UNIFICATION AND FLAVOR WITH EXTRA DIMENSIONS

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Work done in collaboration with:

I. Gogoladze, Y, Mimura, T. Li and K. Tobe

: Gogoladze, Mimura and SN ,(Phys. Lett. B, Phys. ReV Lett. and Phys. ReV. D)

: Gogoladze, Li, Mimura and SN (Phys. Lett. B, and Phys. ReV D)

: Gogoladze, Mimura, SN and Tobe (Phys. Lett. B)

: Y. Mimura and SN (in preparation)

OUTLINE OF THE TALK

Introduction

- Why extra dimension?
- Theoretical motivation
- Experimental Implications

Unification

- What are we trying to achieve?
- Why extra dimension?
- Why supersymmetry?
- What forces (couplings) we are unifying?

A concrete model for gauge, Higgs and matter unification

Phenomenological implication

Models with three families : Flavor symmetries

Conclusions

INTRODUCTION

Why we think there are extra space-like dimension beyond X, Y and Z?

➡ **Theoretical motivation**

How can experiment discover them?

➡ **Experimental implications**

Extra Dimensions

Question:

What are the dimensions of space-time?

Philosopher Immanuel Kant (1781): The Critique of Pure Reason

Has argued that: Space and time are **a priori**

Modern view:

Space-time is **emergent**

must follow from theory

History:

Einstein dream:

Unify gravity **(extremely week)** and electromagnetism

Kaluza (1919), Klein (1926):

Introduce one extra space-like dimension of **finite** size

$$4D: \qquad \text{Gravity}: h_{\mu\nu}$$

$$\text{EM}: A_\mu \,, (\mu, \nu) = 0, 1, 2, 3$$

$$5D: \qquad \text{Gravity}: h_{MN} \quad (M, N) = 0, 1, 2, 3, 5$$

$$h_{MN} \to h_{\mu\nu} \ (\text{gravity in } 4D), \qquad h_{MN} \to h_{\mu 5} \ (\text{EM in } 4D)$$

Current theoretical motivation

Quantum mechanics + Gravity

↓

String Theory

All elementary particles are different vibration of a single entity called string

→ one unifying force

Theoretical consistency requires:

9 spatial dimensions
(6 more than X, Y, Z)

Question:

- **Are the extra spatial dimensions finite or infinite in size?**

→

Finite and very small (because we don't see them)
Current experiments → smaller than *sub-mm*.

- **What are their shapes?**

→ **Shape will determine the number of elementary particles, as well as, their interactions.**

Extra Dimensions: Theory Benefits

- **Can understand why gravity is so weak compared to EM**
  (Arkani-Hamed, Dimopoulos, and Dvali (ADD))

- **True unification of ALL particles:**
  Gauge, Higgs, Matter
  (Gogoladze, Mimura + S. N)

Unification of gauge and Yukawa forces

$$g_1 = g_2 = g_3 = g_t = g_b = g_\tau$$

(works really well)

- **An alternative to Higgs mechanism**
  (Kawamura, Alterarelli & Fergulio; Hall & Nomura)

Theory Benefits (contd.)

- **No Gauge hierarchy problem**

  (Arkani-Hamed, Dimopoullos + Dvali)

- **A mechanism for SUSY breaking**

  (Scherk & Schwarz, Hosotani)

- **Understanding of why** $m_\nu \ll m_q, m_l$

  (Arkani-Hamed, Dimopoulos, Dvali and March-Russell; Dienes, Dudas and Gherghetta)

- **Possibility of Multi-TeV scale GUT**

  (Dienes, Dudas & Gherghetta)

- **Candidate for Cold Dark Matter**

  (Cheng, Matchev& Schmaltz; Tait & Servant, Cheng, Feng & Matchev)

- **Exploring Quantum Gravity**

Experimental Implications:

- Existence of a tower of new particles (KK Excitations)
- Power Law running of gauge couplings
- Deviations from Newton's law of gravity

. Blackholes at colliders

- Astrophysical implications

Unification with Extra Dimensions

True Unification of Elementary Particles and Forces

(**Nandi** with Gogoladze, Li and Mimura)

I. Gogoladze, Y. Mimura and S. Nandi:
Phys. Rev. Lett. 91: 141801 (2003),
Phys. Lett. B562: 307 (2003)
Phys. Rev D69, 075006 (2004)

I. Gogoladze, T. Li, Y. Mimura and S. Nandi:
Phys. Lett. B622: 320 (2005),
Phys. Rev. D72: 055006 (2005)

- An attempt to understand all fundamental forces of Nature as **one** fundamental force
- All fundamental particles having a common origin

  All particles propagate into the extra dimension

Crucial ingrediens → Supersymmetry
→ Extra dimensions

What are we trying to achieve?

SM or MSSM: $SU(3) \times SU(2) \times U(1)$

Gauge couplings: $g_3, g_2, g_1 \Rightarrow$ strong, weak and EM forces

Yukawa couplings: $y_t, y_b, y_\tau, \ldots \Rightarrow$ Yukawa forces $\Rightarrow$ origin of fermion masses

Particles:

- gluons, EW gauge bosons (12) : spin 1
- fermions: $t, b, \tau, c, s, \mu, \ldots$: spin $\frac{1}{2}$
- Higgs bosons: doublets : spin 0

$\longrightarrow$ want to unify forces as well as particles

Grand Unification in 4 dimension:

$$SU(3)\times SU(2)\times U(1) \rightarrow G_{GUT}$$

$$(SU(5),\ SO(10),......)$$

- Unifies 3 gauge couplings and unifies all the gauge bosons
- Unifies also fermions in one family

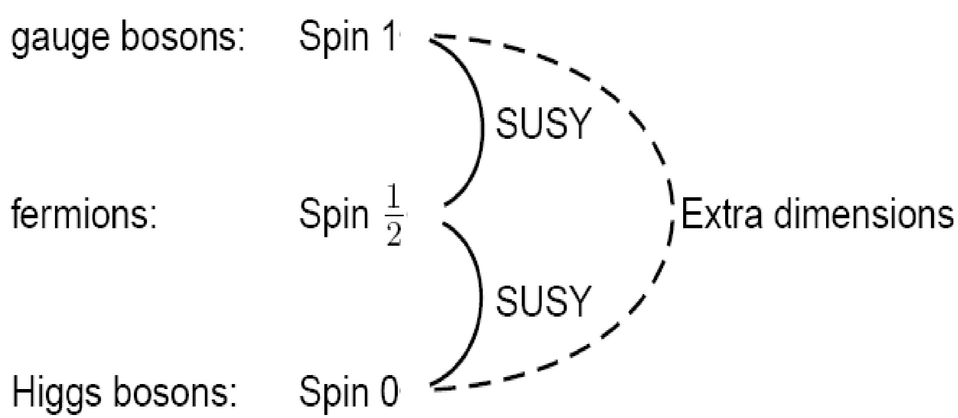
But Grand Unification:

- Does not unify gauge and Yukawa interactions
- Does not unify gauge bosons, Higgs and fermions together

We want complete unification

Why need extra dimensions?

Why need supersymmetry?



$$
\begin{array}{cccc}
A_M & \Rightarrow & A_\mu & A_5 \\
\downarrow & & \downarrow & \downarrow \\
\mu, 5 & & \text{gauge boson} & \text{Higgs scalar}
\end{array}
$$

A concrete model for gauge, Higgs and matter unification
(Gogoladze, Mimura and SN, Phys. Lett. B (2003))

Our model:

- Two extra dimensions
- $\mathcal{N} = 2$ SUSY
- $SU(8)$ gauge symmetry

Two extra dimensions compactified on a torus$/Z_6$ (orbifold)

A CONCRETE MODEL FOR

GAUGE, HIGGS & MATTER UNIFICATION

(3RD FAMILY)

KEY INGREDIENT : EXTRA DIM
+ SUSY

: A 6D, N=2 SUSY MODEL

: SU(8) GAUGE SYMMETRY

: T^2/Z_6 ORBIFOLD

6D, N=2 SUSY $\Longrightarrow$ 4D, N=4 SUSY

$\Downarrow$

ONLY GAUGE SUPERMULTIPLET IN BULK

IN 4D, N=1 LANGUAGE: 6D $\Rightarrow$ 4D

$$V \Longrightarrow V, \underbrace{\Sigma, \Phi, \Phi^c}_{\text{chiral supermultiplets}}$$

6D gauge multiplet

Vector supermultiplet

IN 4D, $N=1$ LANGUAGE,

$$6D \longrightarrow 4D$$

$$\mathbb{V} \Rightarrow V,\ \underbrace{\Sigma,\ \Phi,\ \Phi^c}$$

6D, $N=2$ gauge multiplet in adjoint rep. of the gauge group

Vector super-multiplet

will contain the gauge bosons + gauginos in 4D

Chiral supermultiplets

will contain the Higgs fields, also the quarks and leptons

: UNWANTED FIELDS WILL BE PROJECTED OUT (ZERO MODES) BY SUITABLE ORBIFOLD COMPACTIFICATION.

6D BULK ACTION

$$\overset{6D}{\mathbb{V}} \Longrightarrow \overset{4D}{V,\ \Sigma,\ \Phi,\ \Phi^c}$$

6D BULK ACTION in 4D, $N=1$ LANGUAGE, IN WESS-ZUMINO GAUGE

$$
\begin{aligned}
S = \int d^6x \Big\{ & \mathrm{Tr}\Big[\int d^2\theta \Big(\frac{1}{4kg^2} W^\alpha W_\alpha \\
& + \frac{1}{kg^2}\big(\Phi^c \partial \Phi - \underbrace{\sqrt{2}\,\Sigma[\Phi, \Phi^c]}_{\text{Yukawa coupling}}\big)\Big) + h.c.\Big] \\
& + \int d^4\theta \frac{1}{kg^2} \mathrm{Tr}\Big[\Big(\frac{1}{\sqrt{2}}\partial^\dagger + \Sigma^\dagger\Big) e^{-2V} \Big(-\frac{1}{\sqrt{2}}\partial + \Sigma\Big) e^{2V} \\
& \qquad + \frac{1}{4}\partial^\dagger e^{-2V} \partial e^{2V}\Big] \\
& + \int d^4\theta \frac{1}{kg^2} \mathrm{Tr}\Big[\Phi^\dagger e^{-2V} \Phi e^{2V} + \Phi^{c\dagger} e^{-2V} \Phi^c e^{2V}\Big] \Big\}
\end{aligned}
$$

where $\partial \equiv \partial_5 - i\,\partial_6$

ORBIFOLD COMPACTIFICATION:
2 EXTRA DIMENSIONS

DEFINE $X_5 + i X_6 \equiv z$
$X_5 - i X_6 = \bar{z}$

CONSIDER T^2/Z_n orbifolds,
$z \to \omega z$, $\omega^n = 1$; $n = 2, 3, 4, 6$

IMPOSE THE TRANSFORMATIONS:

$$V(x_\mu, \omega z, \bar{\omega} \bar{z}) = R\, V(x_\mu, z, \bar{z})\, R^{-1}$$
$$\Sigma(x_\mu, \omega z, \bar{\omega} \bar{z}) = \omega^{n-1} R\, \Sigma(x_\mu, z, \bar{z})\, R^{-1}$$
$$\Phi(x_\mu, \omega z, \bar{\omega} \bar{z}) = \omega^{x} R\, \Phi(x_\mu, z, \bar{z})\, R^{-1}$$
$$\Phi^c(x_\mu, \omega z, \bar{\omega} \bar{z}) = \omega^{y} R\, \Phi^c(x_\mu, z, \bar{z})\, R^{-1}$$

where R = an unitary matrix, $R^\dagger R = I$, $R^n = I$.

For the invariance of the action:

$$\partial^\dagger e^{-2V} \Sigma\, e^{2V} \Rightarrow \Sigma \to \omega^{n-1} R\, \Sigma\, R^{-1}$$
$$\Sigma[\Phi, \Phi^c] \Rightarrow x + y = 1 \ (\mathrm{mod.}\ n)$$

NON-TRIVIAL CHOICE OR

$\Rightarrow$ will <u>break</u> $N=4$ SUSY to $N=1$

as well as <u>break</u> gauge <u>symmetry</u>

: OUR CHOICE OF GAUGE SYM: $SU(8)$

$V \Rightarrow$ adjoint rep., $\underset{\sim}{63}$ of $SU(8)$

$R = 8 \times 8$ matrix

: Choice of R determine the $SU(8)$

breaking pattern by T^2/Z_6 compactification

EXAMPLE:

$$R = \left(\begin{array}{cccccc|cc} \omega^a &&&&&&& \\ & \omega^a &&&&&& \\ && \omega^a &&&&& \\ &&& \omega^a &&&& \\ &&&& \omega^a &&& \\ &&&&& \omega^a && \\ \hline &&&&&& \omega^b & \\ &&&&&&& \omega^b \end{array}\right) : SU(8) \Rightarrow SU(6) \times SU(2) \times U(1)$$

$$R = \left(\begin{array}{cccc|cccc} \omega^a &&&&&&& \\ & \omega^a &&&&&& \\ && \omega^a &&&&& \\ &&& \omega^a &&&& \\ \hline &&&& \omega^b &&& \\ &&&&& \omega^b && \\ &&&&&& \omega^b & \\ &&&&&&& \omega^b \end{array}\right) : SU(8) \Rightarrow SU(4) \times SU(4) \times U(1)$$

$$R = \left(\begin{array}{cccc|cc|cc} \omega^a &&&&&&& \\ & \omega^a &&&&&& \\ && \omega^a &&&&& \\ &&& \omega^a &&&& \\ \hline &&&& \omega^b &&& \\ &&&&& \omega^b && \\ \hline &&&&&& \omega^c & \\ &&&&&&& \omega^c \end{array}\right) : SU(8) \Rightarrow SU(4) \times SU(2) \times SU(2) \times U(1) \times U'(1)$$

⇗

We use this to break SU(8) in 6D

$$\Rightarrow \underbrace{SU(4)_c \times SU(2)_L \times SU(2)_R}_{\text{Pati-Salam}} \times U(1) \times U'(1)$$

& N = 4 SUSY to N = 1

$SU(4)_C \times SU(2)_L \times SU(2)_R$ PATI-SALAM :

GAUGE BOSONS : $(15,1,1) + (1,3,1) + (1,1,3)$

FERMIONS : $(4,2,1) + (\bar{4},1,2)$
in each family

HIGGS : $(1,2,2) + \cdots\cdots$
EW

$$SU(8) \xrightarrow{T^2/Z_6} SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1) \times U'(1)$$

$$\underset{\sim}{63} = T_a V_a$$

$$= \begin{array}{c|c} & \begin{array}{ccc} SU(4)_c & SU(2)_L & SU(2)_R \end{array} \\ \begin{array}{c} SU(4)_c \\ \\ SU(2)_L \\ \\ SU(2)_R \end{array} & \left(\begin{array}{c|c|c} \overset{V}{(15,1,1)_{0,0}} & \overset{L}{(4,2,1)_{2,0}} & (4,1,2)_{2,3} \\ \hline (\bar{4},1,2)_{-2,0} & \overset{V}{(1,3,1)_{00}} & \overset{V}{(1,2,2)_{0,3}} \\ \hline \underset{R}{(\bar{4},1,2)_{-2,-3}} & \underset{H}{(1,2,2)_{0,-3}} & \overset{V}{(1,1,3)_{0,0}} \end{array} \right) \end{array}$$

$$+ \underset{U(1)}{(1,1,1)_{0,0}} + \underset{U'(1)}{(1,1,1)_{0,0}}$$

$= 8 \times 8$ matrix

$$V^{6D} \Rightarrow^{4D} 63_V, \; 63_\Sigma, \; 63_\Phi, \; 63_{\Phi^c}$$

$$\Rightarrow 63_X$$

WITH $R = \left(\begin{array}{cccc|cc|cc} \omega^a &&&&&&& \\ & \omega^a &&&&&& \\ && \omega^a &&&&& \\ &&& \omega^a &&&& \\ \hline &&&& \omega^b &&& \\ &&&&& \omega^b && \\ \hline &&&&&& \omega^c & \\ &&&&&&& \omega^c \end{array}\right)$

$$R\, 6_{3_x} R^{-1} = \left(\begin{array}{c|c|c} 1 & \omega^{a-b} & \omega^{a-c} \\ \hline \omega^{b-a} & 1 & \omega^{b-c} \\ \hline \omega^{c-a} & \omega^{c-b} & 1 \end{array}\right)$$

$$\Longleftrightarrow \left(\begin{array}{c|c|c} (15,1,1) & (4,2,1) & (4,1,2) \\ \hline (\bar{4},2,1) & (1,3,1) & (1,2,2) \\ \hline (\bar{4},1,2) & (1,2,2) & (1,1,3) \end{array}\right)$$

$\Rightarrow (4,2,1)_{2,0} : \omega^{a-b}$

$(\bar{4},1,2)_{-2,-3} : \omega^{c-a}$

$(1,2,2)_{0,3} : \omega^{b-c}$

$(4,2,1)_{2,0} \quad : \quad \omega^{a-b}$

$(\bar{4},1,2)_{-2,-3} \quad : \quad \omega^{c-a}$

$(1,2,2)_{0,3} \quad : \quad \omega^{b-c}$

FOR T^2/Z_6:

$$V \to R V R^{-1}$$
$$\Sigma \to \omega^5 R \Sigma R^{-1}$$
$$\Phi \to \omega^x R \Phi R^{-1}$$
$$\Phi^c \to \omega^y R \Phi^c R^{-1}$$

WITH $x+y=1$

SOLUTIONS: (TO OBTAIN CORRECT MASSLESS SPECTRUM + $N=1$ SUSY)

i) $x=1$, $y=0$ ✗

ii) $x=2$, $y=5$ ✗

$\boxed{\begin{array}{l} \text{iii) } x=3,\ y=4 \\ \text{iv) } x=4,\ y=3 \end{array}}$ ✓ ✓

v) $x=5$, $y=2$ ✗

vi) $x=0$, $y=6$ ✗

$\Rightarrow$ $b-c=3$, $a-b=1$, $c-a=2$

MASSLESS SPECTRUM: $SU(4)_c \times SU(2)_L \times SU(2)_R$
$\times U(1) \times U'(1)$

Gauge fields: $(15,1,1) + (1,3,1) + (1,1,3)$
$+ (1,1,1) + (1,1,1)$

Electroweak Higgs: $(1,2,2) + (1,2,2)$
$\Rightarrow$ two bidoublets

Fermions: $(4,2,1) + (\bar{4},1,2) \Leftarrow$ one family
(3rd family)

$$\mathbb{V} \Rightarrow \quad V, \quad \Sigma, \quad \Phi^c, \quad \Phi$$

$$\begin{array}{lcccc}
 & V & \Sigma & \Phi^c & \Phi \\
 & \downarrow & \downarrow & \downarrow & \downarrow \\
(x=3, y=4) & \text{gauge fields of P-S} & (4,2,1)_{2,0} & (\bar{4},1,2) & (1,2,2) + (1,2,2) \\
 & \text{+ two extra } U(1) & \downarrow & \downarrow & \downarrow \qquad\quad \downarrow \\
 & & \psi_L & \psi_L & H_1 \qquad\quad H_2
\end{array}$$

For $x=4, y=3$, $\Phi \Leftrightarrow \Phi^c$

$\Rightarrow$ UNIFICATION OF GAUGE, HIGGS & MATTER
(one family)

IN $D=6$, $N=2$

GAUGE MULTIPLET OF $SU(8)$ ADJOINT.

PHENOMENOLOGICAL IMPLICATIONS

: Gauge-Yukawa unification

$N=1$ chiral multiplets Σ, Φ, Φ^c are <u>all</u> in the same gauge multiplet of $D=6$, $N=2$ SUSY. $\mathbb{V} \Rightarrow (V, \Sigma, \Phi, \Phi^c)$

The gauge interaction term

$$S = \int d^6x \left[d^2\theta \frac{1}{Kg^2} \mathrm{Tr} \left[-\sqrt{2}\, \Sigma [\Phi, \Phi^c] \right] + h.c. \right]$$

include the Yukawa interaction

$$S = \int d^6x \int d^2\theta \;\; \bar{\Psi}_L H_1 \Psi_R \;\; + h.c.$$

$$\Rightarrow y_6 = g_6 \quad \Rightarrow y_4 = g_4$$

$$\Rightarrow g_3 = g_2 = g_1 = \underbrace{y_t = y_b = y_\tau}_{3^{rd}\ \text{family}}$$

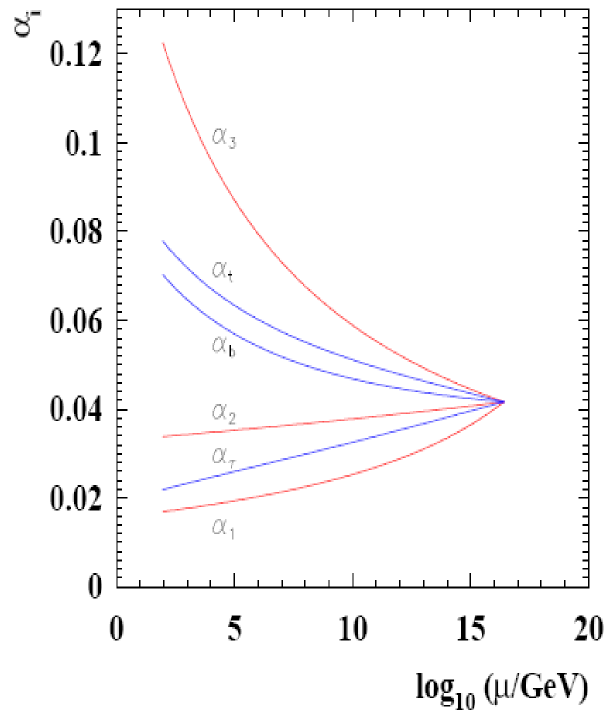
$\Rightarrow$ good experiment with experiment with $M_U \sim 10^{16}$ GeV

Last year, we presented SU(8) model in 6 dimensions

Prediction:

$$\underbrace{g_3 = g_2 = g_1 = y_t = y_b = y_\tau}_{\text{3rd family Yukawa couplings}}$$

$\Rightarrow$ in good agreement with experiment



Other predictions:

$\alpha_3(M_Z) = 0.123,$ $m_t =$ m_t=174 GeV

$m_b/m_t(M_Z) = 1.77,$ $\boxed{\tan\beta = 51}$

MORE FAMILIES & MODEL BUILDING

UNIFICATION WITH <u>TWO</u> FAMILIES IN BULK:

A $SO(16)$ model with $D=6$, $N=2$ SUSY

$$SO(16) \xrightarrow[T^2/Z_6]{\text{broken to}} SU(8) \times U(1)$$

$$\Longrightarrow SU(4) \times SU(2)_L \times SU(2)_R \times U^3(1)$$

$$120 = \underbrace{(63)_0}_{\text{one family}} + \underbrace{(28)_{-1} + (\overline{28})_1}_{2^{nd}\text{ family}} + (1)_0$$

$$\mathbb{V}_{120} \Rightarrow V_{120},\ \Sigma_{120},\ \Phi_{120},\ \Phi^c_{120}$$

$$R = \left(\begin{array}{c|c} \omega^{\frac{a}{2}} R_8 & 0 \\ \hline 0 & \omega^{-\frac{a}{2}} R_8 \end{array}\right)$$

Massless modes :
(Solution with $a = 3$, $x = 4$, $y = 3$)

	63	28	$\overline{28}$
Σ	$L_3 = (4,2,1)_{1,0,0}$	$S_1 = (1,1,1)_{1,1,-1}$ $S_2 = (1,1,1)_{-1,-3,-1}$	$\bar{R}_2 = (\bar{4},1,2)_{0,1,1}$
Φ	$\bar{R}_3 = (\bar{4},1,2)_{-1,-2,0}$	$L_2 = (4,2,1)_{0,1,-1}$	$H_3 = (1,2,2)_{0,1,1}$
Φ^c	$H_1 = (1,2,2)_{0,2,0}$ $H_2 = (1,2,2)_{0,-2,0}$	$C_1 = (6,1,1)_{1,1,-1}$	$C_2 = (6,1,1)_{1,1,-1}$

$\Rightarrow$ <u>Two</u> families in the bulk

Gauge interaction

$$S = \int d^6x \left\{ \int d^2\theta \, \frac{1}{kg^2} \, \mathrm{Tr}\left(-\sqrt{2}\, \Sigma [\Phi, \Phi^c] \right) \right\}$$

$\Rightarrow$ Yukawa interactions for zero modes

$$S = \int d^6x \int d^2\theta \; y_6 \Big[L_3 \bar{R}_3 H_1 + L_2 \bar{R}_2 H_2 + (H_1 S_2 - H_2 S_1) H_3 + \cdots\cdots + h.c. \Big]$$

$\Rightarrow$ $y_4 = g_4$ (gauge-Yukawa Unification)

$S_1, S_2 \Rightarrow$ singlets under Pati-Salam, can have vevs.

$\Rightarrow$ causes mixing of H_1, H_2, H_3

: Only the combination

$$H = \frac{1}{\sqrt{S_1^2 + S_2^2}} (H_1 S_1 + H_2 S_2)$$ remains light,

$$\Rightarrow L_Y = \frac{1}{\sqrt{S_1^2 + S_2^2}} \Big[S_1 \,(L_3 \bar{R}_3 H) + S_2 \,(L_2 \bar{R}_2 H) \Big]$$

$\Rightarrow$ y_2 suppressed compared to y_3 by $\left(\frac{S_2}{S_1}\right)$

$\Rightarrow$ hierarchy between 2nd and 3rd family

MODELS WITH <u>THREE</u> FAMILIES FROM BULK : FLAVOR SYMMETRIES

OBJECTIVE : Can we obtain <u>three</u> Chiral families from bulk with hieaarchial Yukawa couplings ?

: What short of flavor symmetries that lead two ?

1. SIMPLEST MODEL :

6D, N=2 SUSY, SU(8) GAUGE SYMMETRY, T^2/Z_3 ORBIFOLD

$$SU(8) \xrightarrow[T^2/Z_3]{\text{breaks to}} \underbrace{SU(4)_c \times SU(2)_L \times SU(2)_R}_{P-S} \times U^2(1)$$

$$R V_{63} R^{-1} = \begin{pmatrix} (15,1,1), & \omega^{b-c}(4,2,1), & \omega^{b-d}(4,1,2) \\ \omega^{c-b}(\bar{4},2,1), & (1,3,1), & \omega^{c-d}(1,2,2) \\ \omega^{d-b}(\bar{4},1,2), & \omega^{d-c}(1,2,2), & (1,1,3) \end{pmatrix}$$

$$R\Sigma R^{-1} \Rightarrow \omega^2 R\Sigma R^{-1}$$

$$\left.\begin{array}{l} R\phi R^{-1} \Rightarrow \omega^{l} R\phi R^{-1} \\ R\phi^c R^{-1} \Rightarrow \omega^{m} R\phi^c R^{-1} \end{array}\right\} l+m=1$$

With $b=0$, $c=2$, $d=1$

ZERO Modes :

Σ : $(4,2,1)_1$, $(1,2,2)_1$, $(\bar{4},1,2)_1$

ϕ : $(4,2,1)_2$, $(1,2,2)_2$, $(\bar{4},1,2)_2$

ϕ^c : $(4,2,1)_3$, $(1,2,2)_3$, $(\bar{4},1,2)_3$

$\Rightarrow$ Three families + 3 Higgs bi-doublets.

L_{Yukawa} (from bulk)

$= \epsilon_{ijk} \, L_i \bar{R}_j H_k \Rightarrow$ totally antisym.

$\Rightarrow m_1 = 0, \qquad m_2 = - m_3$

good feature (hierarchy between the 1st and 2nd family)

bad feature (no hierarchy between the 2nd and 3rd family)

2. A SECOND MODEL

6D, $N=2$, E_7 gauge sym., T^2/Z_6 orbifold

$E_7 \xRightarrow{\text{break}} SU(8) \Rightarrow P\text{-}S \times U(1)^3$

$133 = 63 + 70$

$\Sigma \to \bar{\omega}\Sigma, \quad \Phi \to \bar{\omega}\Phi, \quad \Phi^c \to \omega^2 \Phi^c$

ZERO MODES :

	63	70
Σ	$L_3, \bar{R}_2, C_1$	C_3
Φ	$L_2, \bar{R}_3, C_2$	
Φ^c	H	L_1, R_1

L_{Yukawa} (from bulk)

$$= (L_3 \bar{R}_3 + L_2 \bar{R}_2) H$$

$\Rightarrow$ 1st family : $y_1 = 0$, but $y_2 = y_3$

good

bad (no hierarchy)

: only <u>one</u> bidoublet of Higgs

3-FAMILY MODEL STARTING FROM E_8

(Y. Mimura + SN, in progress)

$$E_8 \longrightarrow SO(16)$$

$$248 = 120 + 128$$

$$120 \to 63 + \underbrace{28 + \overline{28}}_{} + 1 \qquad 128 \to 70 + \underbrace{28 + \overline{28}}_{} + 1$$

3rd family (63), 2nd family ($28 + \overline{28}$ from 120), 1st family ($28 + \overline{28}$ from 128)

For systematic study, better to look at the branching

$$E_8 \longrightarrow SO(10) \times SU(4)_F$$

$$E_8 \longrightarrow SO(10) \times SU(4)_F$$

$$248 = (16,4) + (\overline{16},\bar{4}) + (45,1) + (10,6) + (1,15)$$

$\Uparrow$

have 4 families of SO(10)

$(L_1, L_2, L_3, L_4 + \bar{R}_1, \bar{R}_2, \bar{R}_3, \bar{R}_4)$

Under Pati-Salam

$$16 = (4,2,1) + (\bar{4},1,2)$$

$$45 = (15,1,1) + (1,3,1) + (1,1,3) + (6,2,2)$$

: Do the Z_n charge assignments for all P-S multiplets

: Apply the orbifold constraints, as well as the inv. of the action

: Look for 3-family solutions

: Calculate Yukawa interaction from Bulk

RESULTS SO FAR :

: THERE ARE FIVE POSSIBLE WAYS TO GET 3 FAMILIES FROM BULK

CASE 1 : Σ, Φ, Φ^c include L_1, L_2, L_3 respectively
(Σ, Φ, Φ^c have different $Z_{n_1} \times Z_{n_2}$ Charge assignments)
: $E_8 \to E_6 \times U(1)$ has this solution

CASE 2 : Σ, Φ, Φ^c include L_1, L_1, L_2 respectively
(Σ and Φ (or any two) have same charge assignment)
: $SO(16)$ and E_7 has this solution.

CASE 3 : Σ, Φ, Φ^c include L_1, L_1, L_1
(Σ, Φ, Φ^c have same charge assignment)
: $SU(8)$ with T^2/Z_3 has this solution

CASE 4: 4D $SU(2)$ Flavor symmetry remains

($\text{Two of } \Sigma, \Phi, \Phi^c$ include (L_1, L_2) and L_3)

: L_1, L_2 have the same charge assignment

: $E_8 \to$ P-S $\times SU(2)_F \times U(1)^2$ has this solution

CASE 5: 4D $SU(3)$ flavor symmetry remains

: One of Σ, Φ, Φ^c include (L_1, L_2, L_3)

(L_1, L_2, L_3 have the same charge assignment)

: $E_8 \to$ (P-S) $\times SU(3)_F \times U(1)$ has this solution

: CALCULATIONS OF Yukawa couplings FOR THESE CASES IS IN PROGRESS

Conclusions

EXTRA DIMENSION HAVE:

- **Good theory motivation**
- **Observable experimental implications**

TRUE UNIFICATION:

- **Can unify all interactions as one gauge interaction**
- **Unify gauge bosons, Higgs, quarks, leptons in one multiplet**
- **Understand the origin of Yukawa interaction $\Rightarrow$ origin of mass**
- **Testable prediction at LHC,** $\tan(\beta) = 50$
- **Can it also lead to three families with predictive flavor symmetry?**