Warped The Empire Strikes Back Penguins

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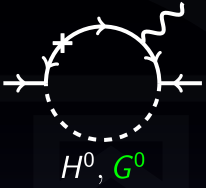
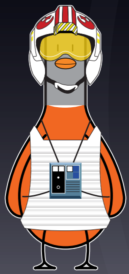
A long time ago in a galaxy far, far away

(One year ago in Newman Lab, Yuhsin's last talk)

- Anarchic RS flavor model
- Loop calculation of $\mu \to e\gamma$
- Mild tension with tree-level constraints
- Matching 5D and KK formalisms





New developments

- Goldstone cancellation* & many more diagrams
- Mild non-tension with tree-level flavor constraints
- **Empire**: 'anarchic' models aren't so anarchic
- Finiteness from 5D power counting
- Comments on two-loop structure.

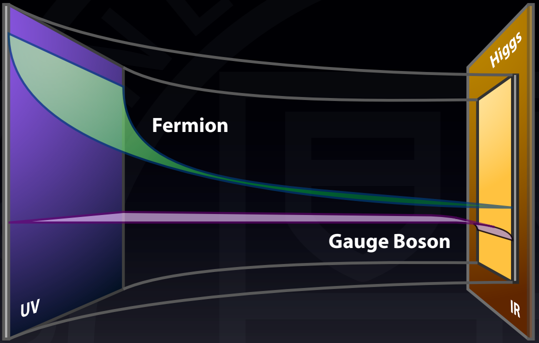
* thanks to M. Blanke, K. Agashe, Y. Hori, T. Okui

Reminder: Randall-Sundrum

$$ds^2 = \left(\frac{R}{z}\right)^2 (dx^2 - dz^2)$$

Randall, Sundrum (99);

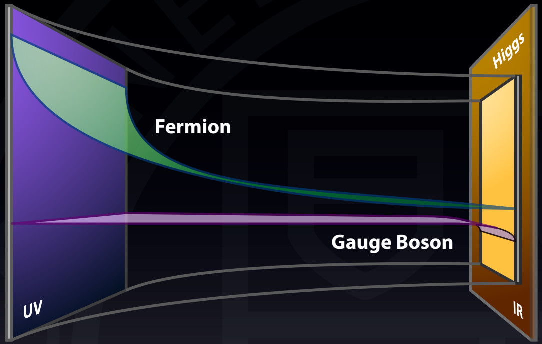
Reminder: Randall-Sundrum



$$ds^2 = \left(\frac{R}{z}\right)^2 \left(dx^2 - dz^2\right)$$

Randall, Sundrum (99); Davoudiasl, Hewett, Rizzo (99); Grossman, Neubert (00); Gherghetta, Pomarol (00); **Bulk Higgs:** Agashe, Contino, Pomarol (04); Davoudiasl, Lille, Rizzo (05)

Reminder: Yukawa matrices



$$Y_{ij}^{(4D)} = f_i Y_{ij}^* f_j \qquad f_i = \sqrt{\frac{1-2c_i}{1-(R/R')^{1-2c_i}}}$$

Flavor: Huber, Shafi (03); Burdman (03); Kalil, Mohapatra (04); Agashe, Perez, Soni (04); Chen (05); Agashe, Blechman, Petriello (06); Davidson, Isidori, Uhlig (07); Csáki, Falkowski, Weiler (08); Chen, H.B. Yu (08); Agashe, Okui, Sundrum (08); Chen, Mahanthappa, F. Yu (09), ...

Anarchic Flavor in RS

Definition: anarchic matrix

All entries $\mathcal{O}(1)$ with arbitrary phase. The product of anarchic matrices is also anarchic. Assumption: this is true in all preferred bases.

$$Y_{ij}^{(4D)} = f_i Y_{ij}^* f_j \qquad f_i = \sqrt{\frac{1-2c_i}{1-(R/R')^{1-2c_i}}}$$

The Y_{ij}^* are anarchic matrices that are 5D parameters,

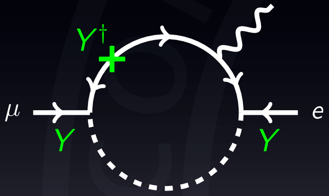
$$Y_{ij}^* = Y_* \,\text{Ⓐ}_{ij}$$

The mass heirarchy $m_i = f_i Y_{ii}^* f_i v$ comes from the exponentially small overlap of the zero-mode fermions with the Higgs vev. This is controlled by the fermion bulk masses, $c_i \sim 0.51 - 0.8$.

Lepton Flavor Violation

Penguin constraints

Assuming the mass hierarchies are controlled by the f_is, we would like to constrain the anarchic and KK scales: Y_* and M_{KK}.

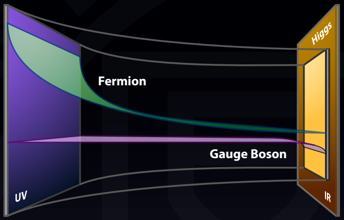


$$\mathcal{M}_{\text{loop}} \sim \left(\frac{1}{M_{\text{KK}}}\right)^2 f_L {Y_*}^3 f_{-E} \sim \left(\frac{1}{M_{\text{KK}}}\right)^2 {Y_*}^2 m$$

Decoupling: $\mathcal{M}$ goes like negative power of M_{KK}.
'No coupling': $\mathcal{M}$ goes like positive power of Y_*.

Lepton Flavor Violation

Tree level constraints

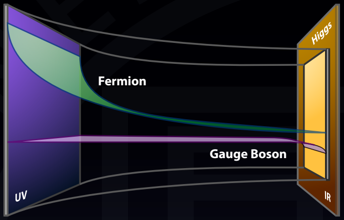


$$\mathcal{M}_{\text{tree}} \sim \left(\frac{1}{M_{\text{KK}}}\right)^2 \left(\frac{1}{Y_*}\right)$$

Must maintain SM spectrum $m_i \sim f_i Y^*_{ii} f_i v$. As Y_* increases, zero-mode fermion profiles are pushed away from the IR brane. This reduces their overlap with the non-universal part of the Z.

Lepton Flavor Violation

A possible tension between tree- and loop-level bounds



- Tree-level bound: $\left(\frac{3\ \text{TeV}}{M_{\text{KK}}}\right)^2 \left(\frac{2}{Y_*}\right) < 0.5,\ 1.6$ (Custodial)
- Penguin bound: $\left| a{Y_*}^2 + b \right| \left(\frac{3\ \text{TeV}}{M_{\text{KK}}}\right)^2 \leq 0.015$

What the heck is this?

Tree: Chang & Ng '05. **Loop NDA**: Agashe et al. '06

Operator analysis of $\mu \to e\gamma$

Match to 4D EFT, integrate over each z_i:

$$R'^2 \frac{e}{16\pi^2} \frac{v}{\sqrt{2}} f_{L_i} \left(a_{k\ell} Y_{ik} Y^\dagger_{k\ell} Y_{\ell j} + b_{ij} Y_{ij} \right) f_{-E_j} \bar{L}_i^{(0)} \sigma^{\mu\nu} E_j^{(0)} F_{\mu\nu}^{(0)}$$

- These may be Y_E or Y_N
- For $c_i = c$, Y_{ij} is a spurion of $U(3)^3$ lepton flavor
- Higher (odd) powers of Y_{ij} suppressed by $vR' \sim 0.1$
- Indices on a_{ij} and b_{ij} encode bulk mass dependence

Operator analysis of $\mu \to e\gamma$: alignment

Definition: anarchic matrix,

All entries $\mathcal{O}(1)$ with arbitrary phase. The product of anarchic matrices is also anarchic. Assumption: this is true in all preferred bases.

$$R'^2 \frac{e}{16\pi^2} \frac{v}{\sqrt{2}} f_{L_i} \left(a_{k\ell} Y_{ik} Y^\dagger_{k\ell} Y_{\ell j} + b_{ij} Y_{ij} \right) f_{-E_j} \bar{L}_i^{(0)} \sigma^{\mu\nu} E_j^{(0)} F_{\mu\nu}^{(0)}$$

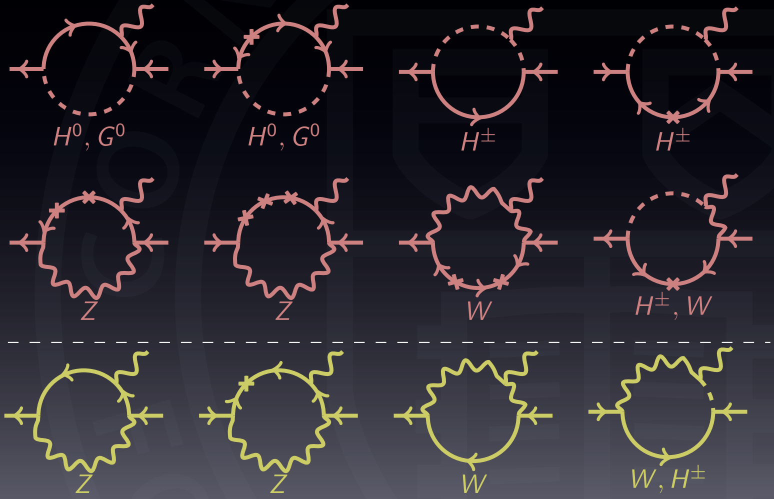
Compare to zero mode mass matrix: $m_{ij} = f_{L_i} Y^*_{ij} f_{-E_j} v$

- Up to the bulk mass non-universality, the b terms have the flavor structure of 4D mass terms
- **Alignment**: b_{ij} term almost diagonalized in the mass basis
- $\Rightarrow$ Structure behind anarchy. The empire strikes back!

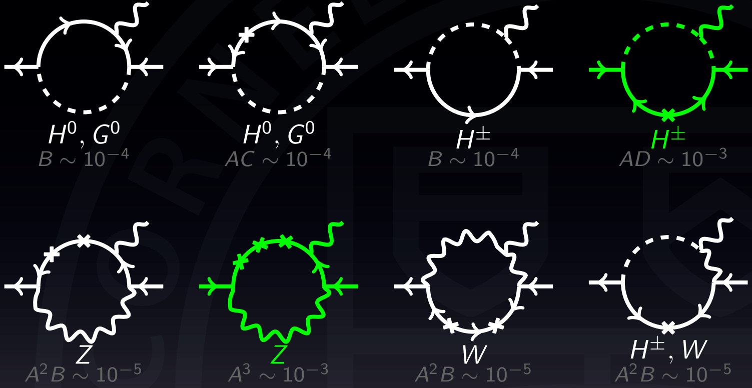
Alignment in RS: Agashe, Perez, Soni '04; Agashe, Azatov, Zhu '08.

A bunch of diagrams: *a* and *b* coefficients

$$R'^2 \frac{e}{16\pi^2} \frac{v}{\sqrt{2}} f_{L_i} \left(a_{k\ell} Y_{ik} Y^{\dagger}_{k\ell} Y_{\ell j} + b_{ij} Y_{ij} \right) f_{-E_j} \bar{L}_i^{(0)} \sigma^{\mu\nu} E_j^{(0)} F_{\mu\nu}^{(0)}$$



The structure of RS penguins: a coefficient



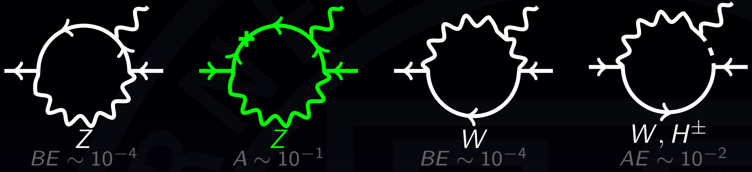
A. Mass insertion $\sim 10^{-1}$ per insertion (cross)

B. Equation of motion $\sim 10^{-4}$ (external arrows point same way)

C. Higgs/Goldstone cancellation $\sim 10^{-3}$ (H^0, G^0 diagram only)

D. Proportional to charged scalar mass $\sim 10^{-2}$

The structure of RS penguins: b coefficient



A. Mass insertion $\sim 10^{-1}$ per insertion (cross)
B. Equation of motion $\sim 10^{-4}$ (external arrows point same way)
E. No sum over internal flavors $\sim 10^{-1}$

Gauge boson diagrams are enhanced by

$$g_5^2/g^2 = \ln R'/R \sim \mathcal{O}(10)$$

This is a common factor for the b diagrams, and ends up being cancelled by a numerical factor of 1/10 in the 3MIZ a diagram.

Leading order diagrams

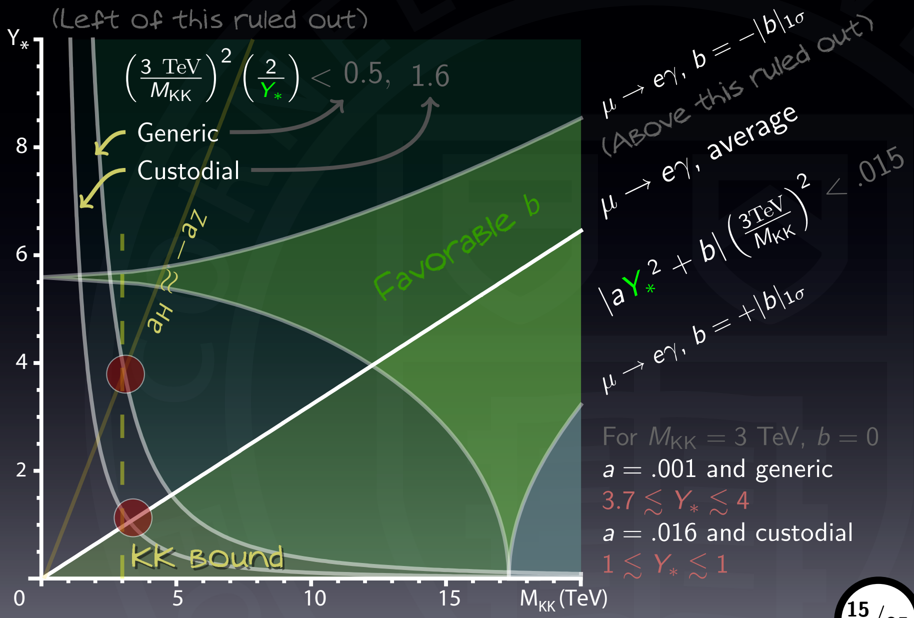
Three coefficients (a_H, a_Z, b) with arbitrary relative signs

Defined $aY_*^3 = \sum_{k,\ell} a_{k\ell} Y_{ik} Y_{k\ell}^\dagger Y_{\ell j}$ and $bY_* = \sum_{k,\ell} (U_L)_{ik} b_{k\ell} Y_{k\ell} (U_R^\dagger)_{\ell j}$

So, 'just calculate' these: (many details in paper)

- 5D position/momentum space: external zero modes
- Mass insertion approximation, but sum over all KK modes
- Gauge invariance: only identify $(p + p')^\mu$ coefficient

Representative Bounds



For $M_{KK} = 3$ TeV, $b = 0$

$a = .001$ and generic

$3.7 \lesssim Y_* \lesssim 4$

$a = .016$ and custodial

$1 \lesssim Y_* \lesssim 1$

Finiteness: naïve dimensional analysis

4D Naïve: $\int d^4k\, \Delta_F \gamma^\mu \Delta_F \Delta_B \sim \log(\Lambda)$

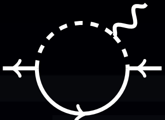
Really log divergent? No, finite. Here's why:

- Gauge invariance: $q_\mu \mathcal{M}^\mu = 0$.
- Lorentz invariance: $\int d^4k\, \frac{\not{k}}{k^{2n}} = 0$.

Indeed, $\mathcal{M}_{4D} \sim \Lambda^{-2}$.
Suspect that $\mathcal{M}_{5D} \sim \Lambda^{-1}$.

5D Bulk, i.e. $d^4k \to d^5k$

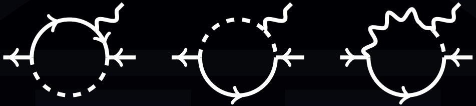
Finiteness: bulk 5D fields

	Neutral	Charged
Loop integral (d^4k)	$+4$	$+4$
Gauge invariance ($p + p'$)	-1	-1
Bulk boson propagator	-1	-2
Bulk vertices (dz)	-3	-3
Overall z-momentum	$+1$	$+1$
Derivative coupling	0	$+1$
Mass insertion/EOM	-1	-1
Total degree of divergence	-1	-1

Note: everything trivially carries over to the KK picture

Finiteness: brane-localized Higgs



	Neutral	Charged	W–$H^\pm$
Loop integral (d^4k)	$+4$	$+4$	$+4$
Gauge invariance ($p+p'$)	-1	-1	-1
Brane boson propagators	-2	-4	-2
Bulk boson propagator	0	0	-1
Bulk vertices (dz)	-1	0	-1
Derivative coupling	0	$+1$	0
Brane chiral cancellation	-1	0	0
Brane M_W^2 cancellation	0	-2	0
Total degree of divergence	-1	-2	-1

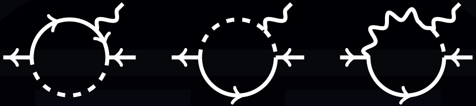
Finiteness: brane-localized Higgs

The $\mathbf{M_W^2}$ **cancellation** comes from the form of the photon coupling to the brane-localized $H^{\pm}$:

$$\frac{(2k-p-p')^\mu}{[(k-p')^2-M_W^2][(k-p)^2-M_W^2]} = \frac{(p+p')^\mu}{(k^2-M_W^2)^2}\left[\frac{k^2}{k^2-M_W^2}-1\right]$$
$$= \frac{M_W^2(p+p')^\mu}{(k^2-M_W^2)^3} \sim \mathcal{O}(1/k^6)$$

We have used the fact that the $(p+p')^\mu$ coefficient gives the complete gauge-invariant contribution.

Finiteness: brane-localized Higgs



	Neutral	Charged	W–$H^\pm$
Loop integral (d^4k)	$+4$	$+4$	$+4$
Gauge invariance ($p+p'$)	-1	-1	-1
Brane boson propagators	-2	-4	-2
Bulk boson propagator	0	0	-1
Bulk vertices (dz)	-1	0	-1
Photon Feynman rule	0	$+1$	0
Brane chiral cancellation	-1	0	0
Brane M_W^2 cancellation	0	-2	0
Total degree of divergence	-1	-2	-1

Finiteness: brane-localized Higgs

The **chiral cancellation** comes from the UV structure of the sum of the two diagrams:

Fermion propagator goes like $\Delta \sim \not{k} + k\gamma^5$, numerator structures are

$$\begin{aligned} \mathcal{M}_a &\sim \not{k}\gamma^\mu\not{k}\not{k} - k\gamma^\mu k\not{k} &= k^2(\not{k}\gamma^\mu - \gamma^\mu\not{k}) \\ \mathcal{M}_b &\sim \not{k}\not{k}\gamma^\mu\not{k} - \not{k}k\gamma^\mu k &= k^2(\gamma^\mu\not{k} - \not{k}\gamma^\mu) \end{aligned}$$

This is hard to see in the KK picture!

See Agashe et al. '06

Perturbativity and the 2-loop result

Yin-yang and double rainbow topologies. Insert a photon and odd number of mass insertions. Dotted line represents gauge or Higgs boson.

Purely bulk fields:

Loop integrals (d^4k)	$+8$
Gauge invariance ($p + p'$)	-1
Bulk boson propagators	-2
Bulk vertices (dz)	-5
Total degree of divergence	0

Log $\Lambda \Rightarrow$ large perturbative regime

Must do full calculation

Like 1-loop, hard to determine brane Higgs power counting. It may not be unreasonable to expect 1-loop cancellations to carry over to 2-loop.

The disappearing KK term

5D Lorentz invariance: must take the $M_n = nM_{\text{KK}}$ and $\Lambda = \lambda M_{\text{KK}}$ cutoffs together. Otherwise might lose leading term!

$$\mathcal{M}_{H^0} = \frac{gv}{16\pi^2} f_\mu f_{-e} \bar{u}_e (p+p')^\mu u_\mu \times \frac{1}{M^2}\left[c_0 + \mathcal{O}\left(\frac{v}{M}\right)^2\right]$$

$$c_0 = -\lambda^2 \sum_{n=1}^{N}\sum_{m=1}^{N} \frac{\lambda^2\left(n^2+m^2\right)+2n^2m^2}{4\left(n^2+\lambda^2\right)^2\left(m^2+\lambda^2\right)^2} \equiv -\frac{1}{\lambda^2}\sum_{n=1}^{N}\sum_{m=1}^{N}\hat{c}_0(n,m),$$

$$\hat{c}_0(n,n) \longrightarrow \left(\frac{n}{\lambda}\right)^2 \quad \text{for } n \ll \lambda$$

$$\hat{c}_0(n,n) \longrightarrow \left(\frac{n}{\lambda}\right)^0 \quad \text{for } n \approx \lambda$$

$$\hat{c}_0(n,n) \longrightarrow \left(\frac{\lambda}{n}\right)^4 \quad \text{for } n \gg \lambda.$$

Dominant contribution from $n \approx \lambda$. Taking $\lambda \to \infty$ for fixed n will lose this term! This is not a non-decoupling effect, just EFT.

Flight of the Warped Penguins

Future directions with local collaborators

1. Bulk Higgs models (integrals are much nastier)
2. $b \to s\gamma$ (operator mixing with $b \to sg$, quark c hierarchy)

H^0, G^0, $H^\pm$

No Goldstone cancellation!

Conclusion

Calculation of $\mu \longrightarrow e\gamma$ in a warped extra dimension:

- Mild non-tension for between loop- and tree-level bounds
- Three separate flavor structures $(Y_E Y_N^\dagger Y_N,\ Y_E Y_E^\dagger Y_E,\ Y_E)$
- Finite at one-loop, reasonable to expect perturbativity
- 5D calculation can make certain features more transparent

Thanks!

Gauge invariance

This is a **dipole operator** and the Ward identity forces the gauge invariant amplitude to take the form

$$\mathcal{M} = \epsilon_\mu \mathcal{M}^\mu \sim \epsilon_\mu \bar{u}_{p'} \left[(p+p')^\mu - (m_\mu + m_e)\gamma^\mu \right] u_p$$

Thus it is sufficient to calculate the coefficient of the $(p+p')^\mu$ term in $\mathcal{M}^\mu$ to determine the overall gauge invariant amplitude.

Diagrams which are not 1PI, such as external photon emissions, are gauge redundant to the 1PI diagrams.

Lavoura '03

The standard $\mu \to e\gamma$ EFT

Traditional parameterization for the $\mu \to e\gamma$ amplitude

$$\frac{-iC_{L,R}}{2m_\mu}\bar{u}_{L,R}\,\sigma^{\mu\nu}\,u_{R,L}F_{\mu\nu},$$

For the case of RS,

$$C_{L,R} = \left(aY_*^3 + bY_*\right)R'^2\frac{e}{16\pi^2}\frac{v}{\sqrt{2}}2m_\mu f_{L_{2,1}}f_{-E_{1,2}}$$

$$\mathrm{Br}(\mu \to e\gamma) = \frac{12\pi^2}{(G_F m_\mu^2)^2}(|C_L|^2 + |C_R|^2) < 1.2\cdot 10^{-11}.$$

Trick: $C_L^2 + C_R^2 \geq 2C_LC_R$

$$\mathrm{Br}(\mu \to e\gamma) \geq 6\left|aY_*^2 + b\right|^2\frac{\alpha}{4\pi}\left(\frac{R'^2}{G_F}\right)^2\frac{m_e}{m_\mu}$$

Operator subtleties

EFT: match amplitude to Wilson coefficient.

Important caveat in higher dimensions

5D amplitudes with 4D external states are non-local.

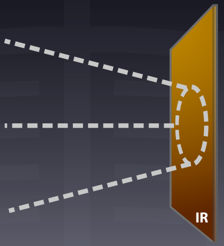
$$\mathcal{M}_{5D} = C(z_H, z_L, z_E, z_A)\, H(z_H) \cdot \bar{L}_i(z_L)\sigma^{MN} E_j(z_E) F_{MN}(z_A)$$

Must integrate over *each* z_i independently.

Pathological e.g.: $H(z) \sim \delta(z - R')$. What happens to operators like $|H|^2$?

Another e.g.: Cannot write a 'naïve' local effective operator for bulk fields coupled through a heavy brane-localized field.

$$\mathcal{O}_{UV} \sim \Phi^3(z)\delta(z - R') \qquad \mathcal{O}_{EFT} \sim \Phi^3(z).$$



UV theory: brane-localized operator.
IR theory: bulk fields ⇒ bulk operator.

Mixed 5D position/momentum space

Mixed position/momentum space: (p^μ, z)

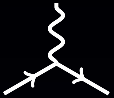
Due to the explicit z-dependence of the geometry and the localization of the Higgs, it is natural to work in mixed space.

$$\int đ^d k \frac{i}{k^2} e^{-ik\cdot(x-x')} \Rightarrow \int đ k_z \frac{i}{k^2 - k_z^2} e^{ik_z(z-z')}$$

- Usual momentum space in Minkowski directions
- Propagator dimension: $[\Delta_{\text{5D}}] = [\Delta_{\text{4D}}] + 1$
- Each vertex: perform dz overlap integral $\sim 1/k$
- External states carry zero-mode z-profile

5D Feynman rules

See our paper for lots of appendices on performing 5D calculations.



$$= ig_5 \left(\frac{R}{z}\right)^4 \gamma^\mu$$

$$= ie_5 (p_+ - p_-)_\mu$$

$$= \frac{i}{2} e_5 g_5 \, v \, \eta^{\mu\nu}$$

$$= i \left(\frac{R}{R'}\right)^3 Y_5$$

$$= \Delta_k(z, z')$$

$$= -i\eta^{\mu\nu} G_k(z, z')$$

$$= \epsilon^\mu(q) f_A^{(0)}$$

$$= \frac{f_c}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{-c} u(p)$$

$$g_5^2 = g_{SM}^2 R \ln R'/R \qquad e_5 f_A^{(0)} = e_{SM} \qquad Y_5 = RY$$

Analytic expressions

$$\mathcal{M}(1\mathrm{MI}H^{\pm}) = \frac{i}{16\pi^2}(R')^2 f_{c_L} Y_E Y_N^\dagger Y_N f_{-c_E} \frac{ev}{\sqrt{2}} \cdot 2I_{1\mathrm{MI}H^{\pm}}$$

$$\mathcal{M}(3\mathrm{MI}Z) = \frac{i}{16\pi^2}(R')^2 f_{c_L} Y_E Y_E^\dagger Y_E f_{-c_E} \frac{ev}{\sqrt{2}} \left(g^2 \ln\frac{R'}{R}\right)\left(\frac{R'v}{\sqrt{2}}\right)^2 \cdot I_{3\mathrm{MI}Z}$$

$$\mathcal{M}(1\mathrm{MI}Z) = \frac{i}{16\pi^2}(R')^2 f_{c_L} Y_E f_{-c_E} \frac{ev}{\sqrt{2}} \left(g^2 \ln\frac{R'}{R}\right) \cdot I_{1\mathrm{MI}Z}.$$

Written in terms of dimensionless integrals. See paper for explicit formulae.

Finiteness in the KK picture

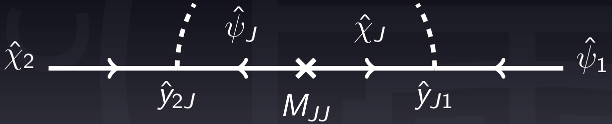
Power counting for the brane-localized Higgs

Charged Higgs: same M_W^2 cancellation argument as 5D
Neutral Higgs: much more subtle!

A basis of chiral KK fermions:

$$\chi = \left(\chi_{L_i}^{(0)}, \chi_{R_i}^{(1)}, \chi_{L_i}^{(1)}\right) \qquad \psi = \left(\psi_{R_i}^{(0)}, \psi_{R_i}^{(1)}, \psi_{L_i}^{(1)}\right)$$

Worry about the following type of diagram:



The (KK) mass term in the propagator can be $\sim \Lambda$.
Have to show that the mixing with large KK numbers is small.

Finiteness in the KK picture

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A basis of chiral KK fermions:

$$\psi = \left(\psi_{R_i}^{(0)}, \psi_{R_i}^{(1)}, \psi_{L_i}^{(1)}\right) \qquad \chi = \left(\chi_{L_i}^{(0)}, \chi_{R_i}^{(1)}, \chi_{L_i}^{(1)}\right)$$

Mass and Yukawa matrices (gauge basis, $\psi M \chi + \mathrm{h.c.}$):

$$M = \begin{pmatrix} m^{11} & 0 & m^{13} \\ m^{21} & M_{\mathrm{KK},1} & m^{23} \\ 0 & 0 & M_{\mathrm{KK},2} \end{pmatrix} \qquad y \sim \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

The zeroes are fixed by gauge invariance.

$$\hat{y}_{1J}\hat{y}_{J2} = 0$$

Indices run from $1, \ldots, 9$ labeling flavor and KK number

Finiteness in the KK picture

Power counting for the brane-localized Higgs

$$\psi = \left(\psi_{R_i}^{(0)}, \psi_{R_i}^{(1)}, \psi_{L_i}^{(1)}\right) \qquad M = \begin{pmatrix} m^{11} & 0 & m^{13} \\ m^{21} & M_{\text{KK},1} & m^{23} \\ 0 & 0 & M_{\text{KK},2} \end{pmatrix}$$

$$\chi = \left(\chi_{L_i}^{(0)}, \chi_{R_i}^{(1)}, \chi_{L_i}^{(1)}\right)$$

Rotating to the mass basis, $\epsilon \sim v/M_{\text{KK}}$:

$$\hat{y} \sim \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & \epsilon & 1 \\ 1 & & \\ \epsilon & & \end{pmatrix}$$

Now we have $y_{1J} y_{J2} \sim \epsilon$, good!

Finiteness in the KK picture

Power counting for the brane-localized Higgs

$$\psi = \left(\psi^{(0)}_{R_i}, \psi^{(1)}_{R_i}, \psi^{(1)}_{L_i}\right) \qquad M = \begin{pmatrix} m^{11} & 0 & m^{13} \\ m^{21} & M_{KK,1} & m^{23} \\ 0 & 0 & M_{KK,2} \end{pmatrix}$$

$$\chi = \left(\chi^{(0)}_{L_i}, \chi^{(1)}_{R_i}, \chi^{(1)}_{L_i}\right)$$

Rotating to the mass basis, $\epsilon \sim v/M_{KK}$:

$$\hat{y} \sim \begin{pmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & \epsilon & 1 \\ 1 & & \\ \epsilon & & \end{pmatrix} \to \begin{pmatrix} 1 & 1+\epsilon & -1+\epsilon \\ 1+\epsilon & & \\ 1-\epsilon & & \end{pmatrix}$$

Must include 'large' rotation of m^{21} and m^{13} blocks representing mixing of chiral zero modes into light Dirac SM fermions. This mixes wrong-chirality states and does not affect the mixing with same-chirality KK modes.

Indeed, $\mathcal{O}(1)$ factors cancel: $y_{1J}y_{J2} \sim \epsilon$, good!

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