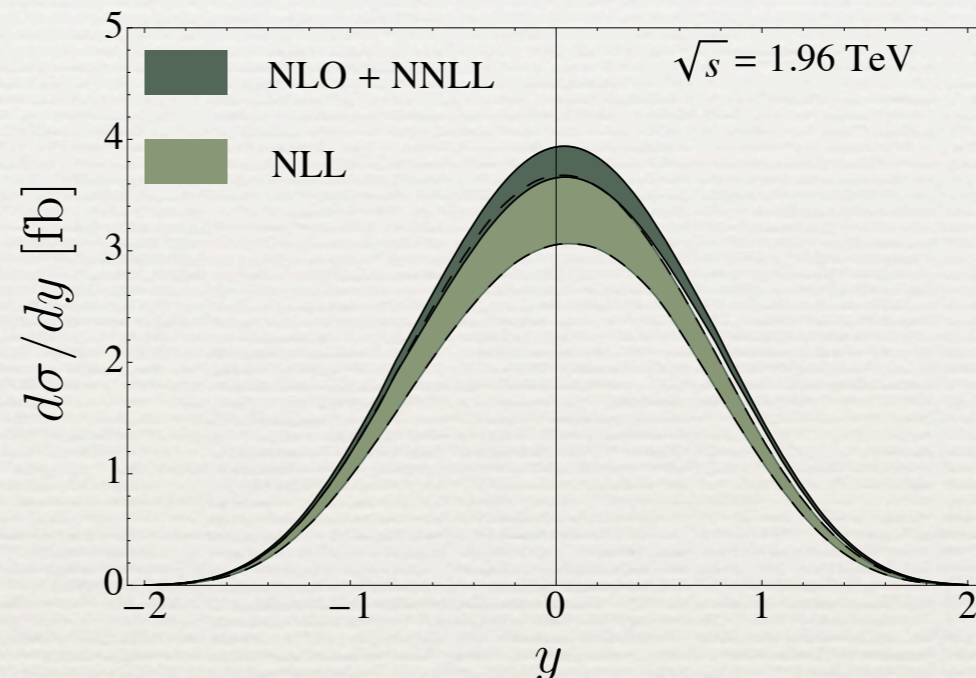
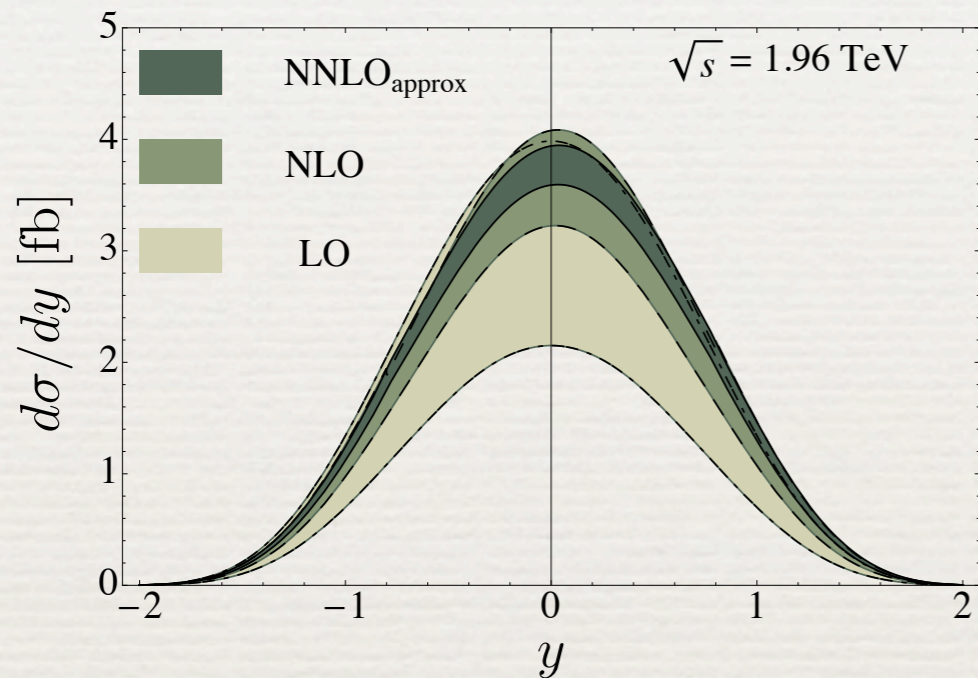


Tevatron

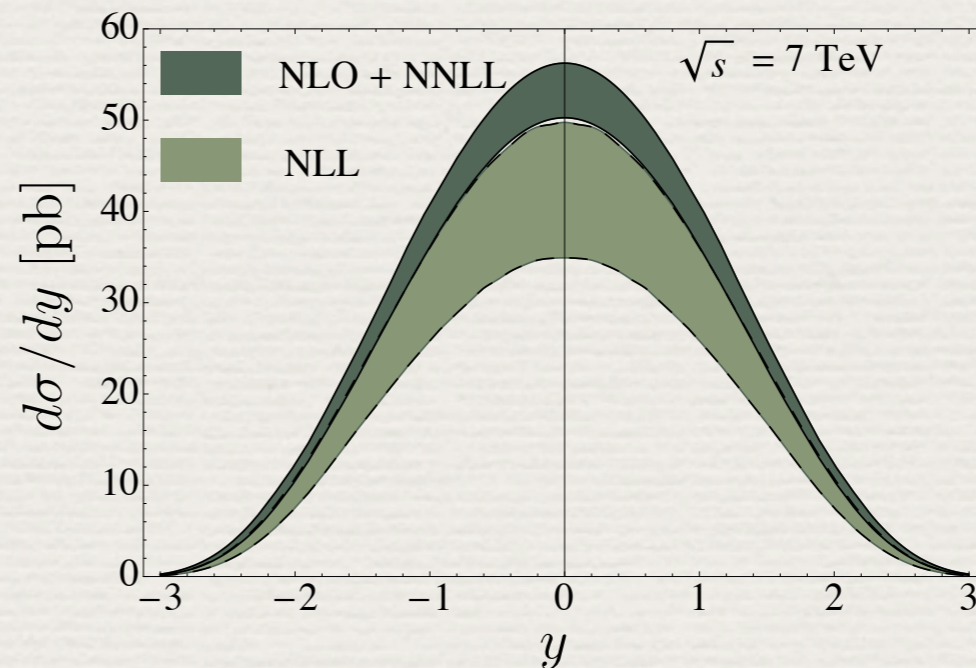
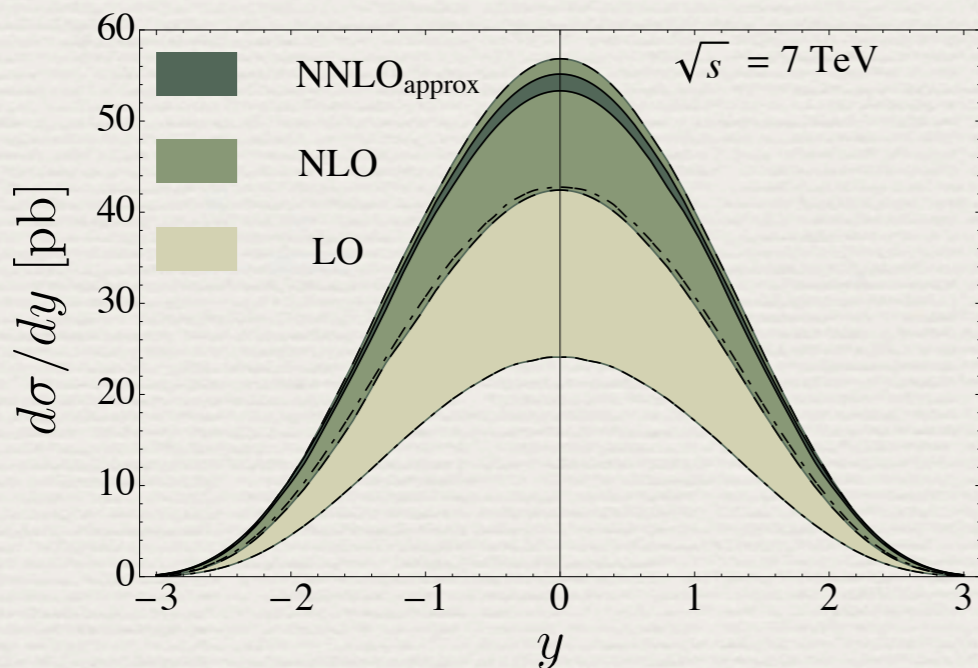


Top-quark rapidity distributions (1PI)

- Fixed-order vs. resummed PT (matched to NLO):



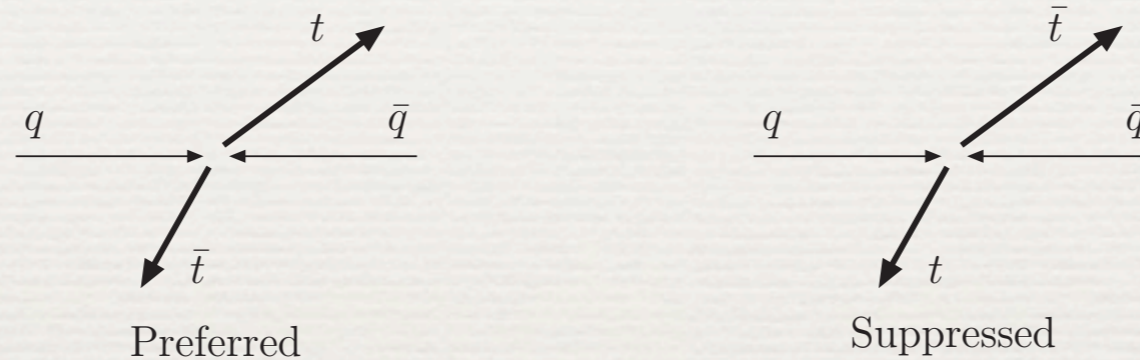
Tevatron



LHC

Forward-backward asymmetry

- At Tevatron, top-quarks are emitted preferably in direction of incoming quark:



- Define inclusive asymmetry:

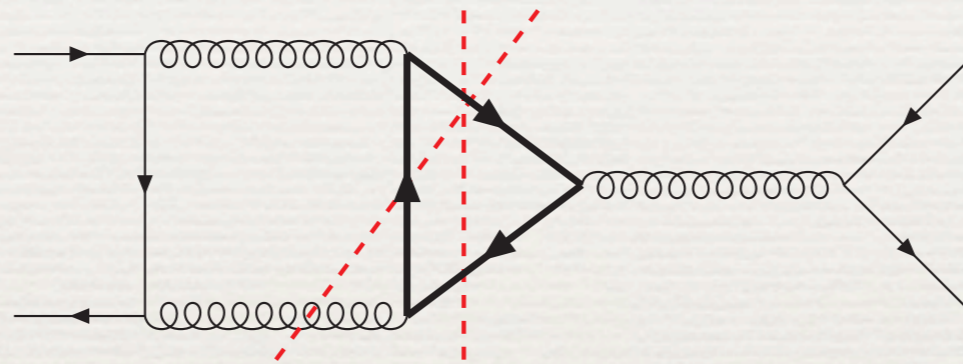
$$A_{\text{FB}}^t \equiv \frac{\int_{4m_t^2}^s dM \left(\int_0^1 d \cos \theta \frac{d^2 \sigma^{N_1 N_2 \rightarrow t \bar{t} X}}{dM d \cos \theta} - \int_{-1}^0 d \cos \theta \frac{d^2 \sigma^{N_1 N_2 \rightarrow t \bar{t} X}}{dM d \cos \theta} \right)}{\int_{4m_t^2}^s dM \left(\int_0^1 d \cos \theta \frac{d^2 \sigma^{N_1 N_2 \rightarrow t \bar{t} X}}{dM d \cos \theta} + \int_{-1}^0 d \cos \theta \frac{d^2 \sigma^{N_1 N_2 \rightarrow t \bar{t} X}}{dM d \cos \theta} \right)}$$

- Most recent exptl. results (ICHEP 2010):

$$A_{\text{FB}}^t |_{\text{CDF}} = (15.8 \pm 7.2_{\text{stat}} \pm 1.7_{\text{sys}})\% \quad (\text{ttbar frame})$$

Forward-backward asymmetry

- Non-zero contributions arise first at one-loop order, from interference terms such as:

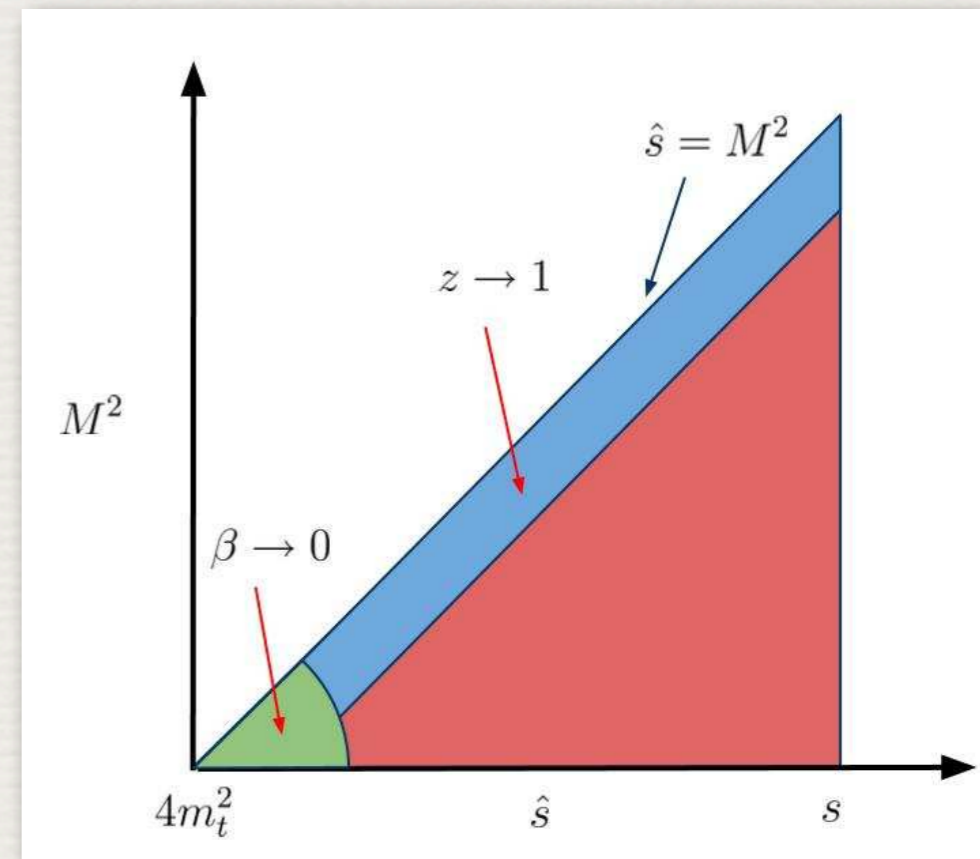


- Predictions:

	$0.2 < \mu_f/\text{TeV} < 0.8$		$m_t/2 < \mu_f < 2m_t$	
	$\Delta\sigma_{\text{FB}}$ [pb]	A_{FB}^t [%]	$\Delta\sigma_{\text{FB}}$ [pb]	A_{FB}^t [%]
NLL	$0.29^{+0.16}_{-0.16}$	$5.8^{+3.3}_{-3.2}$	$0.31^{+0.16}_{-0.17}$	$5.9^{+3.4}_{-3.3}$
NLO, leading	$0.19^{+0.09}_{-0.06}$	$5.2^{+0.4}_{-0.4}$	$0.31^{+0.16}_{-0.10}$	$5.7^{+0.5}_{-0.4}$
NLO	$0.25^{+0.12}_{-0.07}$	$6.7^{+0.6}_{-0.4}$	$0.40^{+0.21}_{-0.13}$	$7.4^{+0.7}_{-0.6}$
NLO+NNLL	$0.40^{+0.06}_{-0.06}$	$6.6^{+0.6}_{-0.5}$	$0.45^{+0.08}_{-0.07}$	$7.3^{+1.1}_{-0.7}$
NNLO, approx (scheme A)	$0.37^{+0.10}_{-0.08}$	$6.4^{+0.9}_{-0.7}$	$0.48^{+0.11}_{-0.10}$	$7.5^{+1.3}_{-0.9}$
NNLO, approx (scheme B)	$0.34^{+0.08}_{-0.07}$	$5.8^{+0.8}_{-0.6}$	$0.45^{+0.09}_{-0.09}$	$6.8^{+1.1}_{-0.8}$

Total cross section

- ♦ Usually, resummation is done around absolute threshold at $\hat{s}=4m_t^2$ (non-relativistic top quarks)
- ♦ Mixed Coulomb and soft gluon singularities arise for $\beta = \sqrt{1 - 4m_t^2/\hat{s}} \rightarrow 0$, which have been resummed at NNLL Moch, Uwer 2008; Beneke et al. 2009
- ♦ In our approach, soft gluon effects are resummed also far above absolute threshold!
- ♦ Important, since top quarks are relativistic, $\beta_t \sim 0.4-0.9$

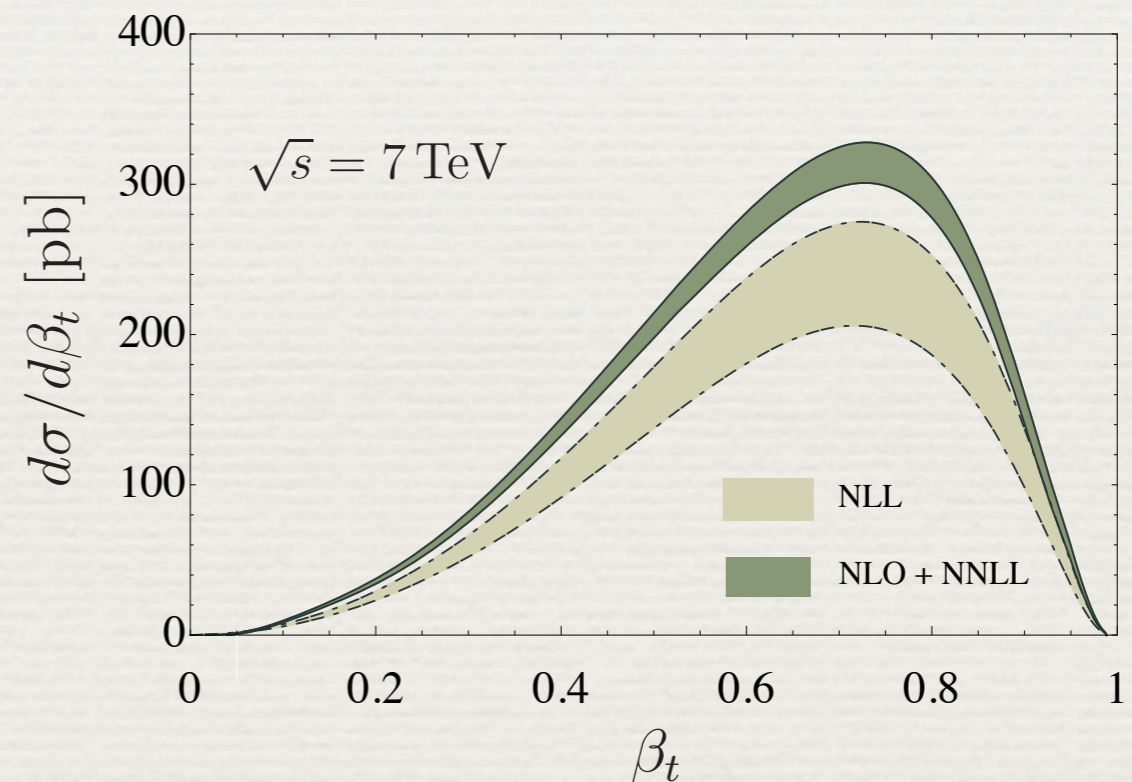
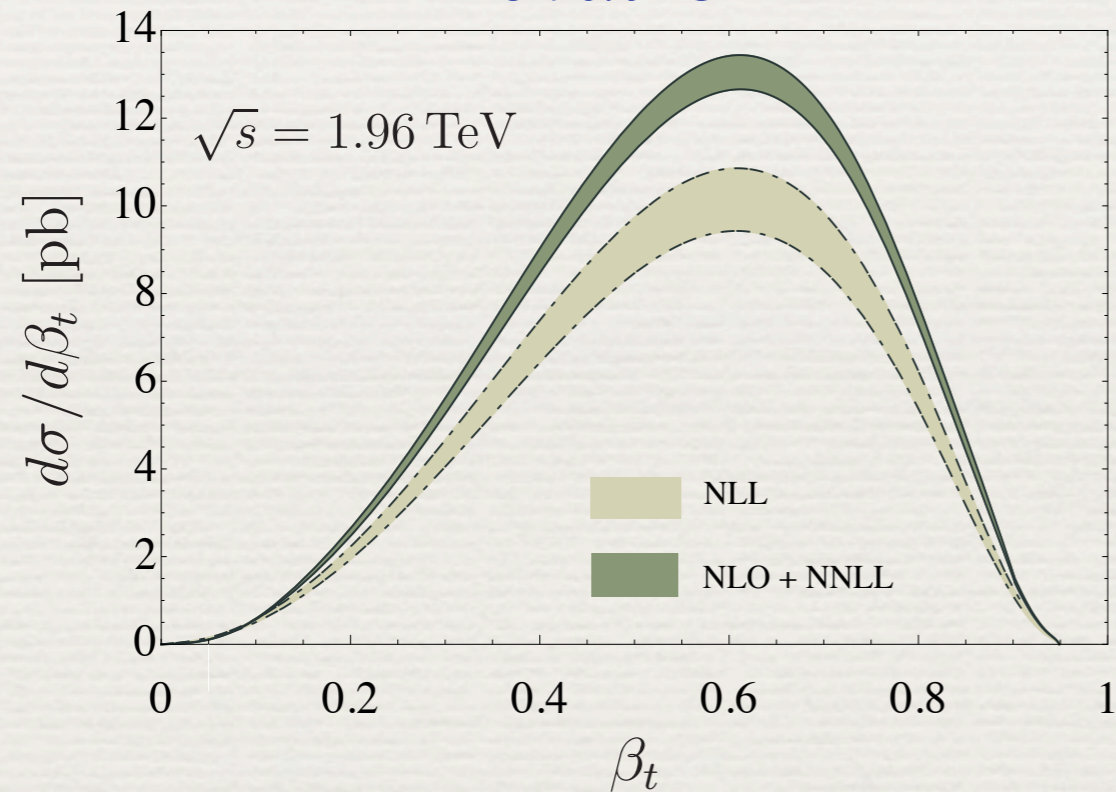


Total cross section

- ◆ Transform from M^2 to relative 3-velocity of top quarks in $t\bar{t}$ rest frame: $\beta_t = \sqrt{1 - \frac{4m_t^2}{M^2}}$

Tevatron

LHC

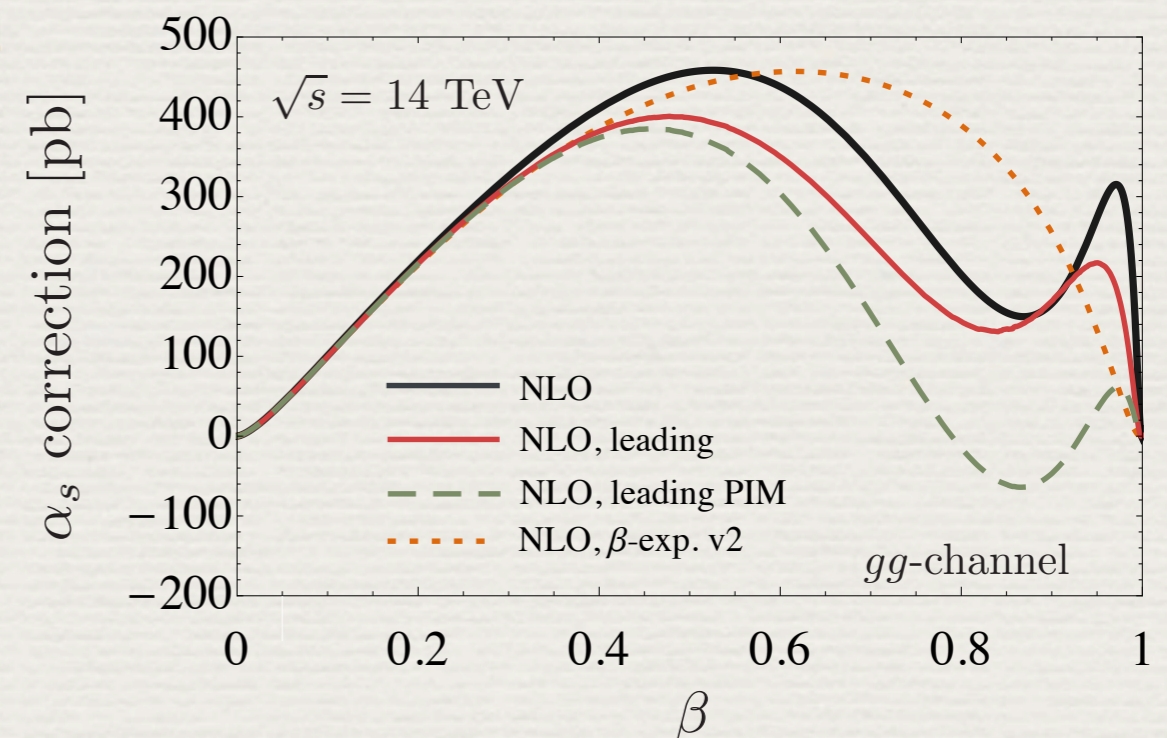
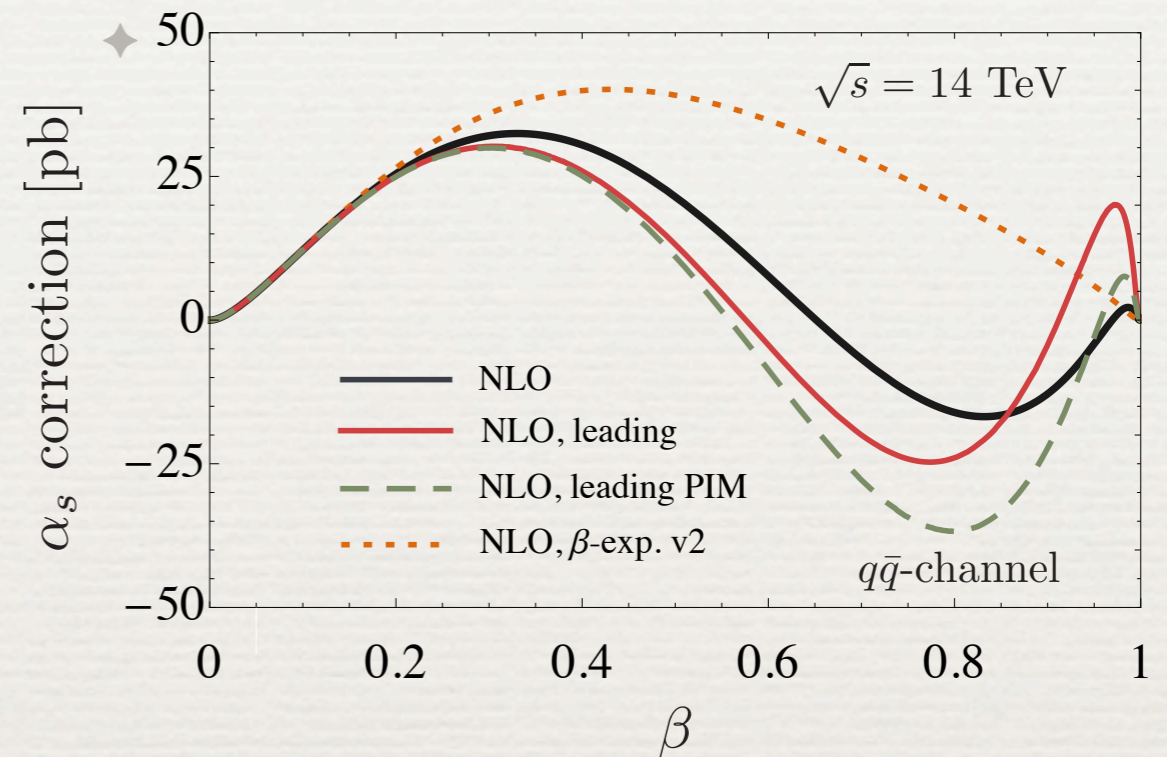


- ◆ Top quarks are relativistic, $\beta_t \sim 0.4-0.9$

Total cross section

Comparison of different approximations to NLO corrections (including parton luminosities):

- ♦ **our approximation** lies much closer to NLO result than **small- β approximation** (Moch, Uwer)
- ♦ reproduces fine details of the curves
- ♦ improvement over **traditional PIM curve** (Kidonakis)



Total cross section (PIM & 1PI)

- ◆ Detailed predictions for total cross sections:

Cross section (pb)	Tevatron	LHC (7 TeV)	LHC (8 TeV)	LHC (14 TeV)
$\sigma_{\text{NLO leading, 1PI}_{\text{SCET}}}$	$6.79^{+0.20+0.38}_{-0.70-0.24}$	$163^{+0}_{-11} \text{ } ^{+9}_{-9}$	$232^{+0}_{-14} \text{ } ^{+11}_{-12}$	$887^{+0}_{-66} \text{ } ^{+30}_{-32}$
$\sigma_{\text{NLO leading, PIM}_{\text{SCET}}}$	$6.42^{+0.42+0.35}_{-0.76-0.23}$	$152^{+7}_{-15} \text{ } ^{+8}_{-8}$	$217^{+8}_{-20} \text{ } ^{+10}_{-11}$	836^{+18+29}_{-60-30}
$\sigma_{\text{NLO, } q\bar{q} + gg}$	$6.80^{+0.27}_{-0.73}$	160^{+5}_{-15}	228^{+6}_{-20}	879^{+21}_{-62}
σ_{NLO}	$6.72^{+0.36+0.37}_{-0.76-0.24}$	159^{+20+8}_{-21-9}	227^{+28+11}_{-30-12}	$889^{+107+31}_{-106-32}$
$\sigma_{\text{NLO+NNLL, 1PI}_{\text{SCET}}}$	$6.55^{+0.16+0.32}_{-0.14-0.24}$	150^{+7+8}_{-7-8}	214^{+10+10}_{-10-11}	824^{+41+28}_{-44-30}
$\sigma_{\text{NLO+NNLL, PIM}_{\text{SCET}}}$	$6.46^{+0.18+0.32}_{-0.19-0.24}$	147^{+7+8}_{-6-8}	210^{+10+10}_{-8-11}	811^{+45+29}_{-42-30}
$\sigma_{\text{NNLO approx, 1PI}_{\text{SCET}}}$	$6.63^{+0.00+0.33}_{-0.27-0.24}$	155^{+3+8}_{-2-9}	222^{+5+11}_{-3-11}	851^{+25+29}_{-5-31}
$\sigma_{\text{NNLO approx, PIM}_{\text{SCET}}}$	$6.62^{+0.05+0.33}_{-0.40-0.24}$	155^{+8+8}_{-8-9}	221^{+12+11}_{-12-12}	860^{+46+30}_{-43-33}

scale uncertainty

PDF uncertainty

$$\mu_f = m_t$$

- ◆ Singular terms dominate NLO corrections
- ◆ Resummation stabilizes scale dependence

Total cross section (PIM & 1PI)

- Comparison with traditional calculations:

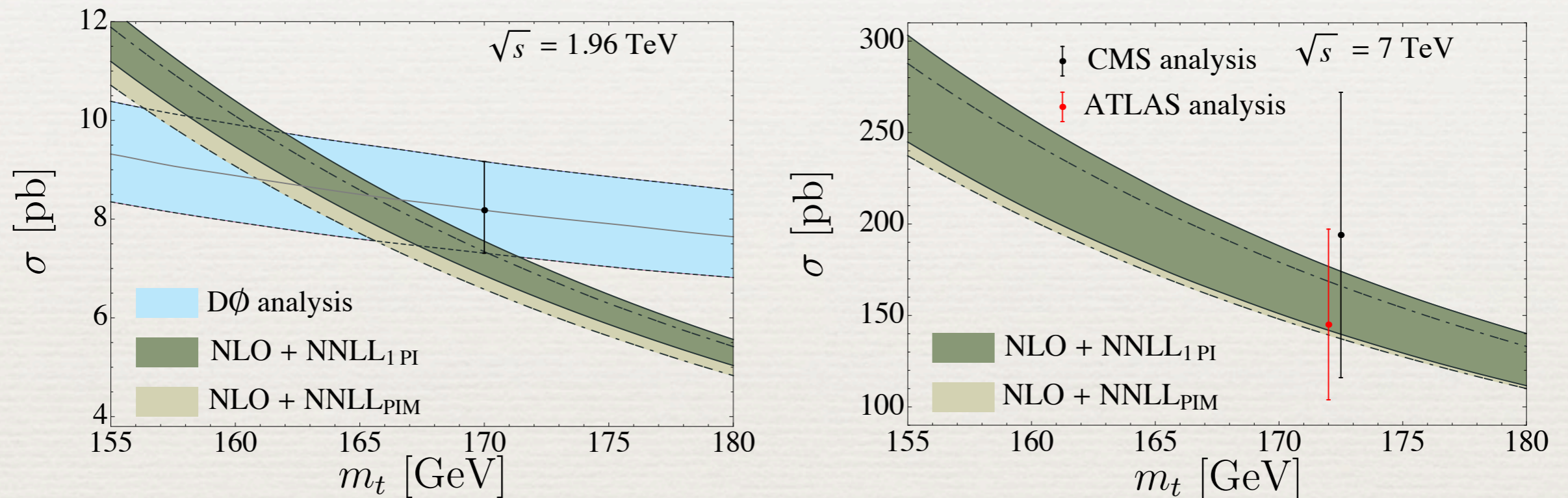
Cross section (pb)	Tevatron	LHC (7 TeV)	LHC (14 TeV)
$\sigma_{\text{NLO leading, 1PI (1PI}_{\text{SCET}})$	$7.23^{+0.45}_{-0.86}$ ($6.79^{+0.20}_{-0.70}$)	183^{+6}_{-18} (163^{+0}_{-11})	1024^{+0}_{-67} (887^{+0}_{-66})
$\sigma_{\text{NLO leading, PIM (PIM}_{\text{SCET}})$	$6.20^{+0.28}_{-0.69}$ ($6.42^{+0.42}_{-0.76}$)	143^{+1}_{-12} (152^{+7}_{-15})	771^{+0}_{-42} (836^{+18}_{-60})
$\sigma_{\text{NNLO approx, 1PI (1PI}_{\text{SCET}})$	$7.06^{+0.00}_{-0.29}$ ($6.63^{+0.00}_{-0.27}$)	180^{+3}_{-8} (155^{+3}_{-2})	1009^{+40}_{-54} (851^{+25}_{-5})
$\sigma_{\text{NNLO approx, PIM (PIM}_{\text{SCET}})$	$6.46^{+0.18}_{-0.45}$ ($6.62^{+0.05}_{-0.40}$)	148^{+14}_{-11} (155^{+8}_{-8})	823^{+78}_{-67} (860^{+46}_{-43})

$$\mu_f = m_t$$

- SCET results include leading power corrections automatically, and thus show much better consistency!
- Legitimate to average results from different schemes (deviations well within errors)

Total cross sections

- ♦ Mass dependence (pole scheme):



- ♦ Extract pole masses in range between 165 and 170 GeV, in fair agreement with world average $m_t = (173.1 \pm 1.3)$ GeV **(which is ill defined!)**
- ♦ Extraction of **short-distance masses** in progress

Conclusions

- ◆ Effective field theory provides **efficient tools** for addressing difficult collider-physics problems
- ◆ Systematic “derivation” of **factorization** theorems and simple, transparent **resummation** techniques
- ◆ Detailed applications exist for **Drell-Yan, Higgs, and top-quark pair production**; first result for jets at hadron colliders emerging recently
- ◆ Longer-term goal is to understand resummation at NNLL+NLO order for **jet processes**, such as $pp \rightarrow n \text{ jets} + V$ (with $n \leq 3$, $V = \gamma, Z, W$)