Quantum Phase Transitions in Holographic Models of Strongly Correlated Systems

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Talk based on:
1006.2387, 1008.1581

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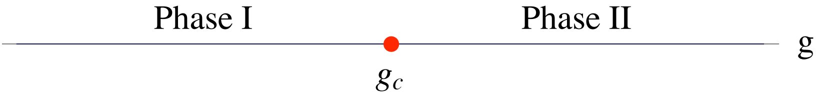
Quantum Phase Transition is

A (continuous) transition between different phases at $T = 0$, as a function of an external parameter (B, Pressure,$x, \ldots$)



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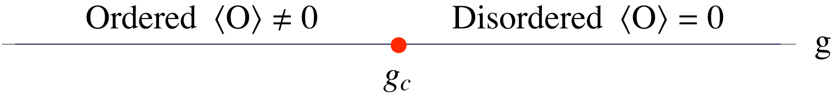


Conventional approach: apply usual ($T \neq 0$) Landau Ginzburg Wilsonian (LGW) symmetry breaking paradigm:

- Phases characterized by different symmetry breaking patterns.
- Fluctuations described by *order parameter* $\mathcal{O}(x)$
- Close to critical point, correlation length diverges

$$\xi \sim (g - g_c)^{-\nu} \qquad \langle 0|\mathcal{O}(x)\mathcal{O}(0)|0\rangle \sim e^{-x/\xi}$$

- Lattice effects wash away leaving a continuum field theory
- At $g = g_c$ one finds a *scale invariant* theory (sometimes a CFT) with an operator $\mathcal{O}$, and one *relevant* perturbation corresponding to $(g - g_c)$

Quantum Phase Transition (QPT)

New phenomena compared to $T \neq 0$ (even within LGW paradigm)

- Dynamical critical exponent of CFT

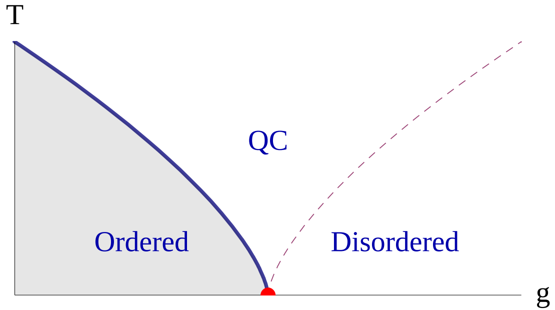
$$\vec{x} \to \lambda \vec{x} \qquad t \to \lambda^z t$$

$$\xi \sim (g - g_c)^{-\nu} \qquad E_g \sim (g - g_c)^{\nu z}$$

Quantum Phase Transition (QPT)

New phenomena compared to $T \neq 0$ (even within LGW paradigm)

- Finite temperature crossovers:



- QPT becomes nonzero T phase transition
- Scale invariance implies $T_c \sim (g - g_c)^{\nu z}$.

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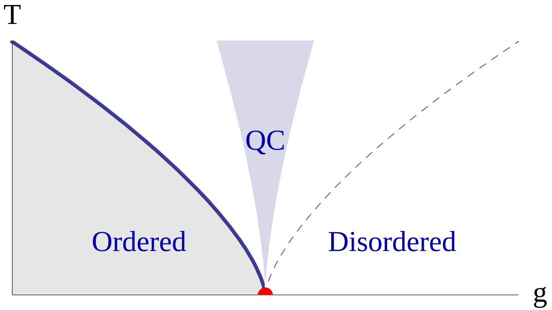

- QC region controlled by CFT in thermal ensemble:
- Two competing energy scales T and $(g - g_c)^{\nu z}$.
- For $T \gg (g - g_c)^{\nu z}$, ignore relevant perturbation, set $g \to g_c$.

Quantum Phase Transition (QPT)

New phenomena compared to $T \neq 0$ (even within LGW paradigm)

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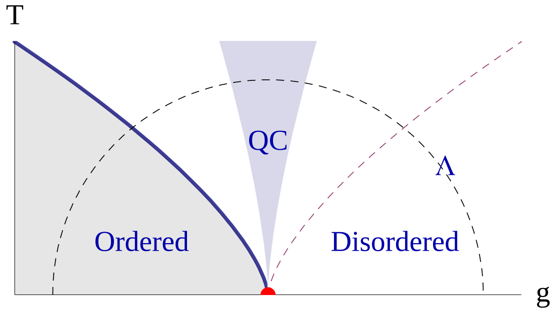
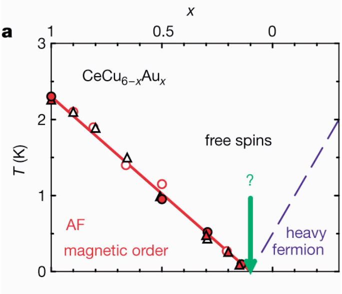


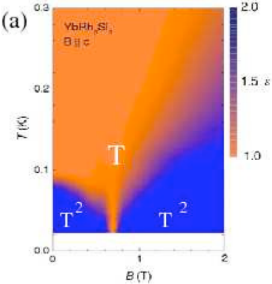
- CFT description only valid up to some cutoff Λ (lattice scale).

Quantum Phase Transitions - Heavy Fermion Criticality

For example:



Schroder et al



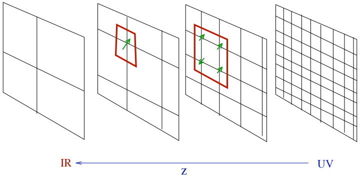
Custers et al.

- Heavy Fermion: mass of electron strongly renormalized. But still conventional Fermi Liquid ($\rho \sim T^2$)
- Ordered phase: Antiferromagnetic (AF) order
- Non fermi liquid occurs in vicinity of QC ($\rho \sim T$)
- LGW fails to describe QCP

Quantum Phase Transitions - AdS/CFT?

Natural to try to use AdS/CFT to describe **QC region**. Sachdev, Muller, Kovtun, Hartnoll, Son, . . .

- Holography (AdS/CFT): Some field theories in d-dimensions dual to a *gravitational* theory in $d + 1$-dimensions
- At the heart of Holographic duality: extra dimension is RG scale (z)



- Challenge: define *local* theory on higher dimensional space
- No general understanding although see: Lee ; Douglas, Mazzucato, Razamat
- Indirect string theory arguments: explicit realizations.

Quantum Phase Transitions - AdS/CFT?

Refined statement:

- Gravity in AdS_{d+1} gives a description of a set of *strongly interacting* CFT_ds
- Use these strongly interacting CFTs as calculable *toy models* of Quantum Criticality
- QC region $\leftrightarrow$ AdS black hole

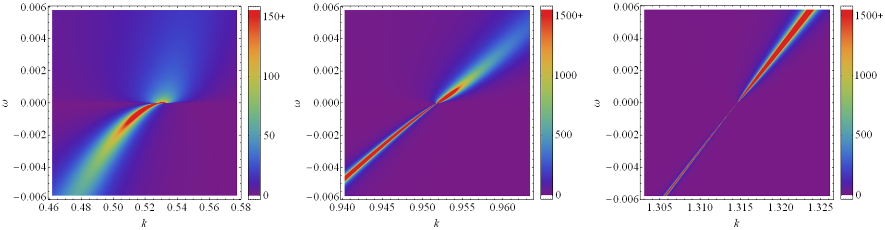
AdS Black Hole has built into it (at leading order):

- Thermodynamics, Dissipation, Hydrodynamics, Response functions
- All of this *without* using quasi-particle description
- Many features *universal* to this set of CFTs
- Extract general lessons/organizing principle?
- Famous example: $(\eta/s)_{BH} = 1/(4\pi)$ compare to $(\eta/s)_{qp} = 1/g^4$

Quantum Phase Transitions - AdS/CFT?

- Non-Fermi Liquid (NFL) **phase** found dual to a charged AdS black hole Lee; Cubrovic, Schalm, Zaanen; TF, Iqbal, Liu, McGreevy, Vegh
  - Fermionic greens functions have distinctly NFL scalings close to Fermi Surface

$$\mathrm{Im}\, G(\omega, k) =$$



$0 < \nu < 1/2$ $\qquad$ $\nu = 1/2$ $\qquad$ $\nu > 1/2$

- NFL a striking feature of heavy fermion QC region - these gravitational toy models may have some relevance here?

Some goals/Outline

Part 0: Simple field theory example of a QPT: Gross-Neveu model

Part 1: extend AdS/CFT program outside of QC region. Especially to QPT.

- general setup for symmetry breaking in *AdS*
- identify useful relevant deformations: *double trace deformation*
- compute critical exponents/ finite-T crossovers

Part 2: extend QPT to non-zero charge density

- relate to previously found NFL behavior?
- bonus: find non-trivial scaling for the order parameter two point function: same form as in heavy fermion criticality!!

Simple field theory example: Gross-Neveu model

- Large-N vector model -N Dirac fermions - d dimensions
- Typical example of a QPT (analogous to our AdS/CFT results)

$$\mathcal{L} = N\left(i\bar{\psi}^i \partial\!\!\!/ \psi_i + \frac{1}{2} g \left(\bar{\psi}^i \psi_i\right)^2\right)$$

- Discrete Z_2 symmetry: $\bar{\psi}^i \psi_i \to -\bar{\psi}^i \psi_i$ (parity in $d = 3$)
- Dimensional analysis (free fixed point):

$$[\psi] = (d-1)/2 \qquad \left[\bar{\psi}\psi\right] = (d-1) \qquad [g] = (2-d)$$

- For $d > 2$ then g is *irrelevant*. Free fermions: **IR fixed point**.
- For $g > g_c$ symmetry breaking occurs. New **UV fixed point** at $g = g_c$.

Simple field theory example: Gross-Neveu model

- To analyze the critical point use Hubbard Stratonovich decoupling

$$\mathcal{L} = N\left(i\bar{\psi}^i \partial\!\!\!/ \psi_i - \alpha\bar{\psi}^i\psi_i - \frac{\alpha^2}{2g}\right)$$

- Integrate out fermions - Effective potential

$$\frac{1}{N}V_{\rm eff}(\alpha) = \frac{\alpha^2}{2g} + \int \frac{d^d p}{(2\pi)^d}\ln(p^2+\alpha^2)$$

- Vacuum: $V'_{\rm eff} = 0 \quad \rightarrow \quad \langle\alpha\rangle = -g\left\langle\bar{\psi}\psi\right\rangle$
- Dimensional regularization:

$$\frac{1}{N}V_{\rm eff}(\alpha) = \frac{1}{2g}|\alpha|^2 + s_d|\alpha|^d \qquad s_d = \frac{\Gamma(-d/2)}{(4\pi)^{d/2}}$$

Simple field theory example: Gross-Neveu model

$$\frac{1}{N}V_{\text{eff}}(\alpha) = \frac{1}{2g}|\alpha|^2 + s_d|\alpha|^d$$

- UV fixed point at $g_c = \infty$
- redefine $\kappa = 1/g$ so $\kappa_c = 0$

Simple field theory example: Gross-Neveu model

$$\frac{1}{N} V_{\text{eff}}(\alpha) = \frac{\kappa}{2}|\alpha|^2 + s_d|\alpha|^d$$

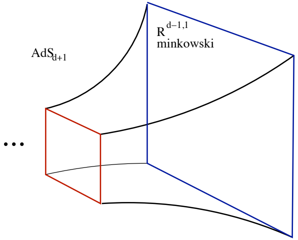
- UV fixed point at $g_c = \infty$
- redefine $\kappa = 1/g$ so $\kappa_c = 0$
- $\exists$ symmetry breaking solutions $V'_{\text{eff}}(\alpha) = 0$ for $\kappa < 0$

$$\langle \bar{\psi}\psi \rangle \sim \alpha \sim (-\kappa)^{1/(d-2)}$$

- Note $[\kappa] = -[g] = (d-2)$ is our *relevant* pertrubation
- $\kappa = 0 \leftrightarrow$ UV fixed point $\leftrightarrow$ QPT $\leftrightarrow$ QCP
- Finite temperature: $T_c \sim (-\kappa)^{1/(d-2)}$

Some aspects of AdS/CFT

Maldacena



$$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$$

$(d-1)+1-$dimensional CFT	$\leftrightarrow$	string theory on AdS_{d+1}
strong coupling, large N	$\leftrightarrow$	classical gravity on AdS_{d+1}
scale invariance	$\leftrightarrow$	$t \to \lambda t$, $\vec{x} \to \lambda \vec{x}$, $z \to z\lambda$
RG scale	$\leftrightarrow$	radial coordinate z
IR (UV)	$\leftrightarrow$	$z \to \infty$ $(z \to 0)$
Field theory space time	$\leftrightarrow$	boundary of AdS $(z = 0)$
Operators	$\leftrightarrow$	Bulk fields

Some aspects of AdS/CFT

Comments:

- Work in $d = 3$ ($3 + 1$ in the *bulk* / $2 + 1$ on the *boundary*)
- Different bulk gravity theories (matter content, bulk couplings etc.) correspond to different CFTs
- CFT deformations correspond to changing boundary conditions at the *AdS* boundary
- Many dual pairs known, typically gauge theories with N colors
  - **Bulk fields $\leftrightarrow$ single trace operators**

$$\mathcal{O} \sim \frac{1}{N} \mathrm{Tr}\,(M \ldots) \qquad M : \text{adjoint of } SU(N)$$

  - Large-N limit: $N^2 \propto 1/G_N \to \infty$
- *We will take phenomenological approach: try to describe large number of CFT's by sticking to general ingredients.*
- Ultimately need to derive these ingredients from string theory

Symmetry breaking in AdS/CFT

A wishlist:

Non perturbative description of an interacting CFT_{2+1}	
Global symmetry G Current: J_μ^a	
Order parameter: $\mathcal{O}_i$	
Relevant deformation: $(g - g_c)$	

Symmetry breaking in AdS/CFT

A wishlist:

Non perturbative description of an interacting CFT_{2+1}	Gravitational theory on AdS_4
Global symmetry G Current: J^a_μ	Guage symmetry G Bulk gauge field: A^a_μ
Order parameter: $\mathcal{O}_i$	Charged field: ϕ_i
Relevant deformation: $(g - g_c)$	??

$$S_{\rm grav} = \frac{1}{G_N}\int d^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F^2 - |D\phi|^2 - V(|\phi|^2)\right)$$

$$D\phi = \partial\phi + iqA\phi \qquad V(|\phi|^2) = m^2|\phi|^2 + \ldots$$

AdS_4 solution is stable for $m^2 > -(3/2)^2$ (BF bound.) So no symmetry breaking yet. Look for a *relevant deformation* which does not explicitly break G.

Possible relevant deformations

$$H \longrightarrow H + \int d^2x \left(\mu J^t + \kappa \mathcal{O}^\dagger \mathcal{O}\right)$$

Note:

- Chemical potential μ, only allowed for $G = U(1)$
- Double trace coupling κ, only relevant for $\Delta(\mathcal{O}) < 3/2$.
- Generally must consider both unless symmetry protects it (say $\mu = 0$ by charge conjugation symmetry or relativistic invariance.)

Double trace deformations

$$H \to H + \int d^2x \kappa \mathcal{O}^\dagger \mathcal{O}$$

Boundary conditions $\leftrightarrow$ CFT deformations

- AdS_4 has a boundary at $z = 0$
- For well defined theory, impose boundary conditions here
- Equation of motion for ϕ

$$\nabla^2\phi - m^2\phi = 0$$

- Solve for small z

$$\phi(r) = \alpha z^{\Delta_-}(1 + \ldots) + \beta z^{\Delta_+}(1 + \ldots)$$

$$\Delta_\pm = 3/2 \pm \sqrt{(3/2)^2 + m^2}$$

- **Take** $m^2 = -2$ **with** $\Delta_+ = 2$ **and** $\Delta_- = 1$ **for concreteness**

Single trace deformation

$$\phi(z) = \alpha z^1(1+\ldots) + \beta z^2(1+\ldots)$$

- Usually fix leading term α as $z \to 0$. Let β fluctuate.
- Defining identity of AdS/CFT:

$$Z_{\rm grav}[\alpha] = Z_{\rm CFT}[\alpha]$$

$$Z_{\rm grav}[\alpha] = \int_{\rm bulk} \mathcal{D}\phi(r,\vec{x})_\alpha e^{-S_{\rm grav}} \qquad Z_{\rm CFT}[\alpha] = \left\langle e^{-\int d^3x \alpha \mathcal{O}} \right\rangle$$

- Classical gravity $G_N \to 0$

$$Z_{\rm grav}[\alpha] = \left(e^{-S_{\rm grav}}\right)_{\phi_{{\rm soln},\alpha}} \equiv e^{W(\alpha)}$$

- Can show that $W'(\alpha) = -\beta$, such that $\langle \mathcal{O} \rangle_\alpha = \beta$
- Dimensions: $[\beta] = 2 \quad \to \quad [\mathcal{O}] = 2$

Double trace deformations

- Previous discussion only works for *single* trace deformation
- Follow the Gross-Neveu example:

$$S_{\mathrm{CFT}} \to S_{\mathrm{CFT}} + \int d^3x\, g\mathcal{O}^\dagger\mathcal{O} = S_{CFT} - \int d^3x \left(\alpha\mathcal{O} + \frac{|\alpha|^2}{2g}\right)$$

- Now can compute using AdS/CFT: ($g = 1/\kappa$)

$$V_{\mathrm{eff}}(\alpha) = \frac{\kappa}{2}|\alpha|^2 + W(\alpha)$$

- Again vacuum: $V'_{\mathrm{eff}}(\alpha) = 0$. Using: $\beta = -W'(\alpha)$ we find:

$$\beta = \kappa\alpha$$

- Well known fact in AdS/CFT: linear boundary conditions $\equiv$ double trace deformations Witten; Aharony, Berkooz, Silverstein
- Note: $\kappa = 0$ corresponds to fixing β ($= 0$)

To summarize:

Again like in the GN model there are two fixed points:

IR Fixed Point (CFT^{+}), $g = 0$ ($\kappa = \infty$) , α fixed

- Characterized by an operator $\mathcal{O}$ with scaling dimension 2 and an *irrelevant* double trace interaction (dimension 4.)

UV Fixed Point (CFT^{-}), $\kappa = 0$ ($g = \infty$), β fixed

- Characterized by an operator $\mathcal{O}$ with scaling dimension 1 and a *relevant* double trace interaction (dimension 2.)

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Again like in the GN model there are two fixed points:

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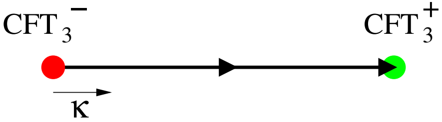
UV Fixed Point (CFT^{-}), $\kappa = 0$ ($g = \infty$), β fixed

- Characterized by an operator $\mathcal{O}$ with scaling dimension 1 and a *relevant* double trace interaction (dimension 2.)

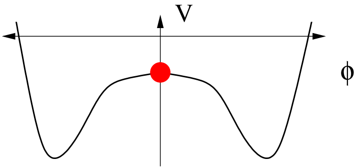
Turning on κ in the **UV** CFT^{-}, one flows to the **IR** CFT^{+}



Claim: CFT_{-} should be associated with a QPT and κ the relevant direction. How do we see the ordered phase when $\kappa < 0$?

Symmetry breaking TF, Horowitz, Roberts

Look for state where $\phi(z) \neq 0$. Consider *bulk* potential:



Solve Einstein's equations with scalar ϕ matter.

- $\phi = 0$ is stable if $m^2 > m^2_{BF} = -(3/2)^2$ *and* $\kappa > 0$.

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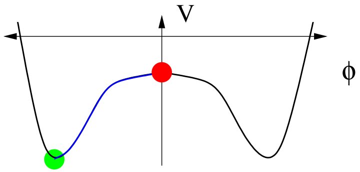

Solve Einstein's equations with scalar ϕ matter.

- $\phi = 0$ is stable if $m^2 > m^2_{BF} = -(3/2)^2$ *and* $\kappa > 0$.
- if $\kappa < 0$: domain wall solution: (only non singular solution)

$$ds^2 = (-dt^2 + d\vec{x}^2 + dz^2)/f(z)$$

$$(z \to 0) \quad f(z) = z^2\,, \;\; \phi(z) = 0 \qquad AdS_4$$

$$(z \to \infty) \quad f(z) = \# z^2\,, \;\; \phi(z) = \phi_0 \qquad \widetilde{AdS}_4$$

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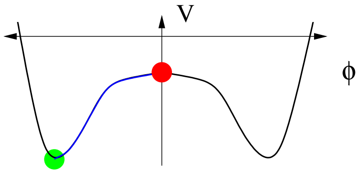

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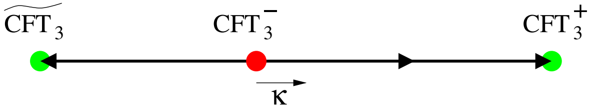
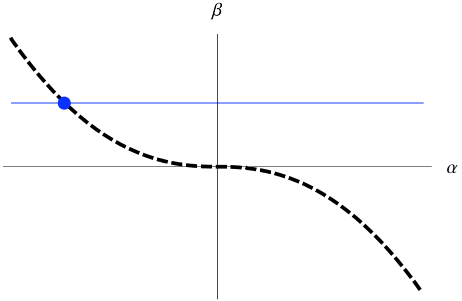


Domain wall flow

- Characterize the flow by ($z \to 0$):

$$\phi \to \alpha z^1 + \beta z^2 + \ldots$$

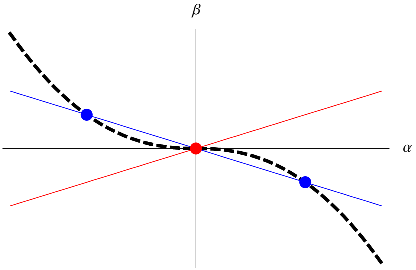
  where the DW solution determines a relationship between the two: $\beta = \beta_{DW}(\alpha)$.
- Previous studies of DW solution looked at *single* trace deformations where one looks for solutions with $\beta = J$ fixed. Explicitly breaks the symmetry.



Domain wall flow

- Reinterpret in terms of double trace coupling
- For a given double trace κ, to find the vev $\langle \mathcal{O} \rangle \sim \alpha$ look for solution of:

$$\beta_{DW}(\alpha) = \kappa\alpha$$

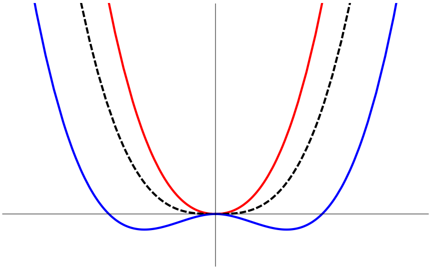


- only solution for $\kappa < 0$. *Consistent with stability analysis.*
- Scale invariance: $\beta_{DW} = -s_c \alpha^2 \mathrm{sign}(\alpha) \to \alpha \sim \pm(-\kappa/s_c)$.

Effective potential

$$W_{DW} = -\int_0^{\alpha} \beta_{DW}(\alpha')d\alpha' \qquad V_{\text{eff}}(\alpha) = W_{DW}(\alpha) + (1/2)\kappa\alpha^2$$

$$W_{DW} = \frac{2s_c}{3}|\alpha|^3$$



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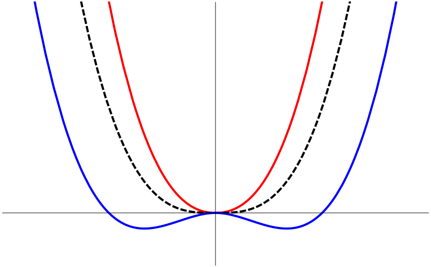
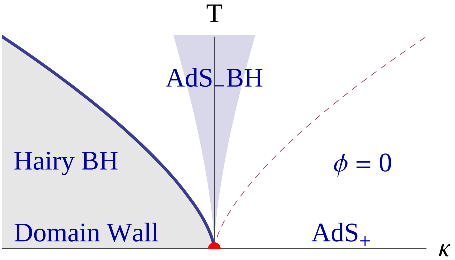


Finite temperature

- Heat up DW solution and restore symmetry for $T > T_c$.
- Ordered state: "hairy black hole"
- Disordered state: AdS-BH with funny boundary conditions
- Phase boundary: look for linearized instability of ϕ fluctuating on the AdS-BH as a function of T, κ
- Scale invariance: $T_c \propto (-\kappa)$



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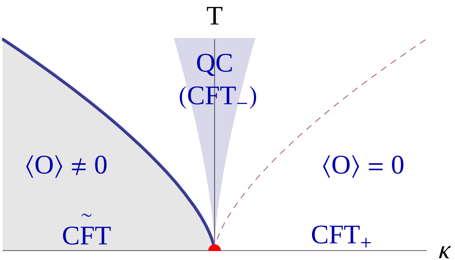
- Phase boundary: look for linearized instability of ϕ fluctuating on the AdS-BH as a function of T, κ
- Scale invariance:
  $T_c \propto (-\kappa)^{1/(\Delta_+ - \Delta_-)} \quad \langle \mathcal{O} \rangle \propto (-\kappa)^{\Delta_-/(\Delta_+ - \Delta_-)}$



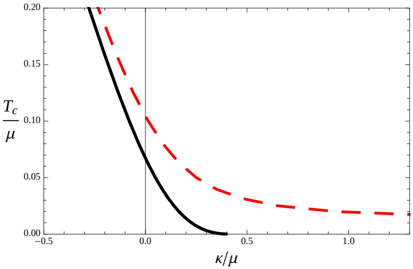
Part 2. Both deformations

$$H \to H + \int d^2x \left(\mu J^t + \kappa \mathcal{O}^\dagger \mathcal{O} \right)$$

New phase diagram

Disordered phase $\phi = 0$ corresponds to the **Charged Black Hole** (Reissner Nordstrom solution)

- Phase boundary: again look for linearized instability of ϕ fluctuations to determine T_c
- Depending on bulk parameters $(q, m^2, \ldots)$, two behaviors:



- QPT shifted to κ_c. Actually QCP dramatically different:

Dissecting new QCP

To study QCP close to $\kappa \approx \kappa_c$ examine physics at small energies ω, momenta $\vec{p}$ and temperature T compared to μ:

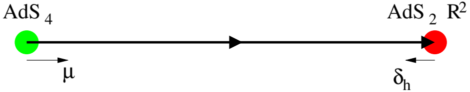
- At $T = 0$, charged black hole close to the horizon $z \to z_\star$ develops an $AdS_2 \times R^2$ throat:

$$ds^2 = \frac{-dt^2 + d\zeta^2}{6\zeta^2} + d\vec{x}^2 \qquad \zeta = 1/(z - z_\star)$$

- From AdS/CFT lore, IR is controlled by AdS_2.
- AdS_2 is dual to a (mysterious) $0+1$ dimensional CFT_1.
- Emergent conformal symmetry:

$$t \to t/\lambda \quad \vec{x} \to \vec{x} \quad \text{and} \quad \zeta \to \zeta/\lambda$$

- μ induces a flow from CFT_{2+1} to CFT_1.



- *Claim: new QCP controlled by CFT_1.*

Dissecting new QCP - Two point function

- To see this examine two point function of $\mathcal{O}$ at low energies

$$G_\kappa(\omega, \vec{p}) \approx \frac{Z}{(\kappa - \kappa_c) - \Sigma(\omega, T)}$$

- where Σ is a scaling function:

$$\Sigma(\omega, T) = T^{2\nu} g(\omega/T) \qquad \Sigma(T = 0) = \#\omega^{2\nu}$$

- these are scale invariant in terms of the CFT_1 scaling:

$$\omega \to \lambda\omega\,, \quad T \to \lambda T\,, \quad \vec{k} \to \vec{k}$$

$$\mathcal{O} \to \lambda^{1/2-\nu}\mathcal{O} \qquad (\kappa - \kappa_c) \to \lambda^{2\nu}(\kappa - \kappa_c)$$

- Suggests under RG:

$$\mathcal{O} \to \mathcal{O}_{0+1} \quad \kappa \to \kappa_{0+1} = (\kappa - \kappa_c)$$

Dissecting new QCP - Two point function

- To see this examine two point function of $\mathcal{O}$ at low energies

$$G_\kappa(\omega, \vec{p}) \approx \frac{Z}{(\kappa - \kappa_c) - \Sigma(\omega, T) + c_p \vec{p}^2 + c_\omega \omega^2 + c_T T}$$

- where Σ is a scaling function:

$$\Sigma(\omega, T) = T^{2\nu} g(\omega/T) \qquad \Sigma(T = 0) = \#\omega^{2\nu}$$

- these are scale invariant in terms of the CFT_1 scaling:

$$\omega \to \lambda\omega \,, \quad T \to \lambda T \,, \quad \vec{k} \to \vec{k}$$

$$\mathcal{O} \to \lambda^{1/2-\nu}\mathcal{O} \qquad (\kappa - \kappa_c) \to \lambda^{2\nu}(\kappa - \kappa_c)$$

- Suggests under RG:

$$\mathcal{O} \to \mathcal{O}_{0+1} \quad \kappa \to \kappa_{0+1} = (\kappa - \kappa_c)$$

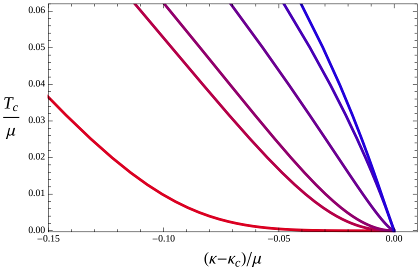
Phase boundary - analytic results

- use G_κ to search for unstable mode (pole in UHP)
- find T_c:

$$(\nu < 1/2) \quad T_c \sim (-\kappa + \kappa_c)^{1/2\nu} \qquad \text{(universal)}$$

$$(\nu > 1/2) \quad T_c \sim (-\kappa + \kappa_c) \qquad \text{(universality spoiled)}$$

- Confirmed numerically:



Dynamical critical exponent

$$G(\omega, \vec{p}) \approx \frac{Z}{(\kappa - \kappa_c) - \#\omega^{2\nu} + c_p \vec{p}^2 + c_\omega \omega^2} \quad (T = 0)$$

- At the critical point $\kappa = \kappa_c$ there is a gapless mode
- Disperses as:

$$\omega \sim |\vec{p}|^z \qquad z = \max\left(1, \frac{1}{\nu}\right)$$

- Away from criticality $\kappa > \kappa_c$ there is a "mass gap":

$$E_g \sim (\kappa - \kappa_c)^{1/2\nu}$$

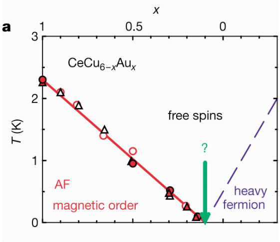
- Correlation length $\xi \sim (\kappa - \kappa_c)^{-1/2} \sim E_g^{-1/z}$

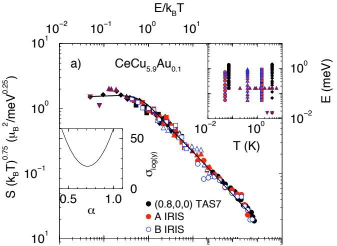
Locally quantum critical

For $\nu < 1/2$ find the 2 point function at the critical point:

$$G(\omega, \vec{p}) \approx \frac{Z}{c_p \vec{p}^2 - T^{2\nu} g(\omega/T)}$$

- Analytic in $\vec{p}^2$, self energy independent of $\vec{p}$. Locally critical.
- Same form as measured for the two point function of the SDW order parameter in Heavy Fermion criticality with $\nu \approx 0.37$ $(z \approx 2.7)$ Schroder et al



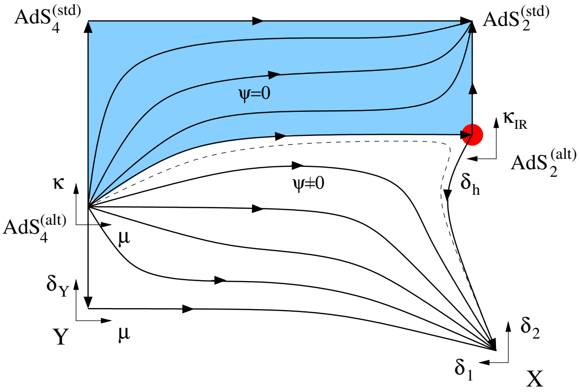


- Similar result found in the Kondo lattice model using $d \to \infty$, Si, Rabello, Ingersent, Smith

Comments and conclusions

- The $AdS_2 \times R^2$ phase which controls the QC region has been previously shown to be associated with Non-Fermi Liquid (NFL) behavior TF, Liu, McGreevy, Vegh
- 2 point function of Fermionic operators in this phase shows gapless fermi surface type excitations with NFL like dispersion
- NFL behavior is generally associated with Heavy Fermion criticality.
- We found a QPT out of this phase which displays other similarities with a certain Heavy Fermion system
- Promising signs! However: where is the Heavy Fermion phase in our system??
- Also what is the connection between AdS_2 and DMFT?

RG Interpretation ($0 < \delta_- < 1/2$)



- matching along dashed line:

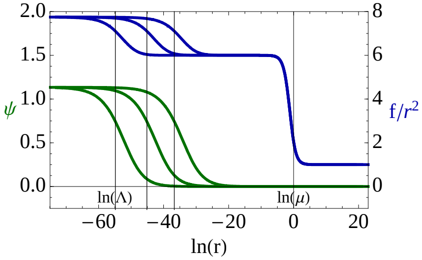
$$\langle \mathcal{O} \rangle = \alpha \sim (-\kappa + \kappa_c)^{\frac{\delta_-}{1-2\delta_-}}$$

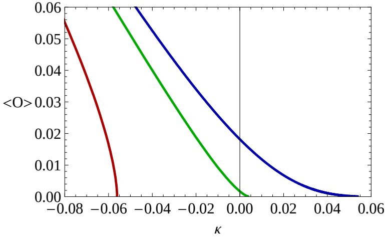
Ordered phase

- Metric ansatz for domain wall solution

$$ds^2 = -f(r)dt^2 + dr^2/f(r) + h^2(r)d\vec{x}^2, \quad A = A_t(r)dt, \quad \varphi = \varphi(r)$$

- Adjust $\delta_{1,2}$ at the (deep) IR fixed point. Shoot (backwards) close to the critical point.





- read off $\beta = \beta_{DW}(\alpha)$. Solve $\beta_{DW}(\alpha) = \kappa\beta$ for $\langle\mathcal{O}\rangle = \alpha(\kappa)$.