Parameter Counting and Symmetry

Mapping out the TeV-scale theory

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arxiv:0811.1019 by JB and Yuval Grossman

Cornell LEPP Theory Seminar

February 13, 2009

Outline

Motivation & Strategy

Motivation

Model building strategy:

$$\text{Data} \rightarrow \text{Particles and Symmetries} \rightarrow \text{Lagrangian}$$

Motivation

Model building strategy:

Data $\rightarrow$ Particles and Symmetries $\rightarrow$ Lagrangian

BUT

- Hidden symmetries
- Unphysical parameters

Our Goal: Find the physical parameters

Quantum Mechanics 101

Hydrogen atom in a uniform $\vec{B}$ field

- B field has three parameters
- Magnetic field breaks $SO(3) \to SO(2)$
- Two broken generators: rotate so that only B_z is non-zero

$\implies$ Number of parameters reduced from 3 to 1

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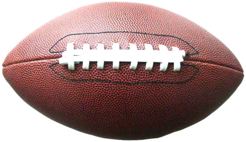

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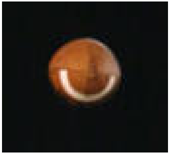

Lessons for any theory

$$N_{\text{phys}} = N_{\text{general}} - N_{\text{broken}}$$

- N_{phys}: Number of measurable parameters
- N_{general}: Number of parameters in an arbitrary coordinate system
- N_{broken}: Change in the number of symmetry generators

$\Longrightarrow$ For H atom in $\vec{B}$ field: $N_{\text{general}} = 3$, $N_{\text{broken}} = 2$, so $N_{\text{phys}} = 3 - 2 = 1$

Example: SM Yukawa Matrices

Most general interactions respecting gauge symmetries:

$$V = Y_{ij}^{U}(\overline{Q_L})_i(U_R)_j H + Y_{ij}^{D}(\overline{Q_L})_i(D_R)_j \tilde{H} + Y_{ij}^{E}(\overline{L_L})_i(E_R)_j \tilde{H}$$

- Three complex, 3×3 interaction matrices: 54 parameters
- $U(3)^5$ symmetry broken to $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$: 41 broken generators
- $N_{\mathrm{phys}} = N_{\mathrm{general}} - N_{\mathrm{broken}} = 54 - 41 = 13$
- 9 fermion masses, 3 CKM angles, 1 CKM phase

New Rules: Partial Flavor Symmetry

Larger global symmetries?

Many models based on broken symmetry in SM flavor sector

$\Longrightarrow$ Strong constraints on new flavor-changing interactions

$\Longrightarrow$ New symmetries can suppress FCNC's

Restoring symmetries: A Toy Model

- Impose $SU(2)$ symmetry on leptons
- Lighter generations doublets; heavy generation singlets

Symmetry implies degenerate mass eigenvalues

Restoring symmetries: Results

- Parameters will cancel in arbitrary basis
- $N_{\text{general}} = 15$ instead of 18
- Symmetry breaking is now $U(3)^2 \to U(2) \times U(1)$:

  $N_{\text{broken}} = 18 - 5 = 13$
- $N_{\text{phys}} = 15 - 13 = 2$

The General Case

Extend the technique to model with n generations

- Two fields in the interaction matrix $\implies SO(n)^2$ symmetry
- Adding a Yukawa coupling to a scalar breaks the symmetry
- Impose an $SU(n_1) \times SU(n_2) \times \cdots \times SU(n_k)$ global symmetry
- In an arbitrary basis, the number of parameters is reduced according to

$$N_{\text{general}} = 2n^2 - \sum_{j=1}^{k}(n_j^2 - 1)$$

- Only requirement: degeneracy of the mass eigenvalues

Parameter Counting in the MSSM

New Flavor Symmetries in MSSM

- Soft SUSY-breaking potential has new, *a priori* general interactions
- There are many possible ways to impose symmetry
  - Different symmetry groups
  - Different sectors on which the symmetry holds
- Need to think of correlations between different interaction matrices

One Illustrative Example

MSSM with $SU(2)$ flavor symmetry imposed on SUSY-violating terms

- Without the added symmetry: 153 parameters for chiral multiplets alone (110 physical)
- Adding symmetry $\implies$ correlations in mass matrices
  - Basis where the symmetry is manifest
- Can account for this and find $N_{\text{general}} = 105$ (62 physical)

Conclusions

Conclusions

- The number of measurable parameters in a theory is not always obvious
- Can be related to the total number of parameters and the the amount of broken symmetry
- Adding flavor symmetry reduces the total number of parameters in calculable way

Backup

Symmetries in the MSSM

Imposed Symmetry	Broken By	$N^{(r)}_{\text{general}}$	$N^{(i)}_{\text{general}}$	$N^{(r)}_{\text{phys}}$	$N^{(i)}_{\text{phys}}$
None		84	69	69	41
Fermion Family	SUSY Int.	66	51	51	23
	Weak Int.	51	36	36	10
	All	48	33	33	9
$SU(2)$ Flavor	SUSY Int.	56	49	41	21
	Weak Int.	37	30	23	6
	All	35	28	22	6
Leading MFV		35	30	20	4
$SU(3)$ Flavor		20	21	11	3