Desensitizing Inflation from the Planck Scale

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arXiv:1004.3801 with Daniel Baumann

Inflation

Explains the flatness and homogeneity of the universe

Explains the origin of structure

COST: Mechanism is sensitive to Planck scale effects.
Easily ruined by "irrelevant" corrections (η problem)

Sensitive to initial conditions (patch problem)

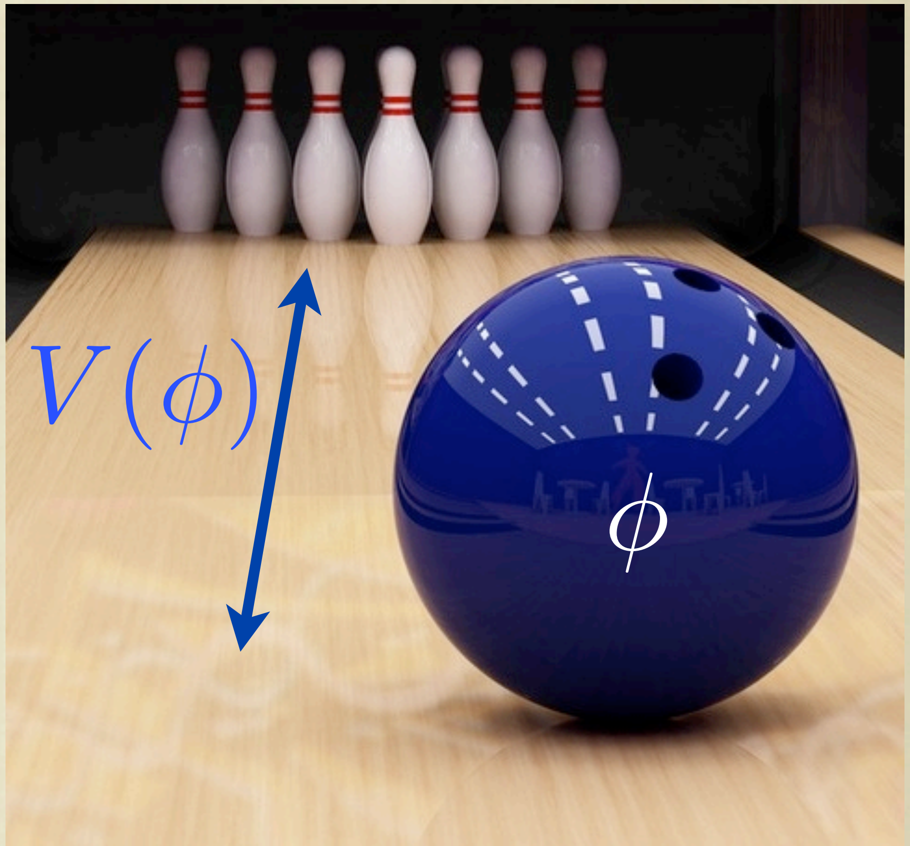
The η Problem

Single field slow roll inflation:

$$S \supset \int d^4x\sqrt{-g}(\partial_\mu\phi\partial^\mu\phi - V(\phi))$$

Inflation = Quasi-de Sitter

$$H^2 \simeq V(\phi)/3M_{pl}^2 \sim V_0/3M_{pl}^2$$



60 e-folds of quasi-de Sitter requires

$$\epsilon \equiv M_{pl}^2(\frac{V'}{V})^2 \ll 1$$

$$\eta \equiv M_{pl}^2\frac{V''}{V} \ll 1$$

The η Problem

Given some $V(\phi)$ with $\eta \equiv M_{pl}^2 \frac{V''}{V} \ll 1$

$$V(\phi) \to V(\phi) + V(\phi)\frac{\phi^2}{M_{pl}^2} \longrightarrow \eta \to \eta + 1$$

More generally, if $V \supset \mathcal{O}_{\Delta=4}\frac{\phi^2}{M_{pl}^2}$

$$\langle \mathcal{O}_\Delta \rangle = cV \to \eta \sim c$$

Inflation sensitive to dimension 5 & 6 Planck suppressed operators (at least)

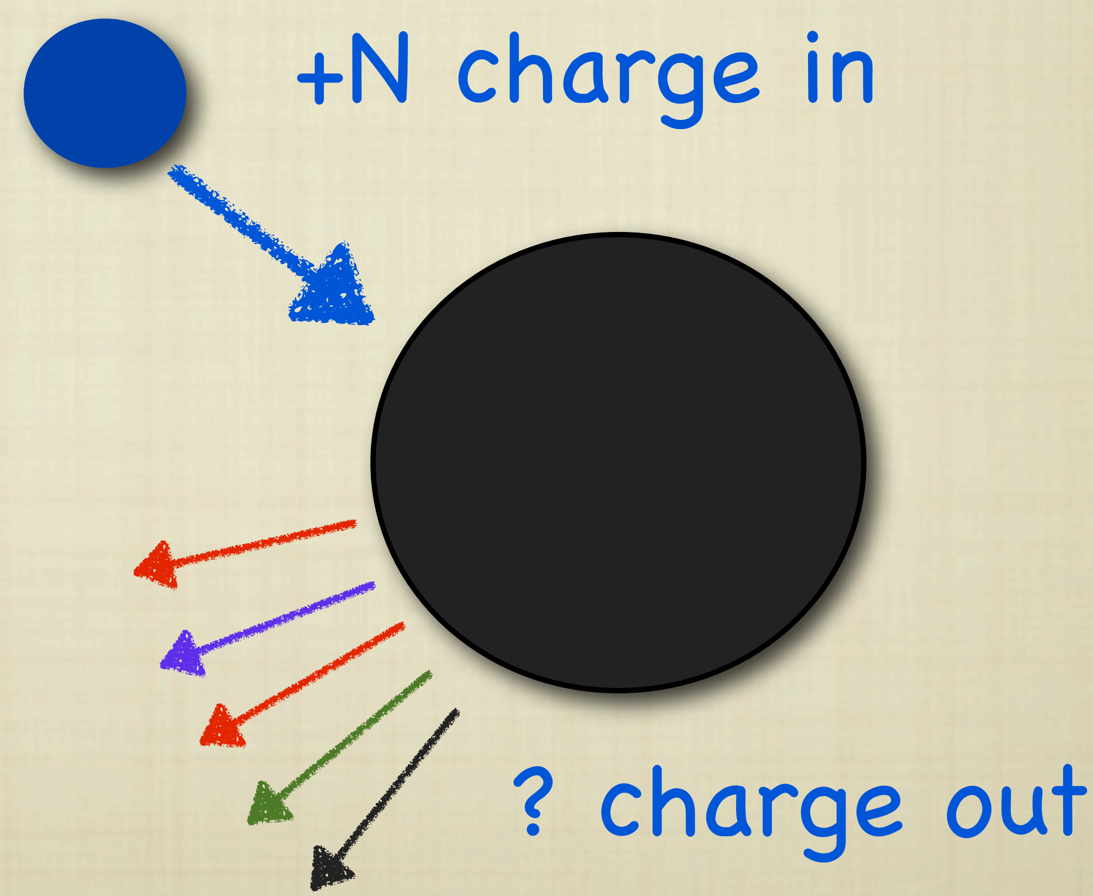
UV Completions

Planck sensitivity of η $\xrightarrow{?}$ Need UV Completion

Global symmetries ?

Forbid dimension 5 & 6 operators that violate symmetry

Black hole evaporation
violates global symmetries



Existence of symmetry requires UV completion

Renormalization

Energy Scale of Inflation $\Lambda_{Inf} \ll M_{pl}$

$$\mathcal{L} \supset c(\mu) V(\phi) \frac{\phi^2}{M_{pl}^2}$$

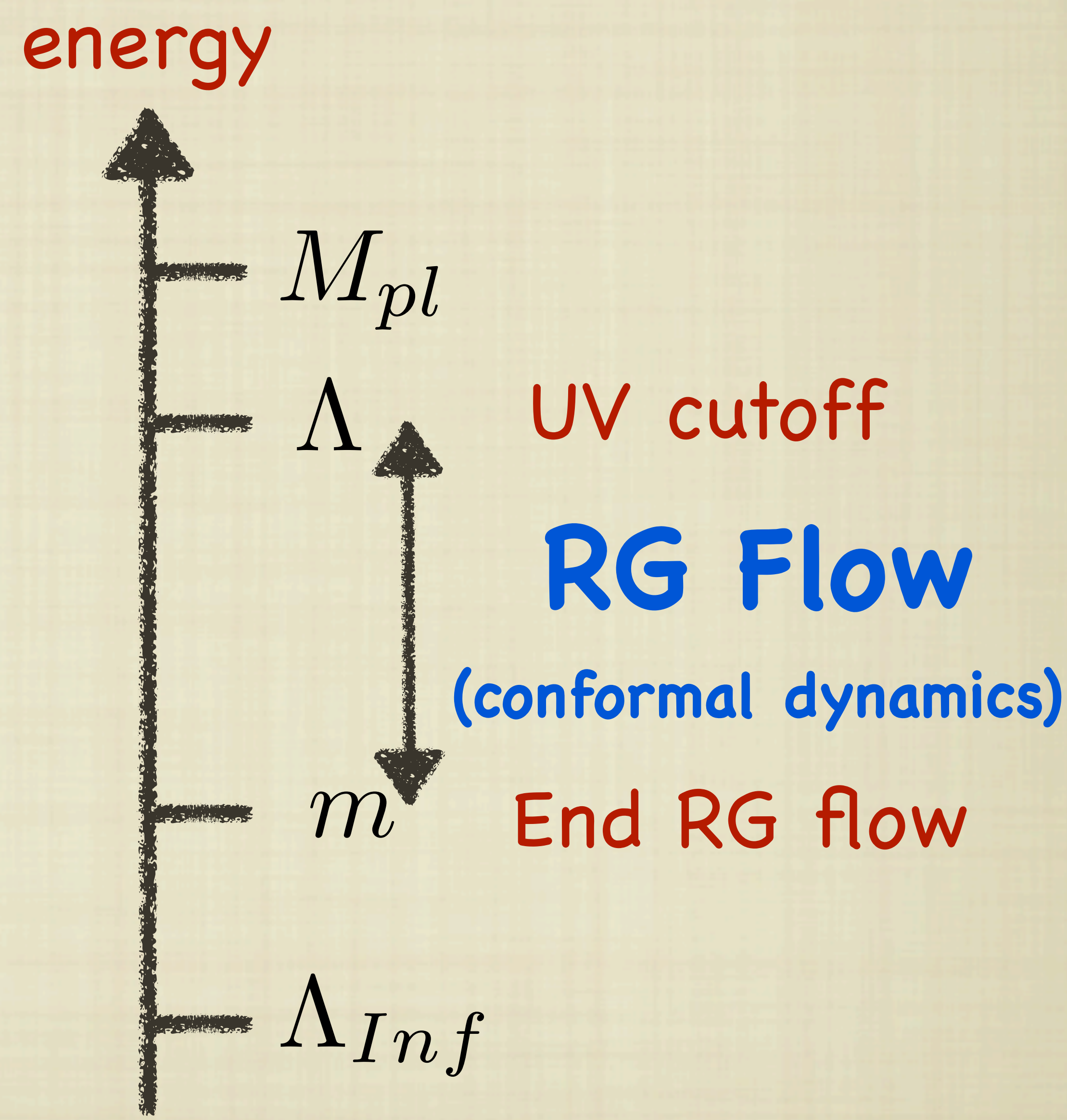
Function of Energy Scale

Possible Solution:

$$c(M_{pl}) \sim \mathcal{O}(1)$$

BUT

$$\eta \sim c(\Lambda_{Inf}) \lesssim 10^{-2}$$



Particle physics: RG used to explain 10^{-3} to 10^{-32}!!

Field Theory "Solution"

Plan: Suppress dim 6 operators by coupling to a CFT

Inflaton acquires anomalous dimension γ

$$c(\Lambda_{Inf}) \simeq (\frac{m}{\Lambda})^{2\gamma} c(M_{pl})$$

Goal: Suppress all dangerous operators

Requirement: No "exotic" field theories

Only phenomena found in the SM

Recipe

RG on it's own is not enough

To eliminate all $\mathcal{O}(1)$ contributions to η

we will need:

- Approximate Global Symmetry (inflaton is PNGB)
- $\mathbb{Z}_2$ symmetry* (eliminate dim 5)
- Couplings that generate RG (suppress dim 6)

* $\mathbb{Z}_2$ not necessary in some strongly coupled models

Outline

- Concrete SUSY model
- Failure modes for general models
- Non-SUSY extensions

The SUSY η Problem

In SUGRA: $$V = e^{K/M_{\rm pl}^2}\left[K^{\phi\bar\phi}D_\phi W\overline{D_\phi W} - \frac{3}{M_{\rm pl}^2}|W|^2\right]$$

F-term vacuum energy drives inflation: $D_X W \simeq \sigma^2$

$$V = \sigma^4\left[1 + K_{\phi\bar\phi}\frac{\phi\phi^\dagger}{M_{\rm pl}^2} + \cdots\right]$$

Copeland et al.

$K(\phi,\phi^\dagger) = \phi^\dagger\phi$ → $\eta = 1 + \ldots$

Cannot be suppressed by RG (tied to kinetic terms)

Can be cancelled (fine tuned solution)

e.g. Baumann et al.

Shift Symmetries and PNGBs

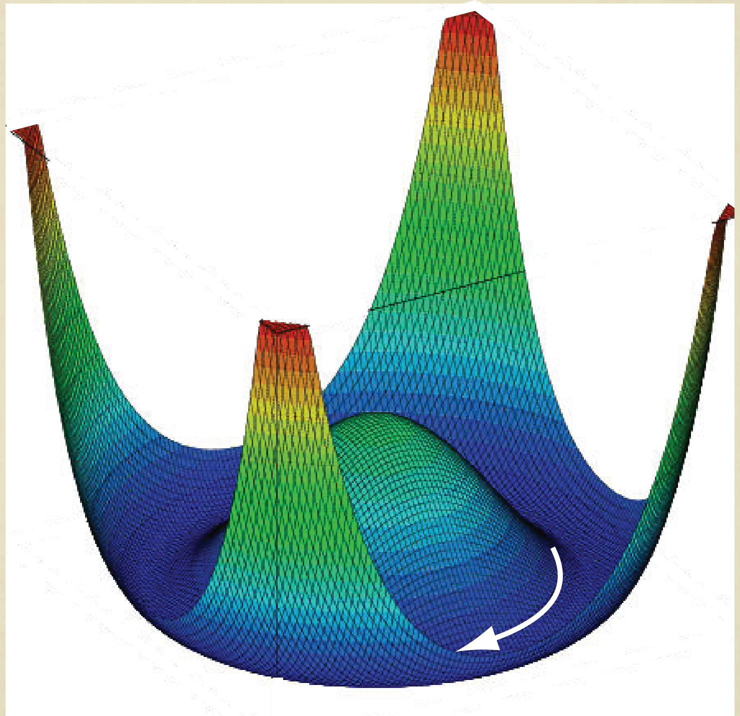
Shift symmetry eliminate SUGRA term, e.g.

Arkani-Hamed et al.; Kaplan & Weiner

$$K(\phi, \phi^\dagger) = (\phi + \phi^\dagger)^2 \longrightarrow \text{No mass for } Im(\phi)$$

Arises naturally in SSB: $\Phi = f e^{\rho + i\phi}$ $K = \Phi^\dagger \Phi \rightarrow f^2 e^{2\rho}$

i.e. a Goldstone nboson coupled to gravity is a Goldstone boson



Mass only generated by explicit symmetry breaking

Use PNGB as inflaton (need approx. symm)

Only need to suppress ~~symmetry~~ operators

A Simple Model

$$W = \lambda_0 S(\Phi\bar{\Phi} - f^2) + \frac{\lambda_1}{2}(\Phi + \bar{\Phi})\psi^2 + \lambda_2 X(\psi^2 - v^2)$$

Arkani-Hamed et al.; Kaplan & Weiner

$$\Phi = (f + \rho)e^{i\varphi/f}$$
$$\bar{\Phi} = (f - \rho)e^{-i\varphi/f}$$

Inflates when

$$\varphi \simeq 0$$

Still more dangerous Kahler potential terms

Dim 5 $\quad c_i \Phi_i \frac{X^\dagger X}{M_{pl}} + c.c. \quad \epsilon \sim c_i \quad \eta \sim c_i \frac{M_{pl}}{f}$

Dim 6 $\quad (k_0 \bar{\Phi}^\dagger \Phi + k_1 \Phi^2 + k_2 \bar{\Phi}^2)\frac{X^\dagger X}{M_{pl}^2} + c.c. \quad \eta \sim k_i$

Discrete Symmetries

Dimension 5 operators give LARGE η

To suppress with RG flow: $(\frac{m}{\Lambda})^\gamma \ll \frac{f}{M_{pl}}$ → $\gamma > 1$

$\gamma > 1$: Inflaton is strongly coupled / composite

Forbid with exact $\mathbb{Z}_2$ where $\Phi_i \rightarrow -\Phi_i$

$$W = \lambda_0(\Phi\bar{\Phi} - f^2) + \frac{\lambda_1}{2}(\Phi + \bar{\Phi})\psi\bar{\psi} + \lambda_2(\psi^2 - v^2)$$

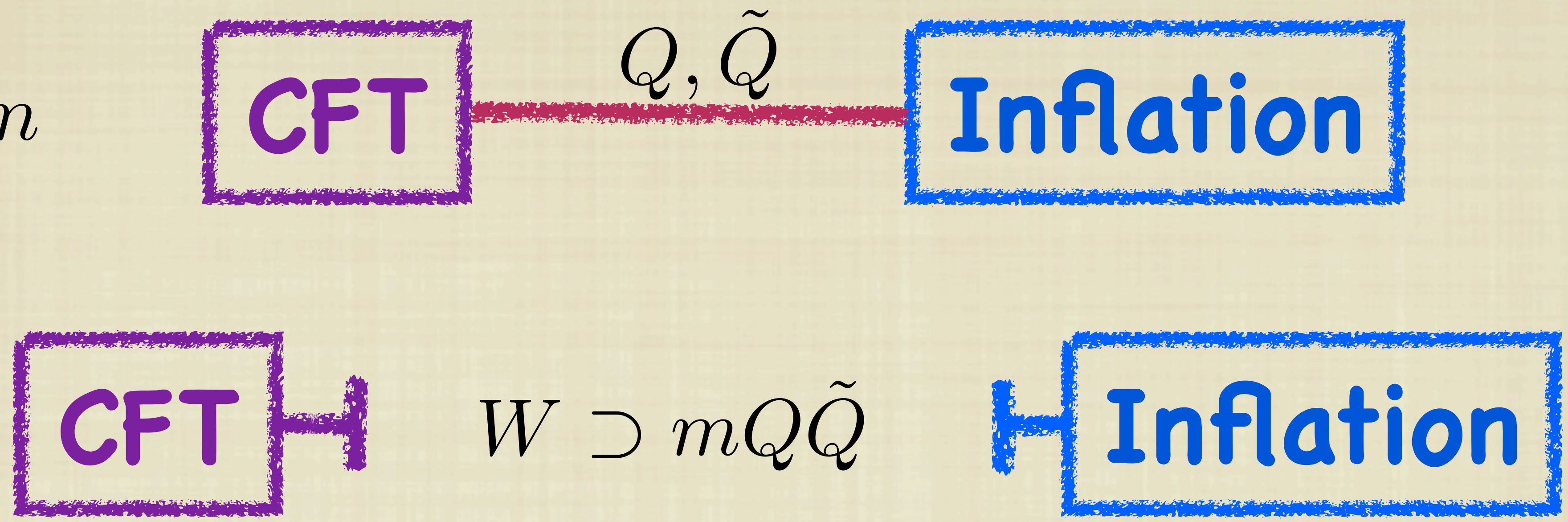
$\Phi, \bar{\Phi} \rightarrow -\Phi, -\bar{\Phi}$ $\qquad$ $\psi \rightarrow -\psi$

UV completions allow exact discrete symmetries

Coupling to CFT

$\Lambda > \mu > m$ $\qquad$ $\gamma_\phi > 0$

$\mu < m$ $\qquad$ $\gamma_\phi \sim 0$



$SU(N_c)$ with $N_f = 3N_c - k$ flavors

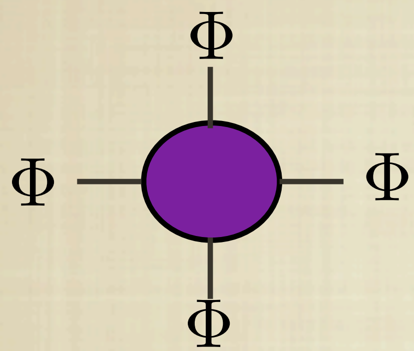
Coupling

$$W_{\rm CFT} = y_1 \sum_{i=1}^{N_1} \tilde{Q}_i Q_i \Phi + y_2 \sum_{j=N_1+1}^{N_2} \tilde{Q}_j Q_j \bar{\Phi} + m \sum_{i=1}^{N_1} \tilde{Q}_i \mathcal{Q}_{N_2+i} + m \sum_{j=N_1+1}^{N_2} \tilde{\mathcal{Q}}_{N_2+j} Q_j$$

$\{Q_i, \tilde{Q}_i\} \to \{-Q_i, +\tilde{Q}_i\}$ $\qquad$ $\{Q_j, \tilde{Q}_j\} \to \{+Q_j, -\tilde{Q}_j\}$ $\qquad$ $\{\mathcal{Q}_k, \tilde{\mathcal{Q}}_k\} \to \{+\mathcal{Q}_k, +\tilde{\mathcal{Q}}_k\}$

Non-Renormalization

Want dangerous couplings to flow to ZERO



$\neq 0 \longrightarrow \quad \lambda\Phi^4 \xrightarrow{\text{RG}} \lambda_* \Phi^4$

Need to know the fixed point, not just dimensions

Can't generate U(1) charged operators: $\Phi^2, \bar{\Phi}^2, \bar{\Phi}^\dagger \Phi$

$$W_{\rm CFT} = y_1^{\bar{i}\bar{j}} \tilde{Q}_i Q_j \Phi + y_2^{\bar{k}\bar{l}} \tilde{Q}_k Q_l \bar{\Phi} + m_1^{\bar{m}\bar{n}} \tilde{Q}_m Q_n + m_2^{\bar{p}\bar{q}} \tilde{Q}_p Q_q$$

Corrections must respect global symmetry

$$(y_1^\dagger y_1)^n (\Phi^\dagger \Phi)^m + (y_2^\dagger y_2)^k (\bar{\Phi}^\dagger \Phi)^l$$

Only non-zero couplings are U(1) invariant!

Model Parameters

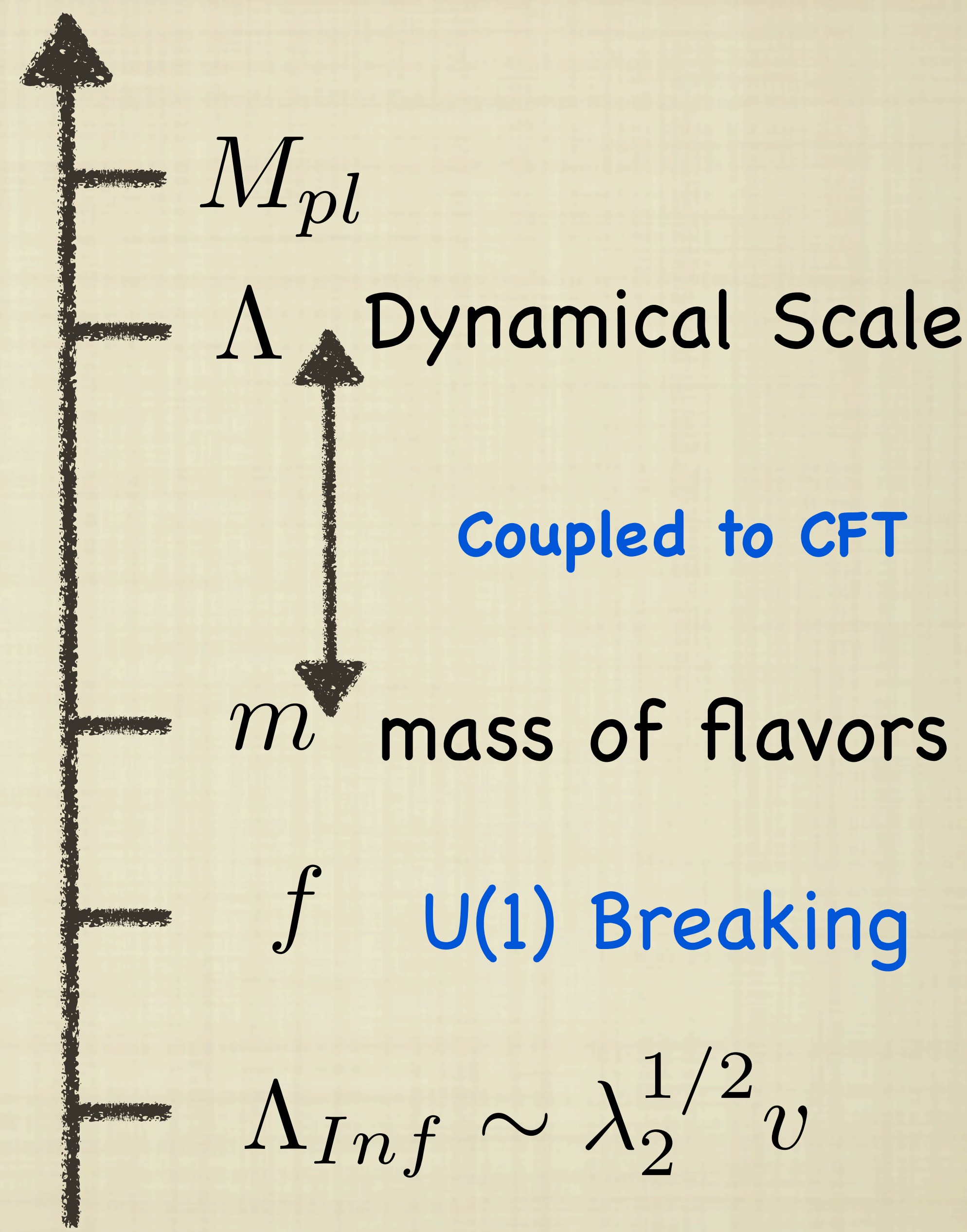
$$\Delta\varepsilon \simeq \frac{(\tilde{c}_2)^2}{2} \frac{f^4}{M_{\rm pl}^4} \left(\frac{m}{\Lambda}\right)^{4\gamma}$$

$$\Delta\eta \simeq 2\tilde{c}_1 \left(\frac{m}{\Lambda}\right)^{2\gamma} \lesssim 10^{-2}$$

Λ_{Inf} is a free parameter

Lots of "room" for RG

$$\gamma \gtrsim \frac{0.1}{1 - \frac{1}{10}\log\left[\left(\frac{m}{\Lambda}\right)/10^{-10}\right]}$$



Anomalous Dimensions

Need to compute the dimensions of $\Phi^2, \bar{\Phi}^2, \bar{\Phi}^\dagger \Phi$

Only contribution from

$$\int d^4\theta Z(\mu)\Phi^\dagger\Phi + \bar{Z}(\mu)\bar{\Phi}^\dagger\bar{\Phi}$$

$$\gamma = -\frac{1}{2}\frac{\partial \log Z}{\partial \log \mu}$$

Can be computed exactly using A-maximization

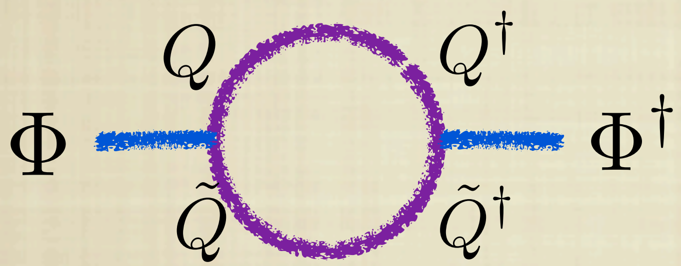
Intriligator & Wecht

Choose: $$N_1 = \frac{N_2}{2} = \frac{N_f}{4} = \frac{3N_c - k}{4}$$

$$\gamma_\Phi = \frac{8+3N_c}{16}\left[1 - \sqrt{1 - \frac{96 N_c}{(8+3N_c)^2}\frac{x}{3-x}}\right] \qquad x \equiv \frac{k}{N_c}$$

Weak Coupling

Weak Coupling ⟷ $\dfrac{k}{N_c} \ll 1$



$$\gamma_\Phi = N_c N_1 \frac{y^2}{8\pi^2} \qquad \frac{y_*^2}{8\pi^2} \sim \frac{3N_c - N_f}{N_1 N_c N_f}$$

1 - Loop

$$\gamma_\Phi = \frac{1}{3}\left(\frac{k}{N_c}\right) + \mathcal{O}\left(g_*^2, y_*^4\right)$$

Exact

$$\gamma_\Phi = \frac{1}{3}\left(\frac{k}{N_c}\right) + \mathcal{O}\left(N_c^{-1}, \frac{k^2}{N_c^2}\right)$$

$\eta \sim 10^{-2}$ → $k \sim N_c/3$

→ $\gamma \sim 1/9$

1-Loop / Exact ≅ 8/9

Comparison to Flavor in AMSB

Approximate flavor symmetry explains K/B physics, etc.

BSM physics: new sources of flavor violation

e.g. Anomaly Mediation

Randall & Sundrum; Luty & Sundrum

$$\int d^4\theta Q_i^\dagger Q_j (c_{ij} \frac{X}{M_{pl}} + c'_{ij} \frac{X^\dagger X}{M_{pl}^2})$$

$$F_X \sim 10^{12}\,\text{GeV}$$

Forbid by $\mathbb{Z}_2$

RG flow makes $c'_{ij} \ll 1$

$$(\frac{\sqrt{F}}{M_{pl}})^\gamma < 10^{-7} \longrightarrow \gamma > 1$$

Beyond the Model

General requirements for model builders

#1: Radiatively Stable Model
where inflaton is a PNGB

#2: Check model has a $\mathbb{Z}_2$
that forbids dim 5

#3: Couple inflaton to CFT
Make sure not to break approx. symm!!

Failure Modes

Can RG improve any model without SUGRA mass?

No.

E.g. Linear Superpotential $W = \sigma^2 \Phi$

$$V(\Phi) = \sigma^4 + \mathcal{O}\Big(\sigma^4 \frac{(\Phi^\dagger \Phi)^2}{M_{\rm pl}^4}\Big)$$

SUGRA terms cancel

Add $K = \Phi^\dagger \Phi + \frac{c_1}{M_{\rm pl}}(\Phi^\dagger \Phi^2 + h.c.) + \frac{c_2}{M_{\rm pl}^2}(\Phi^\dagger \Phi)^2$

Try to suppress c's with RG flow

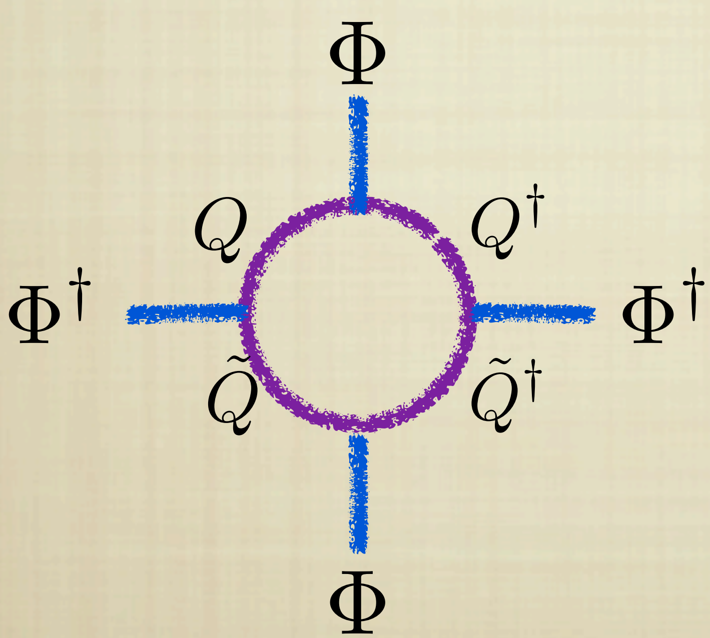
Linear Superpotential

Couple inflaton to CFT as before

When $\langle \Phi \rangle > m$ → $K \sim Z(\Phi/\Lambda)\Phi^\dagger \Phi$

$$V(\Phi) \sim \sigma^4 (\frac{\Phi}{\Lambda})^\gamma \rightarrow \epsilon \sim \gamma \frac{M_{pl}}{\Phi}$$

When $\langle \Phi \rangle < m$ Dangerous operators regenerated



$\neq 0$

$$K \supset \frac{c}{m^2}(\Phi^\dagger \Phi)^2$$

$$\eta \sim c(\frac{M_{pl}}{m})^2 \gg 1$$

Failure Modes

I. Inflating in "CFT" is difficult

Introduces fractions powers into potential,

e.g. $$V \to V \times (\frac{X}{\Lambda})^{\gamma}$$

Small field: $\epsilon << 1$ ⟶ $V = V_0 + f(X)$

Lesson: Must separate V_0 from CFT

OR

Inflate after decoupling from CFT

Failure Modes

II. CFT can (re)generate dangerous operators

X not coupled to CFT, e.g.

$$\int d^4\theta \frac{\Phi^2}{M_{pl}^2} X^\dagger X$$

CFT can't generate X
Flows to zero

X IS coupled to CFT, e.g.

$$\int d^4\theta X^\dagger X$$

CFT generates X
Does NOT flow to zero

Lesson: Operators don't flow to zero unless

Involve fields NOT coupled to CFT

OR

Forbidden by Symmetry

Non-SUSY Models

SUSY makes radiative stability easy

Still possible without SUSY

e.g. Arkani-Hamed et al.; Kaplan & Weiner

Assume a radiatively stable model (inflaton ϕ)

V given such that $\eta \ll 1$ & $\epsilon \ll 1$

Can add curvature couplings:

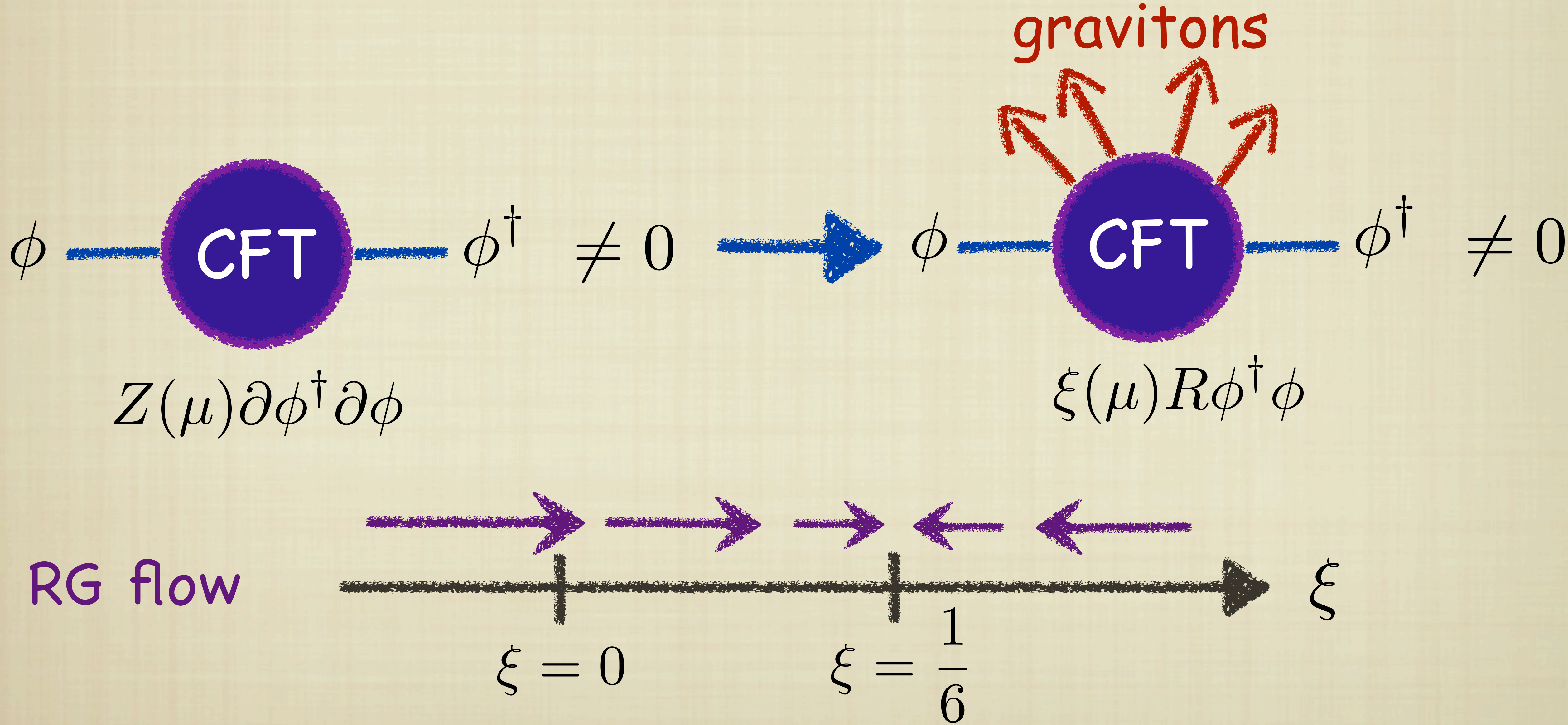
$$\delta S = -\int d^4x\sqrt{-g}\,R[c_1 M_{\rm pl}(\phi+\phi^\dagger)+c_2(\phi^2+\phi^{\dagger 2})+c_3\phi^\dagger\phi]$$

During inflation: $R \sim \dfrac{V}{M_{pl}^2}$ → $\epsilon \sim c_1^2$ → $\eta \sim c_2 + c_3$

RG Flow

Want to suppress curvature couplings with RG

Problem: Gravity also coupled to CFT



Eliminating Curvature Couplings

RG flow will not suppress all curvature couplings

$\partial_\mu \phi^\dagger \partial^\mu \phi - \frac{1}{6} R \phi^\dagger \phi$	Conformally Invariant
$R(\phi^2 + \phi^{\dagger\, 2})$	May or may not run to zero

Solution: #1 Inflaton a PNSB, $\phi = f e^{i\varphi}$

(Conformal Coupling is invariant)

#2 Find $\mathbb{Z}_2$ that forbids $M_{pl} R(\phi^\dagger + \phi)$

#3 Add U(1) invariant coupling to CFT

(U(1) breaking couplings flow to zero)

Comments on Model Building

Focus on SUSY models

At very least: moved problem to superpotential

$$K \supset \mathcal{O}\frac{\phi^\dagger \phi}{M_{pl}^2} \longrightarrow W \sim S(\phi\bar{\phi} - f^2) + \mu^2 X$$

Planck slop
Hard to compute → Approximate Symm.
Holomorphic

So far, no explanation of origin of symmetry

UV completion may be fine tuned

Comments on Model Building

Approach #1 : look for UV completion of field theory

Approach #2: Add gauge symmetry

Standard Model has approximates symmetries
e.g. Baryon / Lepton number

Reason: Gauge invariance forbids breaking terms with dim < 6

Goal: Inflaton = PNGB is **consequence** of gauge symmetry

Summary

Approach to η problem

Both SUSY and Non-SUSY examples

#1: Radiatively Stable Model
where inflaton is a PNGB

#2: Check model has a $\mathbb{Z}_2$
that forbids dim 5

#3: Couple inflaton to CFT
Make sure not to break approx. symm!!