How to get a superconductor out of a black hole

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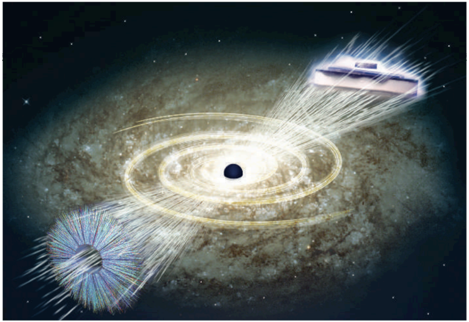

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Quantum Phase Transition:

a phase transition between different quantum phases (phases of matter at $T = 0$). Quantum phase transitions can only be accessed by varying a physical parameter — such as magnetic field or pressure — at $T = 0$.

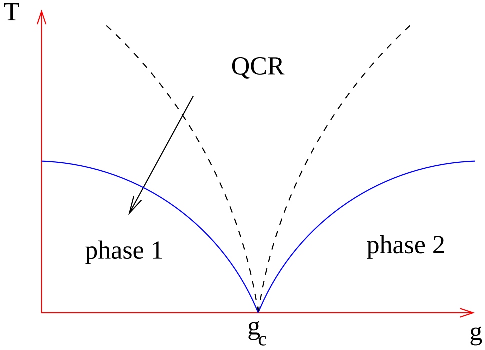


Figure: Phase diagram paradigm

Experimental relevance

Many important physical systems may have quantum critical points (QCPs). The QCP has an effective field theory description which continues to be valid at small "distances" away from the QCP. This quantum critical region may be in an experimentally accessible regime.

Examples:

- superfluid-insulator transition in thin films
- high temperature, under-doped superconductors at $T > T_c$ and the Nernst effect

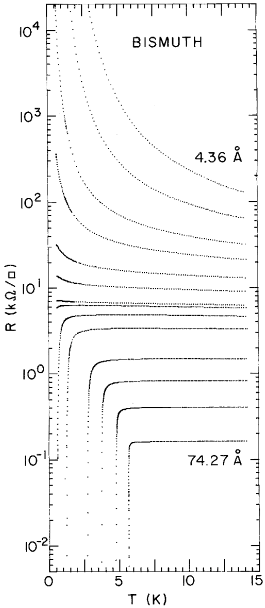
Thin Films

Conductivity σ

$\sigma_{\text{thick}}(T \to 0) = \infty$

$\sigma_{\text{thin}}(T \to 0) = 0$

Haviland, Liu, and Goldman, *Phys. Rev. Lett.*, **62**, 2180 (1989)



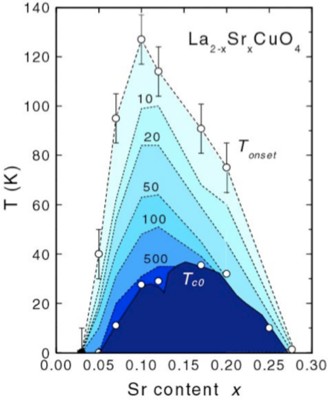
High T_c superconductors

- La_2CuO_4 is an antiferromagnetic insulator
- 2d physics: The Cu atoms arrange themselves into a square lattice on separated sheets.
- Hole doping: substitute some of the La with Sr, $La_{2-x}Sr_xCuO_4$

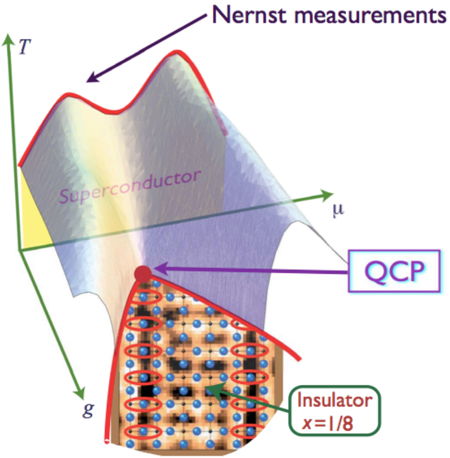
The Nernst effect

- Apply ∇T
- Apply $B \perp \nabla T$
- Measure $E \parallel B \times \nabla T$
- The Nernst coefficient is

$$\nu = \frac{E}{B|\nabla T|}$$



High T_c superconductors and quantum criticality



Comments about Scale Invariance

At the quantum critical point, the system is invariant under

$$t \to \lambda^z t \qquad \text{and} \qquad x \to \lambda x \ .$$

where z is the dynamical critical exponent.

- The Lorentzian case $z = 1$:
  - insulating quantum antiferromagnets (relevant for high T_c)
  - Bose Hubbard-like models at p/q filling (optical lattices)
- The case $z = 2$ is more common (Galilean, Schrödinger, and Lifshitz scaling symmetries)
- Other z, e.g. $z = 3$ for the heavy fermion compounds.

How do we analyze strongly interacting, scale invariant field theories?

The role of AdS/CFT

The AdS/CFT correspondence provides a tool to study a class of strongly interacting field theories with Lorentzian symmetry in d dimensions by mapping the field theories to classical gravity in $d + 1$ dimensions.

- equation of state
- real time correlation functions
- transport properties — conductivities, diffusion constants, etc.

The ambitious program: There may be an example in this class of field theories which describes the quantum critical region of a real world material such as a high T_c superconductor.

The less ambitious program: By learning about this class of field theories, we may find universal features that could hold more generally for QCPs ($\eta/s = \hbar/4\pi k_B$).

There are a few entries in the AdS/CFT dictionary for $z > 1$ (Kachru, Balasubramanian, McGreevy, ...), but here we consider only $z = 1$.

Some Recent History

The AdS/CFT approach to superconductors and superfluids has been evolving:

2007 The first papers modeled the quantum critical region only. (Herzog, Kovtun, Sachdev, Son; Hartnoll, Kovtun, Mueller, Sachdev; Hartnoll, Herzog)

2008 Next we added the physics of a classical thermal phase transition. These simple gravity models were phenomenological in nature. (Gubser; Hartnoll, Herzog, Horowitz)

2009 Do field theory duals to these simple gravity models exist? What is the nature of these field theories? To answer these questions, we have been looking for string embeddings. (Gubser, Herzog, Pufu, Tesileanu; Gauntlett, Sonner, Wiseman)

What happened in 2008

Holographic Phase Transitions

Old Goal: To have a simple holographic model of a phase transition where we can calculate the phase diagram and transport coefficients.

$$S = \frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{-g}(R-2\Lambda) - \frac{1}{4g^2}\int d^{d+1}x\sqrt{-g}F_{\mu\nu}F^{\mu\nu}$$

$$-\int d^{d+1}x\sqrt{-g}\left(|(\partial - iqA)\Psi|^2 + V(|\Psi|)\right) \ .$$

- Einstein-Hilbert produces correlators of the stress tensor $T^{\mu\nu}$ in the boundary theory. ($\Lambda < 0$)
- Maxwell produces correlators of a global current J^μ in the boundary.
- The order parameter is the boundary value of Ψ.

The Phase Structure

Different types of black holes correspond to different phases.

- The high temperature phase is an electrically charged black hole in AdS_{d+1} with $\Psi = 0$. In this phase, the system is not superconducting. In our QCP paradigm, we would be in the quantum critical regime where the most important scales are T and charge density ρ.
- The low temperature phase is an electrically charged black hole in AdS_{d+1} with hair, $\Psi \neq 0$. The system is superconducting/superfluid.

The Hawking temperature of the black hole is T of the field theory. The charge on the black hole translates into ρ in the field theory. These parameters are tuneable, and we can calculate the transport coefficients as a function of T and ρ!

Why there is a phase transition

Assuming $V(\Psi) = m^2|\Psi|^2$, Gubser observed an instability for the scalar to condense when ρ gets too large:

$$m_{\text{eff}}^2 = m^2 + g^{tt} A_t^2$$

where

$$g_{tt} < 0 \; ; \quad A_t \sim \rho \; .$$

The effective mass becomes tachyonic and the scalar condenses in a narrow region of radial coordinate r.

There is no need for a Ψ^4 term because curvature in the radial direction stabilizes the runaway.

There is only one other scale in the problem, the temperature, so large ρ corresponds to small T.

Superfluid or superconductor?

Two interpretations of the instability

- This $U(1)$ symmetry in the field theory is global, and strictly speaking we have only spontaneous symmetry breaking — a superfluid phase transition.
- We can think of the $U(1)$ as being weakly gauged, in which case we have a superconductor.

The phase transition

Given $m^2L^2 = -2$ (above the BF bound), we can choose a scalar in the field theory with scaling dimension one or two.

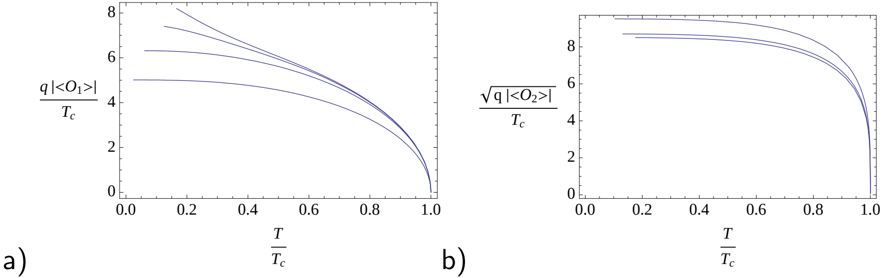


Figure: The value of the condensate as a function of temperature for the two different boundary conditions: a) from bottom to top, the various curves correspond to $q = 1$, 3, 6, and 12; b) from top to bottom, the curves correspond to $q = 3$, 6, and 12. Note that $T_c \sim \sqrt{\rho}$.
Probe limit is large q.

The Conductivity from BCS Theory

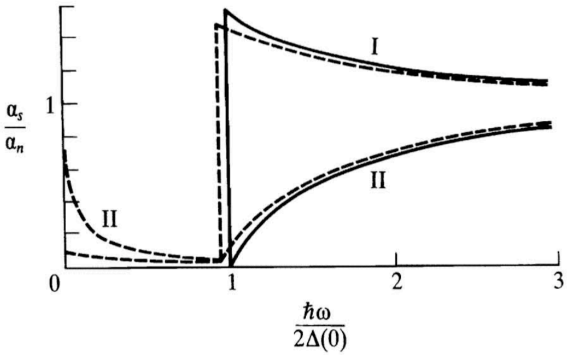


Figure: Frequency dependence of absorption processes obeying case I and II coherence factors at $T = 0$ (solid curves) and $T \approx \frac{1}{2} T_c$ (dashed curves). [Tinkham, Superconductivity, 2nd edition]

Conductivity for dimension one case, probe limit

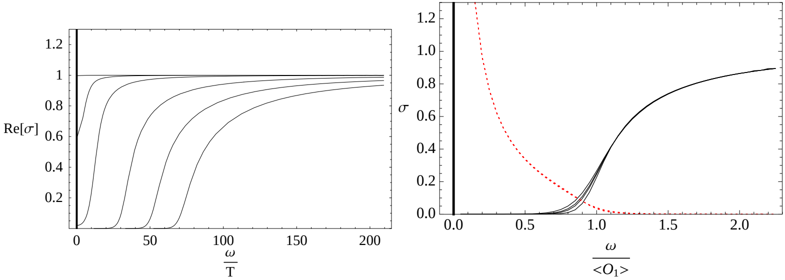


Figure: Left: Plots of the real part of the conductivity versus frequency for various temperatures. Right: Plots of the conductivity versus frequency at very low temperature. The dotted red curve is the $\mathrm{Im}(\sigma)/5$.

$\mathrm{Re}[\sigma(\omega)]$ contains a delta function $\pi n_s \delta(\omega)$ which leads to superconductivity where n_s is the superfluid density.

Conductivity for dimension two case, probe limit

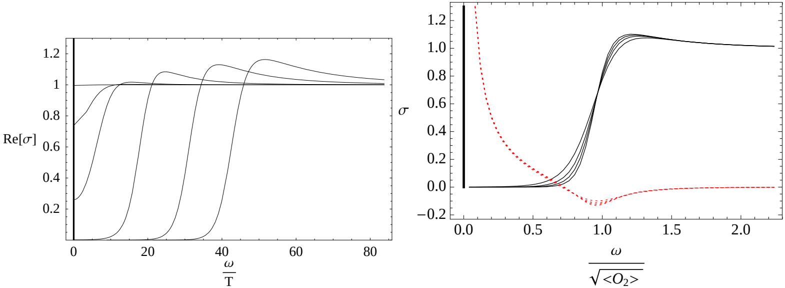


Figure: Left: Plots of the real part of the conductivity versus frequency for various temperatures. Right: Plots of the conductivity versus frequency at very low temperature. The dotted red curve is the $\mathrm{Im}(\sigma)/5$.

One conclusion that may be drawn from these plots is that $\langle O_1 \rangle$ and $\sqrt{\langle O_2 \rangle}$ can be interpreted as twice the superconducting gap.

What's happening in 2009

Finding a Stringy Embedding

The construction of a large class of AdS_5/CFT_4 dualities starts by placing N D3-branes at the tip of a six dimensional Calabi-Yau cone in type IIB string theory.

Generically, the resulting superconformal field theory has a collection of $SU(N)$ gauge groups and bifundamental matter fields that can be described by a quiver.

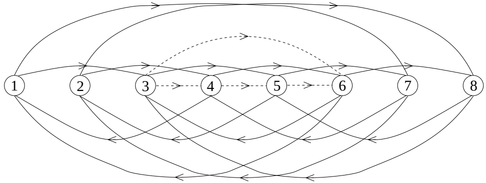


Figure: L^{263}

An important point is that each $SU(N)$ gauge group implies the existence of a gluino with $R = 1$.

A Universal Mode

There exists a gauge invariant, chiral primary operator in all of these quiver gauge theories with $\Delta = 3$ and $R = 2$.

$$\mathcal{O} = \mathcal{W}(\phi_i) + \frac{1}{32\pi i} \sum_j \tau_j \operatorname{tr} \lambda_{j\alpha}^2 \, ,$$

where $\mathcal{W}$ is the superpotential, the gluino field λ_α is the lowest component of the superfield W_α, and the complex scalar fields ϕ_i are the lowest component of the chiral matter superfields Φ_i. The $\tau_j = \theta_j/2\pi + 4\pi i/g_j^2$ are the complexified gauge couplings, and the sum j runs over the gauge groups in the quiver.

Can we figure out what this operator is dual to in gravity?

A Consistent Truncation

A solution to the following 5d effective action lifts to a solution of type IIB supergravity in 10 dimensions.

$$\mathcal{L} = \mathcal{L}_{\rm EM} + \mathcal{L}_{\rm scalar} \quad ; \quad \mathcal{L}_{\rm EM} = R - \frac{L^2}{3} F_{\mu\nu} F^{\mu\nu} + CS \ ,$$

$$\mathcal{L}_{\rm scalar} = -\frac{1}{2}\left[(\partial_\mu \eta)^2 + \sinh^2 \eta (\partial_\mu \theta - 2A_\mu)^2 - \frac{6}{L^2} \cosh^2 \frac{\eta}{2} (5 - \cosh \eta) \right]$$

The real fields η and θ are the modulus and phase of a complex scalar dual to the gluino bilinear $\mathcal{O}$.

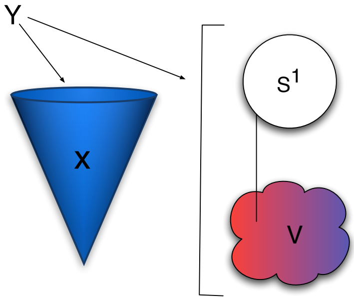
The potential for the scalars looks like a Mexican hat. A simple phenomenological model with this type of potential was already studied in 2008 by Gubser and de la Rocha.

The 10d Lift: The metric

The metric looks like

$$ds^2 = \cosh\frac{\eta}{2}\, ds_M^2 + \frac{L^2}{\cosh\frac{\eta}{2}}\left[ds_V^2 + \cosh^2\frac{\eta}{2}(\zeta^A)^2\right] .$$

Recall that the level surface of a Calabi-Yau cone X is a Sasaki-Einstein manifold Y. Locally, Y looks like a U(1) fibration over a Kähler-Einstein manifold V.



The 10d Lift: The RR 5-form

The guess work was made easier by previous consistent truncations where only the scalars or only the U(1) gauge field were kept.

$$F_5 = \frac{1}{g_s}\left(\mathcal{F} + *\mathcal{F}\right) \ ,$$

$$\begin{aligned}
\mathcal{F} &\equiv -\frac{1}{L}\cosh^2\frac{\eta}{2}\left(\cosh\eta - 5\right)\mathrm{vol}_M - \frac{2L^3}{3}(*_M F)\wedge\omega \\
&\quad + \frac{L^4}{4\cosh^4\frac{\eta}{2}} J\wedge\omega^2 \ , \\
*\mathcal{F} &= L^4\frac{(\cosh\eta - 5)}{2\cosh^2\frac{\eta}{2}}\zeta^A\wedge\omega^2 + \frac{2L^4}{3}F\wedge\zeta^A\wedge\omega \\
&\quad + \frac{L}{2}(*_M J)\wedge\zeta^A \, .
\end{aligned}$$

$J = \sinh^2\eta(d\theta - 2A)$ and ω is the Kähler form on V.

The 10d Lift: The 3-forms

$\hat{\Omega}_3$ is the holomorphic 3-form on the Calabi-Yau X.

$$\hat{\Omega}_3 = r^3 \left(\frac{dr}{r} \wedge \Omega_2 + \Omega_3 \right) ,$$

$$F_2 = L^2 \tanh \frac{\eta}{2} e^{i\theta} \Omega_2 \, , \qquad F_2 \equiv B_2 + i g_s C_2 \, .$$

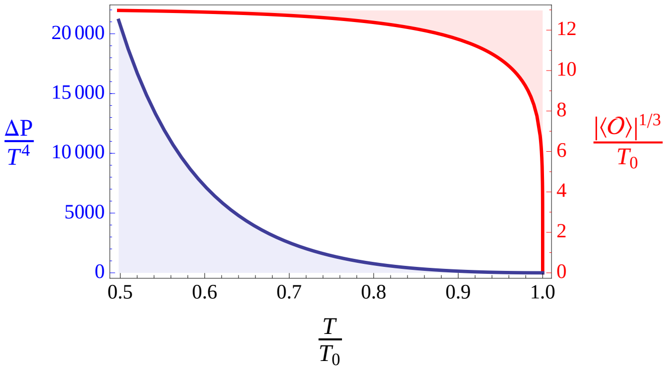


Figure: Upper right plot: $|\langle\mathcal{O}\rangle|^{1/3}/T_0$ vs. T/T_0, where $\langle\mathcal{O}\rangle$ is expressed as multiples of L^3/κ_5^2. The critical temperature is $T_0 \approx 0.0607\mu$. Near T_0, $\langle\mathcal{O}\rangle \sim |T - T_0|^{1/2}$, indicating a mean field critical exponent. Lower left plot: $\Delta P/T^4$ vs. T/T_0, where ΔP is the difference in pressure between the broken and unbroken phases, calculated in the grand canonical ensemble. Near T_0, one has $\Delta P \sim (T - T_0)^2$, so the phase transition is second order.

Does the instability drive the phase transition?

Are there other modes with instabilities that set in at a $T > T_0$?

- Let's ignore vector and tensor modes.
- Let's look at an arbitrary scalar with charge R and mass $m^2 L^2 = \Delta(\Delta - 4)$. (We just assumed m^2 is not corrected by the background charged black hole solution!)
- We can determine the $T_p(R, \Delta)$ at which these other scalars become perturbatively unstable. (If the transition is first order, $T_p < T_c$.)
- What region of the $R\Delta$ plane do we need to worry about?

Hard argument!

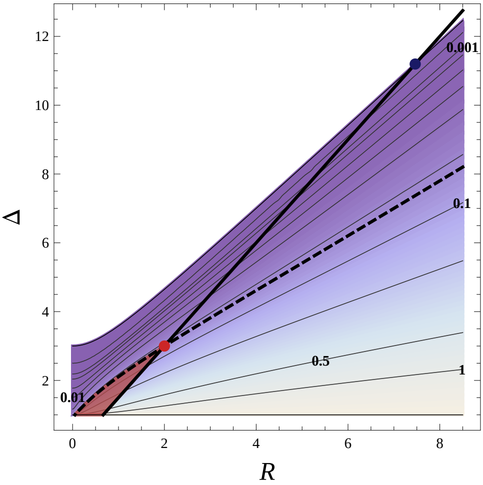


Figure: A contour plot of T_p/μ as a function of Δ and R. The numbers next to the contour lines represent T_p/μ. We need only consider scalars above the unitarity bound, $\Delta \geq 1$. The dark solid line is the BPS bound $\Delta = 3R/2$. Scalars which are less stable than the operator $\mathcal{O}$ are restricted to the triangular, shaded region near the lower-left corner.

Discussion

- Stringy models with an instability to condense a gluino bilinear.
- The condensation involves fermion pairing, morally.
- It is not clear if we have a model where this instability is dominant. The "beta deformation" exists in all quiver theories arising from placing D3-branes at the tip of a toric Calabi-Yau cone. This mode also has $\Delta = 3$, $R = 2$.
- There are some reasons to suspect that nonzero R-charge chemical potential means these quiver theories can only be metastable ($\mathcal{N} = 4$ SYM).
- Reasons for optimism: One might hope that other symmetries, $U(1)_B$ for instance, might allow for similar phase transitions but where one has greater control. Stay tuned!