Holographic non-Fermi liquids:

Strange metal from black holes

John McGreevy, MIT

based on:

Hong Liu, JM, David Vegh, 0903.2477

Tom Faulkner, HL, JM, DV, 0907.2694

TF, Gary Horowitz, JM, Matthew Roberts, DV, 0911.3402

TF, Nabil Iqbal, HL, JM, DV, 1003.1728 and in progress

see also: Sung-Sik Lee, 0809.3402

Cubrovic, Zaanen, Schalm, 0904.1933

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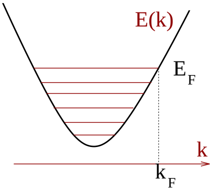
if we had *free* fermions, we would fill single-particle energy levels $E(k)$ until we ran out of fermions: $\longrightarrow$



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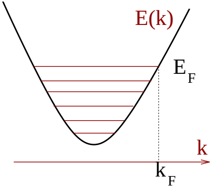

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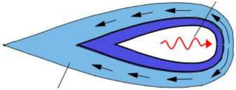
Idea [Landau]: The low-energy excitations of the interacting theory are still weakly-interacting fermionic, charged 'quasiparticles'

Elementary excitations are free fermions with some dressing:

$\xrightarrow{\textit{in medium}}$

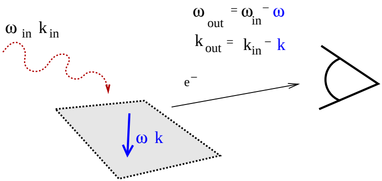


The standard description of metals

The metallic states of a finite density of fermions that we understand well are described by Landau's Fermi liquid theory.
Landau quasiparticles $\to$ poles in single-fermion Green function G_R

at $k_\perp \equiv |\vec{k}| - k_F = 0$, $\omega = \omega_\star(k_\perp) \sim 0$: $G_R \sim \dfrac{Z}{\omega - v_F k_\perp + i\Gamma}$

Measurable by ARPES (angle-resolved photoemission):



Intensity $\propto$ spectral density : $A(\omega, k) \equiv \operatorname{Im} G_R(\omega, k) \stackrel{k_\perp \to 0}{\to} Z\delta(\omega - v_F k_\perp)$

Landau quasiparticles are long-lived: width is $\Gamma \sim \omega_\star^2$.
residue Z (overlap with external e^-) is finite on Fermi surface.
Reliable calculation of thermodynamics and transport relies on this.

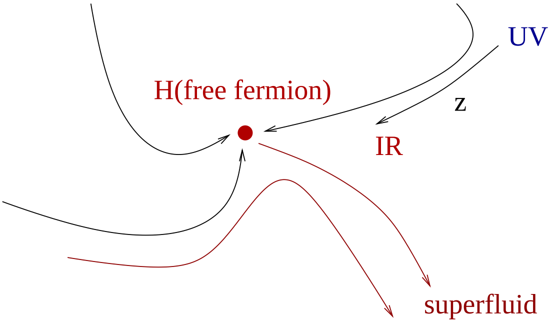
Ubiquity of Landau fermi liquid

Physical origin of lore:

1. Landau FL successfully describes ^{3}He, all metals studied before $\sim 1980s$, ...

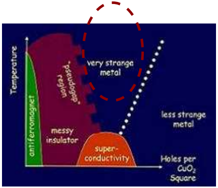
2. RG: Landau FL is stable under almost all perturbations.

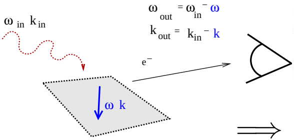
[Shankar, Polchinski, Benfatto-Gallivotti 92]

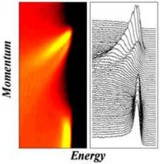


Non-Fermi liquids exist, but are mysterious

e.g.: 'normal' phase of optimally-doped cuprates: ('strange metal')







among other anomalies: ARPES shows gapless modes at finite k (FS!)

with width $\Gamma(\omega_\star) \sim \omega_\star$, vanishing residue $Z \overset{k_\perp \to 0}{\longrightarrow} 0$.

Working defintion of NFL:

Still a sharp Fermi surface (nonanalyticity of $A(\omega \sim 0, k \sim k_F)$)

but no long-lived quasiparticles.

[Anderson, Senthil] 'critical fermi surface'

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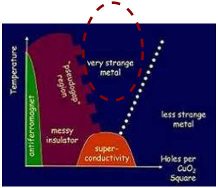

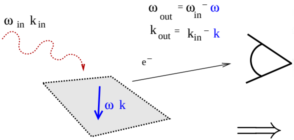

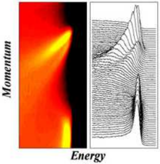

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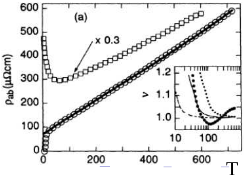
Most prominent

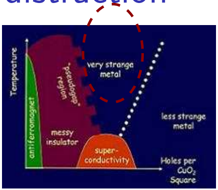
mystery of the strange metal phase:

e-e scattering: $\rho \sim T^2$, e-phonon: $\rho \sim T^5$,

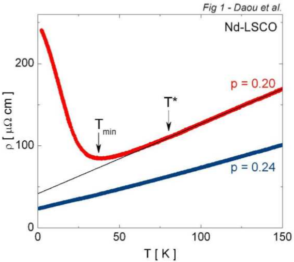
no known robust effective theory: $\rho \sim T$.



Superconductivity is a distraction



Look 'behind' superconducting dome by turning on magnetic field:



(Magnetoresistance is negligible: evidence that B doesn't alter normal state.)

Strange metal persists to $T \sim 0$!

This is the state we will be comparing to later on.

My understanding of the theoretical status of NFL

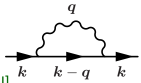
• Luttinger liquid (1+1-d) $G(k,\omega) \sim (k-\omega)^{2g}$ ✓

• loophole in RG argument:

couple a Landau FL perturbatively to a bosonic mode

(magnetic photon, slave-boson gauge field, statistical gauge field, ferromagnetism, SDW, Pomeranchuk order parameter...)

[Holstein et al, Baym et al, ... Halperin-Lee-Read, Polchinski, Altshuler-Ioffe-Millis, Nayak-Wilczek, Schafer-Schwenzer, Chubukov et al, Fradkin et al, Metzner et al, S-S Lee, Metlitski-Sachdev, Mross et al]



$\rightarrow$ nonanalytic behavior in $G^R(\omega) \equiv \frac{1}{v_F k_\perp + \Sigma(\omega,k)}$ at FS:

$$\Sigma(\omega) \sim \begin{cases} \omega^{2/3} & d = 2+1 \\ \omega \log \omega & d = 3+1 \end{cases} \implies Z \overset{k_\perp \to 0}{\rightarrow} 0, \qquad \frac{\Gamma(k_\perp)}{\omega_\star(k_\perp)} \overset{k_\perp \to 0}{\rightarrow} \text{const}$$

Fermi liquid killed by gapless boson

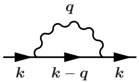
1. In these perturbative calculations, non-analytic terms $\propto$ control parameter $\Longrightarrow$
perturbative answer is parametrically reliable $\leftrightarrow$

effect is visible only at parametrically low temperatures.

2. Recently, the validity of the $1/N$ expansion has been questioned.
[Sung-Sik Lee 0905, Metlitski-Sachdev 1001]

A controlled perturbation expansion does exist. [Mross, JM, Liu, Senthil, 1003.0894]

3. These NFLs are not strange metals in terms of transport.
FL killed by gapless bosons: small-angle scattering dominates $\Longrightarrow$
(forward scattering does not degrade current)
'transport lifetime' $\neq$ 'single-particle lifetime'
i.e. in models with $\Gamma(\omega_\star) \sim \omega_\star$, $\rho \sim T^{\alpha > 1}$.



Can string theory be useful here?

It would be valuable to have a non-perturbative description of such states in more than one dimension.

Gravity dual?

We're not going to look for a gravity dual of the whole material.

Rather: lessons for principles of "non-Fermi liquid".

Basic question for the holographic descripion:

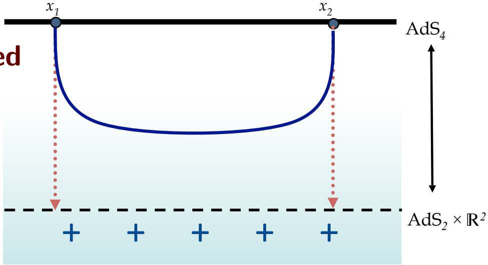
How to make a finite density of fermions?

Outline

1. Introduction: ‘post-particle physics of metals’
2. Strategy for holographic description
3. Fermion green functions, numerically
4. Analytic understanding of Fermi surface behavior
5. Charge transport
6. Stability of the groundstate
7. A framework for strange metal

Strategy to find a holographic Fermi surface

Consider any relativistic CFT with a gravity dual
a conserved $U(1)$ symmetry proxy for fermion number $\longrightarrow A_\mu$
and a charged fermion proxy for bare electrons $\longrightarrow \psi$.
Any $d > 1 + 1$, focus on $d = 2 + 1$.



CFT at finite density: **charged** black hole (BH) in AdS.
To find FS: [Sung-Sik Lee 0809.3402]
look for sharp features
in fermion Green functions
at finite momentum
and small frequency.

To compute G_R: solve Dirac equation in charged BH geometry.

What we are doing, more precisely

Consider any relativistic CFT_d with

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- a conserved $U(1)$ current (proxy for fermion number)

$\rightarrow$ gauge field $F = dA$ in the bulk.

An ensemble with finite chemical potential for that current is described by the AdS Reissner-Nordstrom black hole:

$$ds^2 = \frac{r^2}{L^2}\left(-f dt^2 + d\vec{x}^2\right) + L^2 \frac{dr^2}{r^2 f}, \qquad A = \mu\left(1 - \left(\frac{r_0}{r}\right)^{d-2}\right) dt$$

$$f(r) = 1 + \frac{Q^2}{r^{2d-2}} - \frac{M}{r^d}, \quad f(r_0) = 0, \qquad \mu = \frac{g_F Q}{c_d L^2 r_0^{d-1}},$$

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- a charged fermion operator $\mathcal{O}_{\mathrm{F}}$ (proxy for bare electrons)

$\rightarrow$ spinor field ψ in the bulk $\mathcal{L}_{d+1} \ni \bar{\psi}\left(D_M \Gamma^M - m\right)\psi + \text{interactions}$

with $D_\mu \psi = \partial_\mu \psi - iqA_\mu \psi$ ($\Delta = \frac{d}{2} \pm mL, \quad q = q$)

'Bulk universality': for two-point functions, the interaction terms don't matter!

Results only depend on q, Δ.

Comments about the strategy

- There are many string theory vacua with these ingredients. In specific examples of dual pairs (*e.g.* M2-branes $\Leftrightarrow$ M th on $AdS_4 \times S^7$), interactions and $\{q, m\}$ are specified.
  which sets $\{q, m\}$ are possible and what correlations there are is not clear.
- This is a large complicated system ($\rho \sim N^2$), of which we are probing a tiny part ($\rho_\Psi \sim N^0$).
- It would be surprising if we could describe a Fermi liquid (= a weakly coupled QFT).
- In general, both bosons and fermions of the dual field theory are charged under the $U(1)$ current: this is a Bose-Fermi mixture.

Notes: frequencies ω below are measured from the chemical potential.

Results are in units of μ.

Computing G_R

Translation invariance in $\vec{x}, t \implies$ ODE in r.
Rotation invariance: $k_i = \delta_i^1 k$
Near the boundary, solutions behave as $(\Gamma^{\underline{r}} = -\sigma^3 \otimes 1)$

$$\psi \overset{r\to\infty}{\approx} a_\alpha r^m \begin{pmatrix} 0 \\ 1 \end{pmatrix} + b_\alpha r^{-m} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Matrix of Green's functions, has two independent eigenvalues:

$$G_\alpha(\omega, \vec{k}) = \frac{b_\alpha}{a_\alpha}, \quad \alpha = 1, 2$$

To compute G_R: solve Dirac equation in BH geometry,
impose infalling boundary conditions at horizon [Son-Starinets, Iqbal-Liu].
Like retarded response, falling into the BH is something that *happens*.

Dirac equation

$$\Gamma^a {e_a}^M \left(\partial_M + \frac{1}{4}\omega_{abM}\Gamma^{ab} - iqA_M\right)\psi - m\psi = 0$$

$$\Phi_\alpha \equiv (-gg^{rr})^{-1/4}\Pi_\alpha^{\hat{k}}\psi, \quad \psi = e^{-i\omega t + ik_i x^i}\psi_{\omega,k},$$

$$\boxed{\left(\partial_r + M\sigma^3\right)\Phi_\alpha = \left((-1)^\alpha K\sigma^1 + Wi\sigma^2\right)\Phi_\alpha, \quad \alpha = 1, 2}$$

with

$$M \equiv m\sqrt{g_{rr}} = \frac{m}{r\sqrt{f}}, \quad K \equiv k\sqrt{\frac{g_{rr}}{g_{ii}}} = \frac{k}{r^2\sqrt{f}}, \quad W \equiv u\sqrt{\frac{g_{rr}}{g_{ii}}} = \frac{u}{r^2\sqrt{f}}.$$

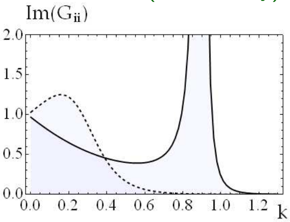
$$u \equiv \sqrt{\frac{-g^{tt}}{g^{ii}}}\left(\omega + \mu_q\left(1 - \left(\frac{r_0}{r}\right)^{d-2}\right)\right)$$

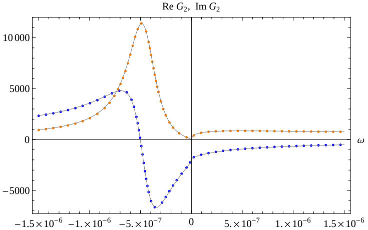
Eqn depends on q and μ only through $\mu_q \equiv \mu q$

$\rightarrow$ ω is measured from the effective chemical potential, μ_q.

Fermi surface!

At $T = 0$, we find (numerically):





'MDC': $G(\omega = -0.001, k)$ 'EDC': $G(\omega, k = 0.9)$

For $q = 1, m = 0$: $k_F \approx 0.918528499$

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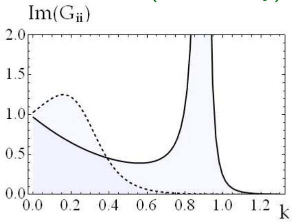

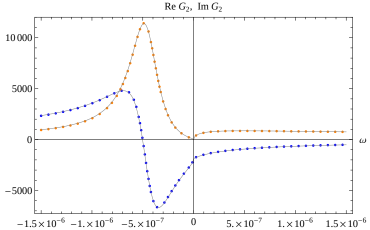

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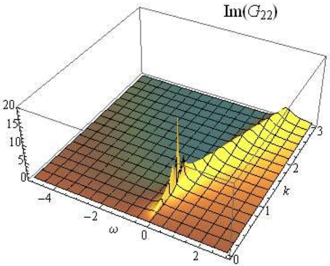


But it's not a Fermi liquid:

The peak moves
with dispersion relation $\omega \sim k_\perp^z$ with

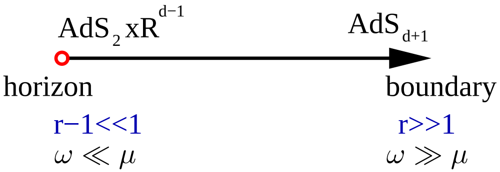
$$z = 2.09 \text{ for } q = 1, \Delta = 3/2.$$
$$z = 5.32 \text{ for } q = 0.6, \Delta = 3/2$$

and the residue vanishes.

Emergent quantum criticality

Whence these exponents?

Near-horizon geometry of black hole is $AdS_2 \times \mathbb{R}^{d-1}$.
The conformal invariance of this metric is **emergent**.
(We broke the microscopic conformal invariance with finite density.)



AdS/CFT says that the low-energy physics is governed by the dual **IR CFT**.

The bulk geometry is a picture of the RG flow from the CFT_d to this NRCFT.

Analytic understanding of Fermi surface behavior: idea

$T = 0$: Expanding the wave equation in ω is delicate.

The ω-term dominates near the horizon.

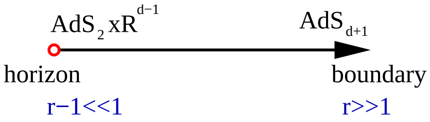
Method of matched asymptotic expansions:

Find solution (in ω-expansion) in two regions of BH geometry (IR and UV),

match their behavior in the region of overlap.

Familiar from the brane absorption calculations which led to AdS/CFT.

[Klebanov, Gubser, Maldacena, Strominger...]



Here: this ‘matching’ can be interpreted in the QFT as RG matching between UV and IR CFTs.

Analytic understanding of Fermi surface behavior: results

$$G_R(\omega, k) = \frac{b_+^{(0)} + \omega b_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(b_-^{(0)} + \omega b_-^{(1)} + O(\omega^2)\right)}{a_+^{(0)} + \omega a_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left(a_-^{(0)} + \omega a_-^{(1)} + O(\omega^2)\right)}$$

The location of the Fermi surface ($a_+^{(0)}(k = k_F) = 0$) is determined by short-distance physics (analogous to band structure –

$a, b \in \mathbb{R}$ from normalizable sol'n of $\omega = 0$ Dirac equation in full BH)

but the low-frequency scaling behavior near the FS is universal

(determined by near-horizon region – IR CFT $\mathcal{G}$).

$\mathcal{G} = c(k)\omega^{2\nu}$ is the retarded G_R of the op to which $\mathcal{O}_{\mathrm{F}}$ matches.

its scaling dimension is $\nu + \frac{1}{2}$, with (for $d = 2 + 1$)

$$\nu \equiv L_2\sqrt{m^2 + k^2 - q^2/2}$$

L_2 is the 'AdS radius' of the IR AdS_2.

Inner region (IR data) in more detail

$$\mathcal{G}_R(\omega) = \overbrace{e^{-i\pi\nu} \frac{\Gamma(-2\nu)\,\Gamma\left(1+\nu-iqe_d\right)}{\Gamma(2\nu)\,\Gamma\left(1-\nu-iqe_d\right)} \cdot \frac{(m+i\tilde{m})\,R_2 - iqe_d - \nu}{(m+i\tilde{m})\,R_2 - iqe_d + \nu}}^{c(k)} (2\omega)^{2\nu}$$

The AdS_2 Green's functions look like DLCQ of 1+1d CFT.

Leftmoving bit depends on q, rightmoving bit depends on ω.

qv [Azeyanagi et al, Guica et al, de Boer et al]

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$T \neq 0$: near-horizon geometry is a BH in AdS_2

$\omega^{2\nu}$ is the $T \to 0$ limit of

$$T^{2\nu}g(\omega/T) = (2\pi T)^{2\nu}\frac{\Gamma\left(\frac{1}{2}+\nu-\frac{i\omega}{2\pi T}+iqe_d\right)}{\Gamma\left(\frac{1}{2}-\nu-\frac{i\omega}{2\pi T}+iqe_d\right)}$$

DLCQ of 1+1d CFT at $T > 0$.

Consequences for Fermi surface

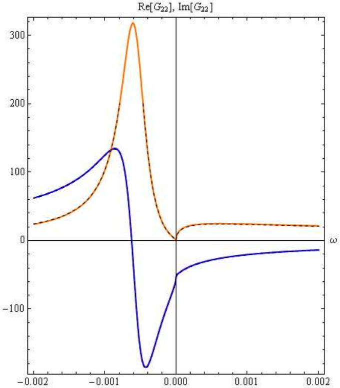
$$G_R(\omega, k) = \frac{h_1}{k_\perp - \frac{1}{v_F}\omega - h_2 c(k)\omega^{2\nu_{k_F}}}$$

$h_{1,2}, v_F$ real, UV data.

The AdS_2 Green's function

is the self-energy $\Sigma = \mathcal{G} = c(k)\omega^{2\nu}$!

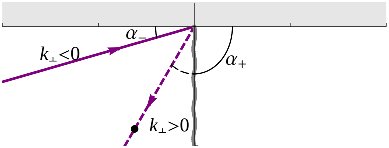
Correctly fits numerics near FS:



$\nu < \frac{1}{2}$: non-Fermi liquid

$$G_R(\omega, k) = \frac{h_1}{k_\perp - \frac{1}{v_F}\omega - h_2 \omega^{2\nu_{k_F}}}$$

$$\text{if } \nu_{k_F} < \frac{1}{2}, \quad \omega_\star(k) \sim k_\perp^z, \qquad z = \frac{1}{2\nu_{k_F}} > 1$$



$$\frac{\Gamma(k)}{\omega_\star(k)} \overset{k_\perp \to 0}{\longrightarrow} \text{const}, \qquad Z \propto k_\perp^{\frac{1-2\nu_{k_F}}{2\nu_{k_F}}} \overset{k_\perp \to 0}{\longrightarrow} 0.$$

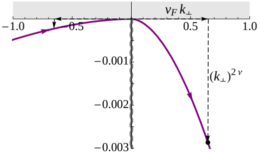
Not a stable quasiparticle.

$\nu > \frac{1}{2}$: Fermi liquid

Suppose $\nu_{k_F} > \frac{1}{2}$: $\longrightarrow$ $\mathcal{O}_{k_F}$ is **irrelevant** $\quad \delta_k = \frac{1}{2} + \nu_k > 1$.

$$G_R(\omega, k) = \frac{h_1}{k_\perp + \frac{1}{v_F}\omega + c_k \omega^{2\nu_{k_F}}}$$

$$\omega_\star(k) \sim v_F k_\perp$$

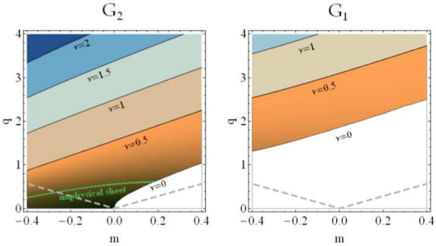


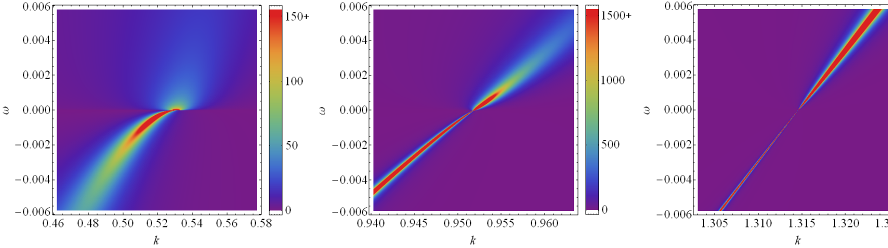
$$\frac{\Gamma(k)}{\omega_\star(k)} \propto k_\perp^{2\nu_{k_F}-1} \overset{k_\perp \to 0}{\longrightarrow} 0 \qquad Z \overset{k_\perp \to 0}{\longrightarrow} h_1 v_F .$$

A stable quasiparticle, but never Landau Fermi liquid.
(different thermo, transport.)

summary

Depending on the dimension of the operator $(\nu + \frac{1}{2})$ in the IR CFT, we find Fermi liquid behavior (but not Landau) or non-Fermi liquid behavior:





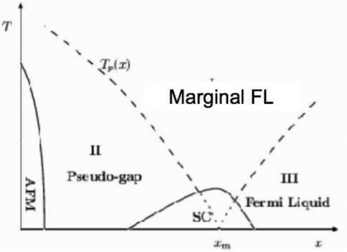
$\nu < \frac{1}{2}$, $\nu = \frac{1}{2}$, $\nu > \frac{1}{2}$

$\nu = \frac{1}{2}$: Marginal Fermi liquid

$$G_R \approx \frac{h_1}{k_\perp + \tilde{c}_1 \omega \ln \omega + c_1 \omega}, \quad \tilde{c}_1 \in \mathbb{R}, \quad c_1 \in \mathbb{C}$$

$$\frac{\Gamma(k)}{\omega_\star(k)} \overset{k_\perp \to 0}{\to} \text{const}, \qquad Z \sim \frac{1}{|\ln \omega_\star|} \overset{k_\perp \to 0}{\to} 0.$$

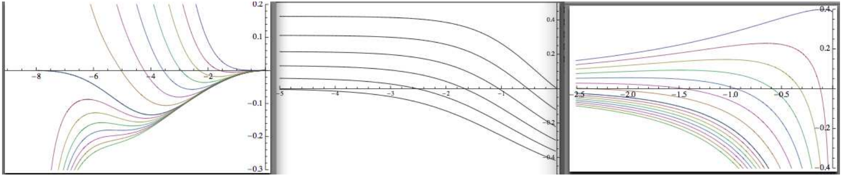
A well-named phenomenological model of high-T_c cuprates near optimal doping



[Varma et al, 1989].

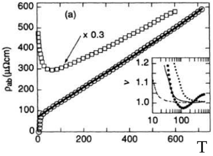
UV data: where are the Fermi sufaces?

Above we supposed $a(k_F)_+^{(0)} = 0$. This happens at k_F: k s.t. $\exists$ normalizable, incoming solution at $\omega = 0$:
This black hole can acquire 'inhomogenous fermionic hair'



Schrodinger potential $V(\tau)/k^2$ at $\omega = 0$ for $m < 0, m = 0, m > 0$.
τ is the tortoise coordinate   Right ($\tau = 0$) is boundary; left is horizon.
$k > qe_d$: Potential is always positive
$k < k_{osc} \equiv \sqrt{(qe_d)^2 - m^2}$: near the horizon $V(x) = \frac{\alpha}{\tau^2}$, with $\alpha < -\frac{1}{4}$ ("oscillatory region")
$k \in (qe_d, k_{osc})$: the potential develops a potential well, indicating possible existence of a zero energy bound state.
Note: *can* exist on asymp. flat BH [Hartman-Song-Strominger 0912]

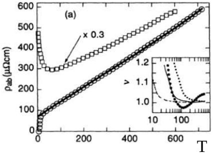
Charge transport



Most prominent mystery $\longrightarrow$
of strange metal phase: $\sigma_{\rm DC} \sim T^{-1}$
$(j = \sigma E)$

e-e scattering: $\sigma \sim T^{-2}$, e-phonon scattering: $\sigma \sim T^{-5}$, **nothing**: $\sigma \sim T^{-1}$

Charge transport



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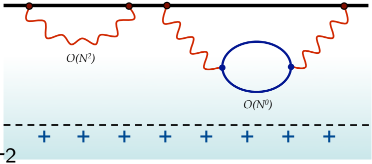
We can compute the contribution
to the conductivity from
the Fermi surface. [Faulkner, Iqbal, Liu, JM, Vegh]



Note: this is not the dominant contribution. $\rightarrow$

$$\sigma_{\rm DC} = \lim_{\omega \to 0} {\rm Im} \frac{1}{\omega} \langle j^x j^x \rangle (\omega, \vec{0}) = N^2 \frac{T^2}{\mu^2} + N^0 \left(\sigma_{\rm DC}^{\rm FS} + \ldots \right)$$

Charge transport by holographic non-Fermi liquids

slight complication: gauge field a_x mixes with metric perturbations.

There's a big charge density. Pulling on it with $\vec{E}$ leads to momentum flow.

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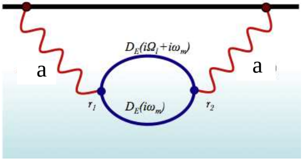
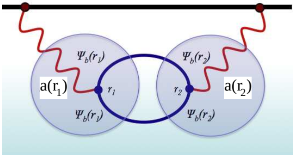


key step: $\operatorname{Im} D_{\alpha\beta}(\Omega, k; r_1, r_2) = \frac{\psi^b_\alpha(\Omega,k,r_1)\bar{\psi}^b_\beta(\Omega,k,r_2)}{W_{ab}} A(\Omega, k)$

bulk spectral density $\operatorname{Im} D$ ...

1. ... is determined by bdy fermion spectral density, $A(\omega, k) = \operatorname{Im} G_R(\omega, k)$
2. ... factorizes on normalizable bulk sol'ns ψ^b



Charge transport by holographic non-Fermi liquids

like Fermi liquid calculation

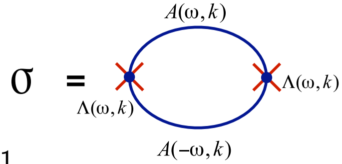
but with extra integrals over r, and no vertex corrections.

$$\sigma_{\mathrm{DC}}^{\mathrm{FS}} = C\int_0^\infty dk k \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{df}{d\omega} \Lambda^2(k,\omega) A^2(\omega,k)$$

$f(\omega) = \frac{1}{e^{\frac{\omega}{T}}+1}$: the Fermi distribution function

Λ: an effective vertex, data analogous to $v_F, h_{1,2}$.

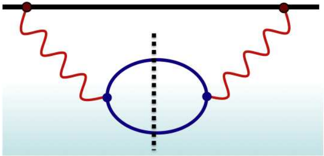
$\Lambda \sim q\int_{r_0}^{\infty} dr \sqrt{g} g^{xx} a_x(r,0) \frac{\bar{\psi}^b(r)\Gamma^x\psi^b(r)}{W_{ab}} \sim$ const.

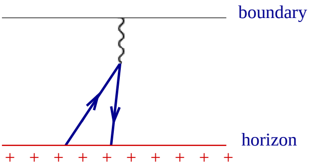


$$\int dk A(k,\omega)^2 \sim \frac{1}{T^{2\nu} g(\omega/T)}$$

scale out T-dependence $\implies \sigma^{DC} \sim T^{-2\nu}$.

Dissipation mechanism

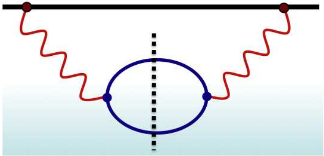


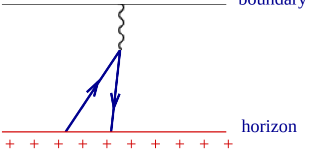


$\sigma_{\mathrm{DC}} \propto \mathrm{Im}\langle jj \rangle$ comes from fermions falling into the horizon.
dissipation of current is controlled by the decay of the fermions into the AdS_2 DoFs.
$\Longrightarrow$ single-particle lifetime controls transport.

marginal Fermi liquid: $\nu = \frac{1}{2} \Longrightarrow$ $\boxed{\rho_{FS} = \left(\sigma^{DC}\right)^{-1} \sim T}$.

Dissipation mechanism





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marginal Fermi liquid: $\nu = \frac{1}{2} \Longrightarrow$ $\boxed{\rho_{FS} = \left(\sigma^{DC}\right)^{-1} \sim T}$.

The optical conductivity $\sigma(\Omega)$ can distinguish the existence of quasiparticles ($\nu > \frac{1}{2}$) through the presence of a transport peak.

Questions regarding the stability of this state

Charged AdS black holes and frustration

Entropy density of black hole:

$$s(T=0) = \frac{1}{V_{d-1}} \frac{A}{4G_N} = 2\pi e_d \rho. \qquad (e_d \equiv \frac{g_F}{\sqrt{2d(d-1)}})$$

This is a large low-energy density of states!

not supersymmetric ... lifted at finite N

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pessimism: $S(T=0) \neq 0$ violates third law of thermodynamics, unphysical, weird string-theorist nonsense.

optimism:

we're describing the state where the SC instability is removed by hand

(**here:** don't include charged scalars, **expt:** large $\vec{B}$).

Can we get this behavior w/o the large low-E density of states?

Presumably: Small-freq behavior depended on existence of IR CFT, not large $c \propto s(T=0)$ of IR CFT.

[Hartnoll-Polchinski-Silverstein-Tong, 0912.]: bulk density of fermions modifies extreme near-horizon region (out to $\delta r \sim e^{-N^2}$), removes residual entropy.

Stability of the groundstate

Charged bosons: In many explicit dual pairs, $\exists$ charged scalars.

• At small T, they can condense spontaneously breaking the $U(1)$ symmetry, changing the background [Gubser, Hartnoll-Herzog-Horowitz].

spinor: $G_R(\omega)$ has poles only in LHP of ω [Faulkner-Liu-JM-Vegh, 0907]

scalar: $\exists$ poles in UHP $\quad \langle \mathcal{O}(t) \rangle \sim e^{i\omega_\star t} \propto e^{+\mathrm{Im}\,\omega_\star t}$

$\Longrightarrow$ growing modes of charged operator: holographic superconductor

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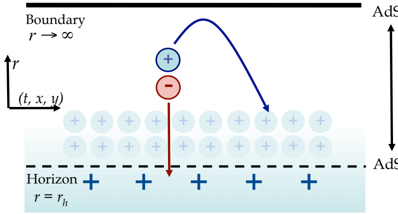
$\Longrightarrow$ growing modes of charged operator: holographic superconductor

why: black hole *spontaneously* emits charged particles [Starobinsky, Unruh, Hawking].
AdS is like a box: they can't escape.
Fermi: negative energy states get filled.
Bose: the created particles then cause *stimulated emission* (superradiance).
A holographic superconductor is a "black hole laser".



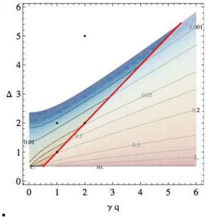
Stability of the groundstate, cont'd

- If their mass/charge is big enough, they don't condense.

[Denef-Hartnoll]:

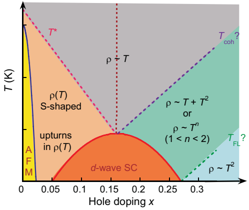
(vs: a weakly-coupled charged boson at $\mu \neq 0$ will condense.)

Finding such string vacua is like moduli stabilization.



- Many systems to which we'd like to apply this also have a superconducting region.

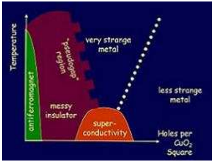
Other light bulk modes (*e.g.* neutral scalars) can also have an important effect on the groundstate [Mulligan, Kachru, Polchinski].

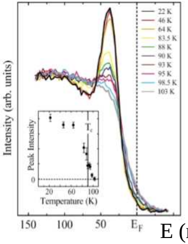


Photoemission 'exp'ts' on holographic superconductors

So far: a model of
some features of the normal state.

In SC state: a sharp peak forms in $A(k, \omega)$.





Photoemission ‘exp’ts’ on holographic superconductors

So far: a model of
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In SC state: a sharp peak forms in $A(k, \omega)$.

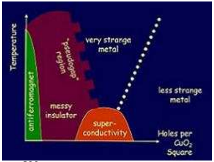
With a suitable coupling between ψ and φ,
the superconducting condensate
opens a gap in the fermion spectrum.

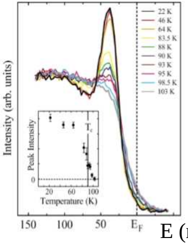
[Faulkner, Horowitz, JM, Roberts, Vegh]

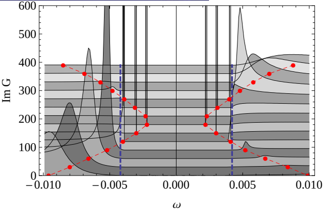
if $q_\varphi = 2 q_\psi$ we can have

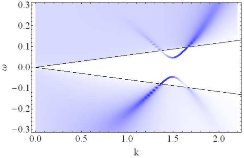
$$L_{\rm bulk} \ni \eta_5 \varphi \bar\psi \mathcal{C} \Gamma^5 \bar\psi^T + {\rm h.c}$$

The (gapped) quasiparticles
are exactly stable in a certain
kinematical regime
(outside the lightcone of the IR CFT) –
the condensate lifts the IR CFT modes
into which they decay.

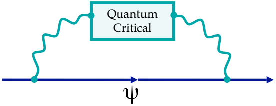








framework for strange metal

a cartoon of the mechanism: 

a similar picture has been advocated by [Varma et al]

comparison

- a Fermi surface coupled to a critical boson field

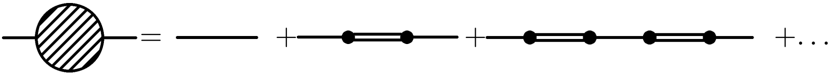
$$L = \bar{\psi}\,(\omega - v_F k)\,\psi + \bar{\psi}\psi a + L(a)$$

small-angle scattering dominates.

- a Fermi surface mixing with a bath of critical fermionic fluctuations with large dynamical exponent [FLMV 0907.2694, Faulkner-Polchinski 1001.5049, FLMV+Iqbal 1003.1728]

$$L = \bar{\psi}\,(\omega - v_F k)\,\psi + \bar{\psi}\chi + \psi\bar{\chi} + \bar{\chi}\mathcal{G}^{-1}\chi$$

χ: IR CFT operator



$$\langle\bar{\psi}\psi\rangle = \frac{1}{\omega - v_F k - \mathcal{G}} \qquad \mathcal{G} = \langle\bar{\chi}\chi\rangle = c(k)\omega^{2\nu}$$

$\nu \le \frac{1}{2}$: $\bar{\psi}\chi$ coupling is a relevant perturbation.

Concluding remarks

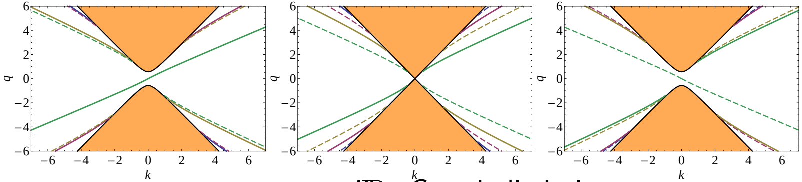
1. The green's function near the FS is of the form ('local quantum criticality', analytic in k.) found previously in perturbative calculations, but the nonanalyticity can be order one.
2. This is an *input* of many studies (dynamical mean field theory)
3. The knowledge of quantum statistics displayed by the classical wave equations is remarkable and necessary for AdS/CFT to be consistent with basic facts about many-body physics.
4. [Denef-Hartnoll-Sachdev] The leading N^{-1} contribution to the free energy exhibits quantum oscillations in a magnetic field.
5. Main challenge: step away from large N. So far:
   - Fermi surface is a small part of a big system.
   - Fermi surface does not back-react on IR CFT.
   - IR CFT has $z = \infty$.

The end.

Thanks for listening.

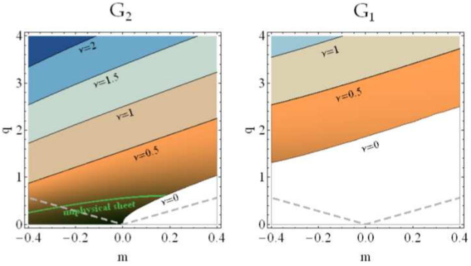
Where are the Fermi sufaces?

$m = -0.4,\ 0,\ 0.4$:

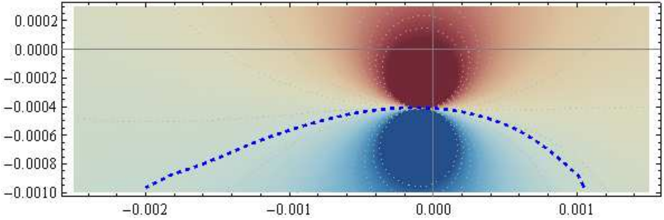


orange: 'oscillatory region': $\nu \in i\mathbb{R}$, G periodic in $\log \omega$

$$\delta_k = \frac{1}{2} + \nu_k, \qquad \nu_k = \frac{1}{\sqrt{6}}\sqrt{m^2 + k^2 - q^2/2}$$



finite temperature



The complex omega plane for $T = 4.13 \times 10^{-4}$:
the quasi-particle pole is a finite distance *below* the real ω-axis.
dashed line: trajectory of the pole between
$k = 0.87$(left)$\ldots 0.93$(right). $\min_k (\mathrm{Im}\,\omega_c) \simeq T$ (up to 1% accuracy).
In background: density plot for $\mathrm{Im}\, G_{22}(\omega)$ at $k = 0.90$
where the corresponding pole is closest to the real axis.
There is a numerical instability for $\mathrm{Im}\,\omega < -\pi T$
(can also be seen directly from the wave equation:
the outgoing solution near the horizon dominates exponentially over the desired incoming solution.)

Fermion poles always in LHP!

$$\arg c_k = \arg\left(e^{2\pi i\nu} \pm e^{-2\pi q e_d}\right) \qquad \mathcal{G} = c_k \omega^{2\nu}$$

$\pm$ for boson/fermion.

$$\omega_c^{2\nu} = \mathrm{real} \cdot \left(e^{-2\pi i\nu} - e^{-2\pi q e_d}\right).$$

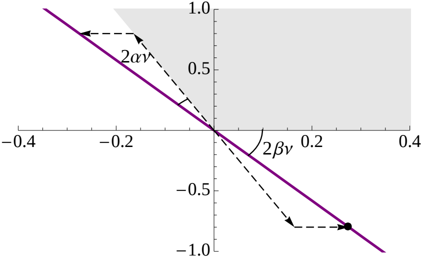


Figure: A geometric argument that poles of the fermion Green function always appear in the lower-half ω-plane: Depicted here is the $\omega^{2\nu}$ covering space on which the Green function is single-valued. The shaded region is the image of the upper-half ω-plane of the physical sheet.

fermi velocity

Think of $\omega = 0$ Dirac eqn as Schrödinger problem.
Like Feynman-Hellmann theorem: $\partial_k \langle H \rangle = \langle \partial_k H \rangle$
we can derive a formula for v_F in terms of expectation values in the bound-state wavefunction $\Phi^+_{(0)}$.
Let:

$$\langle \mathcal{O} \rangle \equiv \int_{r_\star}^{\infty} dr \sqrt{g_{rr}} \mathcal{O} \ ,$$

$$J^\mu \equiv \bar{\Phi}^+_{(0)} \ \partial_{k_\mu} \not{D}_{0,k_F} \Phi^+_{(0)} = \bar{\Phi}^+_{(0)} \Gamma^\mu \Phi^+_{(0)}$$

is the bulk particle-number current.

$$v_F = \frac{\langle J^1 \rangle}{\langle J^0 \rangle} = \frac{\int dr \sqrt{g_{rr} g^{ii}} \left(|y|^2 - |z|^2 \right)}{\int dr \sqrt{g_{rr}(-g^{tt})} \left(|y|^2 + |z|^2 \right)} .$$

$$\Phi = \begin{pmatrix} y \\ z \end{pmatrix}$$

Note: $\frac{g^{ii}}{-g^{tt}} = f(r) \leq 1$ implies that $v_F \leq c$.

fermi velocity

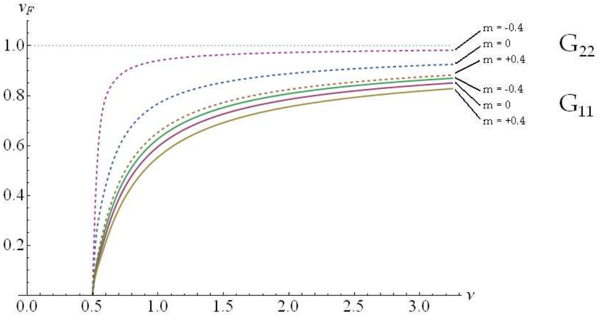


Figure: The Fermi velocity of the primary Fermi surface of various components as a function of $2\nu > 1$ for various values of m.

An explanation for the particle-hole symmetry

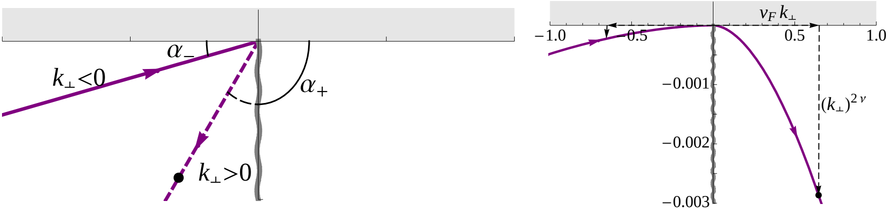


Figure: Left: Motion of poles in the $\nu < \frac{1}{2}$ regime. As k varies towards k_F, the pole moves in a straight line (hence $\Gamma \sim \omega_c$), and hits the branch point at the origin at $k = k_F$. After that, depending on $\gamma(k_F)$, it may move to another Riemann sheet of the ω-plane, as depicted here. In that case, no resonance will be visible in the spectral weight for $k > k_F$. Right: Motion of poles in the $\nu > \frac{1}{2}$ regime, which is more like a Fermi liquid in that the dispersion is linear in $k_\perp$; the lifetime is still never of the Landau form.

Note: the location of the branch cut is determined by physics:

at $T > 0$, it is resolved to a line of poles.

Oscillatory region

Above we assumed $\nu = R_2\sqrt{m^2 + k^2 - (qe_d)^2} \in \mathbb{R}$

$$\nu = i\lambda \quad \Leftrightarrow \text{Oscillatory region.}$$

This is when particle production occurs in AdS_2. [Pioline-Troost]
Effective mass below BF bound *in* AdS_2. [Hartnoll-Herzog-Horowitz]
$\mathrm{Re}\,\omega^{i2\lambda} = \sin 2\lambda \log \omega \implies$ periodic in $\log \omega$ with period $\frac{\pi}{|\nu|}$.
comments about boson case:
Net flux into the outer region $> 0 =$ superradiance of AdS RN black hole (rotating brane solution in 10d)
Classical equations know quantum statistics!
like: statistics functions in greybody factors
Required for consistency of AdS/CFT!
boson: particles emitted from near-horizon region, bounce off AdS_{d+1} boundary and return, causing further stimulated emission.
spinor: there is particle production in AdS_2 region, but net flux into the outer region is negative ('no superradiance for spinors').

oscillatory region and log-periodicity

When $\nu(k)$ is imaginary, $\mathcal{G} \sim \omega^{\nu}$ is periodic in $\log \omega$.

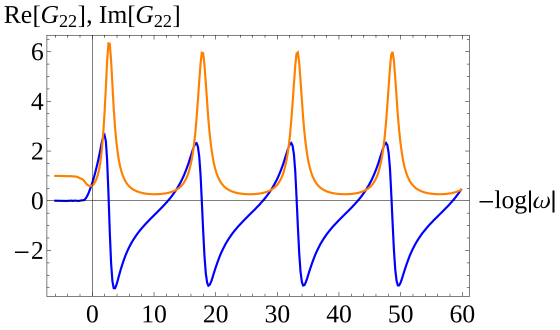


Figure: Both $\mathrm{Re}\, G_{22}(\omega, k = 0.5)$ (blue curve) and $\mathrm{Im}\, G_{22}(\omega, k = 0.5)$ (orange) are periodic in $\log \omega$ as $\omega \to 0$.

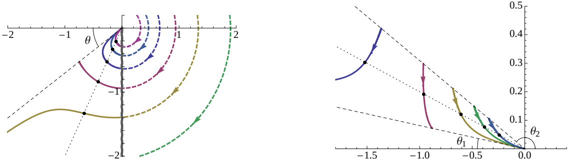
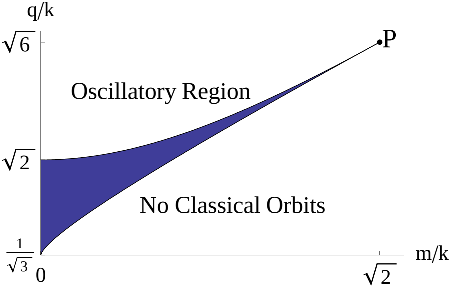


Figure: The motion of poles of the Green functions of spinors (left) and scalars (right) in the complex frequency plane. Both plots are for parameter values in the oscillatory region ($q = 1, m = 0$). In order to give a better global picture, the coordinate used on the complex frequency plane is $s = |\omega|^{\frac{1}{20}} \exp(i \arg(\omega))$. The dotted line intersects the locations of the poles at $k = k_0 = ...$, and its angle with respect to the real axis is determined by $\mathcal{G}(k, \omega)$. The dashed lines in the left figure indicate the motion of poles on another sheet of the complex frequency plane at smaller values of $k < k_0$. As k approaches the boundary of the oscillatory region, most of the poles join the branch cut. It seems that one pole that becomes the Fermi surface actually manages to stay in place. These plots are only to be trusted near $\omega = 0$.



Information from WKB. At large q, m, the primary Fermi momentum is given by the WKB quantization formula: $k_F \int_{s_-}^{s^+} ds \sqrt{V(s;\alpha,\beta)} = \pi$, where $\alpha \equiv \frac{q}{k}, \beta \equiv \frac{m}{k}$, s is the tortoise coordinate, and $s_\pm$ are turning points surrounding the classically-allowed region. For $k < q/\sqrt{3}$, the potential is everywhere positive, and hence there is no zero-energy boundstate. This line intersects the boundary of the oscillatory region at $k^2 + m^2 = q^2/2$ at the point $P = (\alpha, \beta) = (\sqrt{6}, \sqrt{2})$. Hence, only in the shaded (blue) region is there a Fermi surface. The exponent $\nu(k_F)$ is then given by $\nu(k_F) = \frac{\pi\sqrt{1+\beta^2-\alpha^2/2}}{\int ds \sqrt{V(s;\alpha,\beta)}}$. This becomes ill-defined at the point P, and interpolates between $\nu = 0$ at the boundary of the oscillatory region, and $\nu = \infty$ at $k = q/\sqrt{3}$.