Three perspectives on eternal inflation

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Based on:

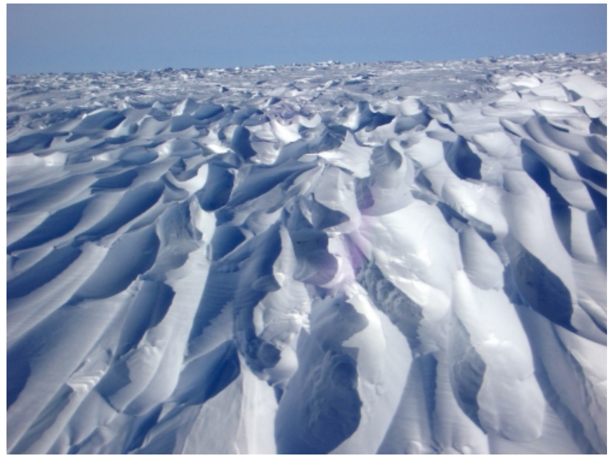
① BC, M. Kleban, K. Larjo, T. Levi, K. Sigurdson
JCAP **1102**, 023 (2010) [arXiv:1006.0382 - astro-ph.CO]

② V. Balasubramanian, BC, K. Larjo, T. Levi
Phys. Rev. D **84**, 025019 (2011) [arXiv:1012.2065 - hep-th]

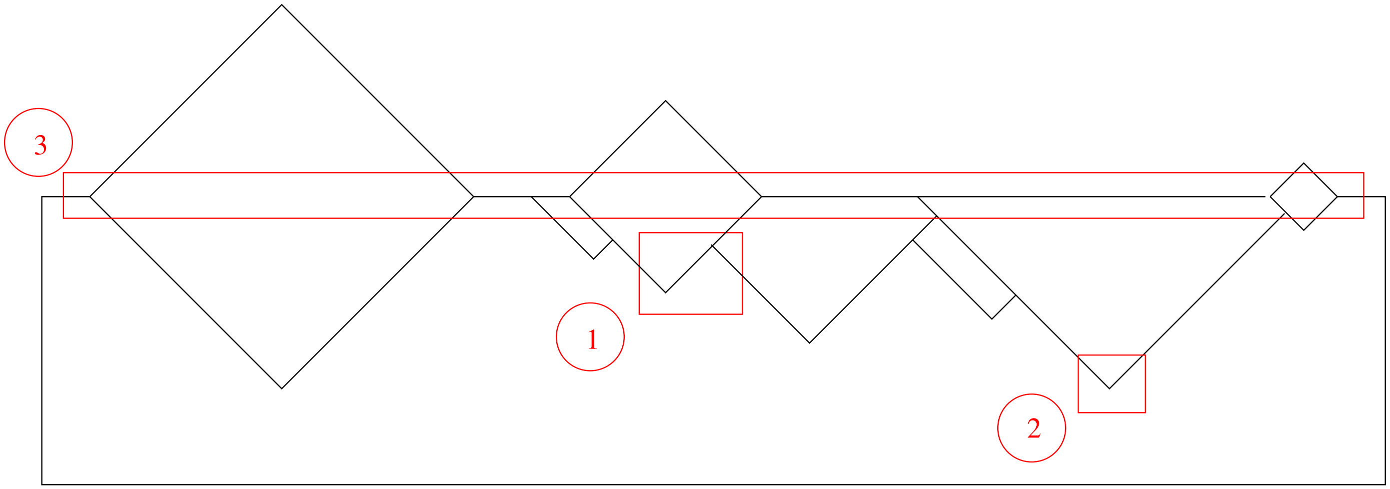
③ BC
Phys. Rev. D **84**, 064021 (2011) [arXiv:1102.1007]

Three perspectives on eternal inflation

- String theory predicts a potential landscape with many vacua
- CDL instantons mediate nucleations of bubbles filled with lower energy vacua
- Resulting bubbles contain open FRW universes



This leads to the following picture of eternal inflation:



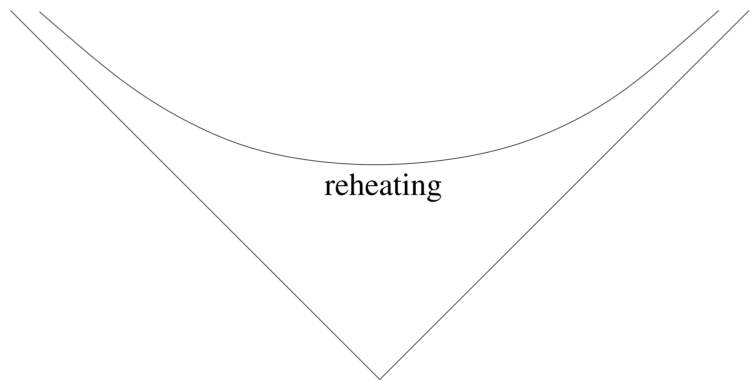
Plan: zoom in on this picture in 3 ways $\leftrightarrow$ 3 perspectives:

① On the interior of a bubble after collision $\rightarrow$ observational prediction

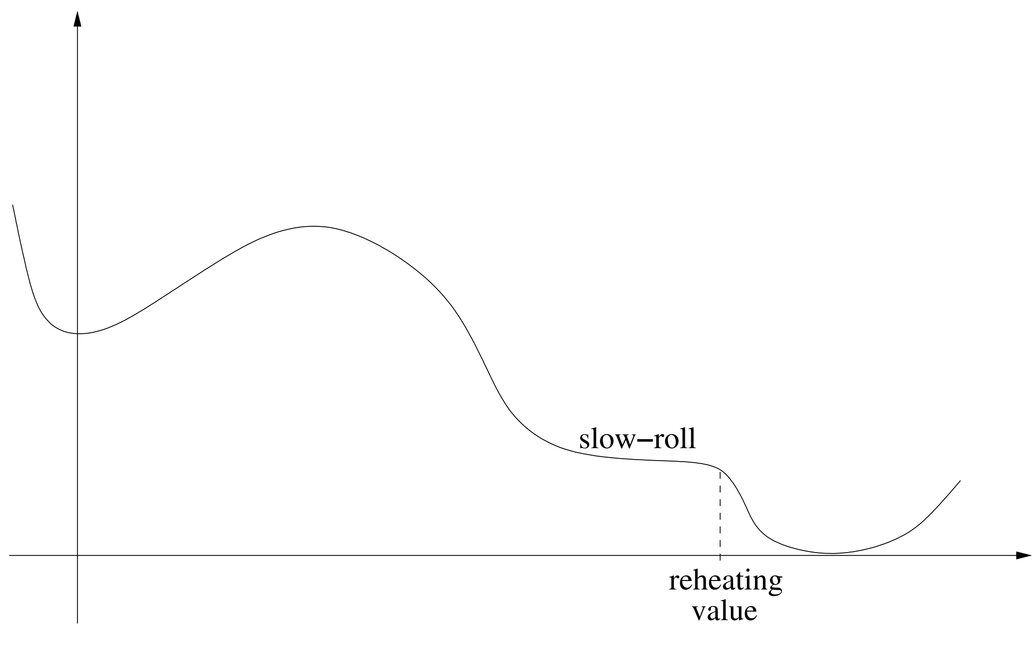
② On the instanton mediating the nucleation $\rightarrow$ to explore more general bubbles

③ On future infinity $\rightarrow$ for theoretical insight

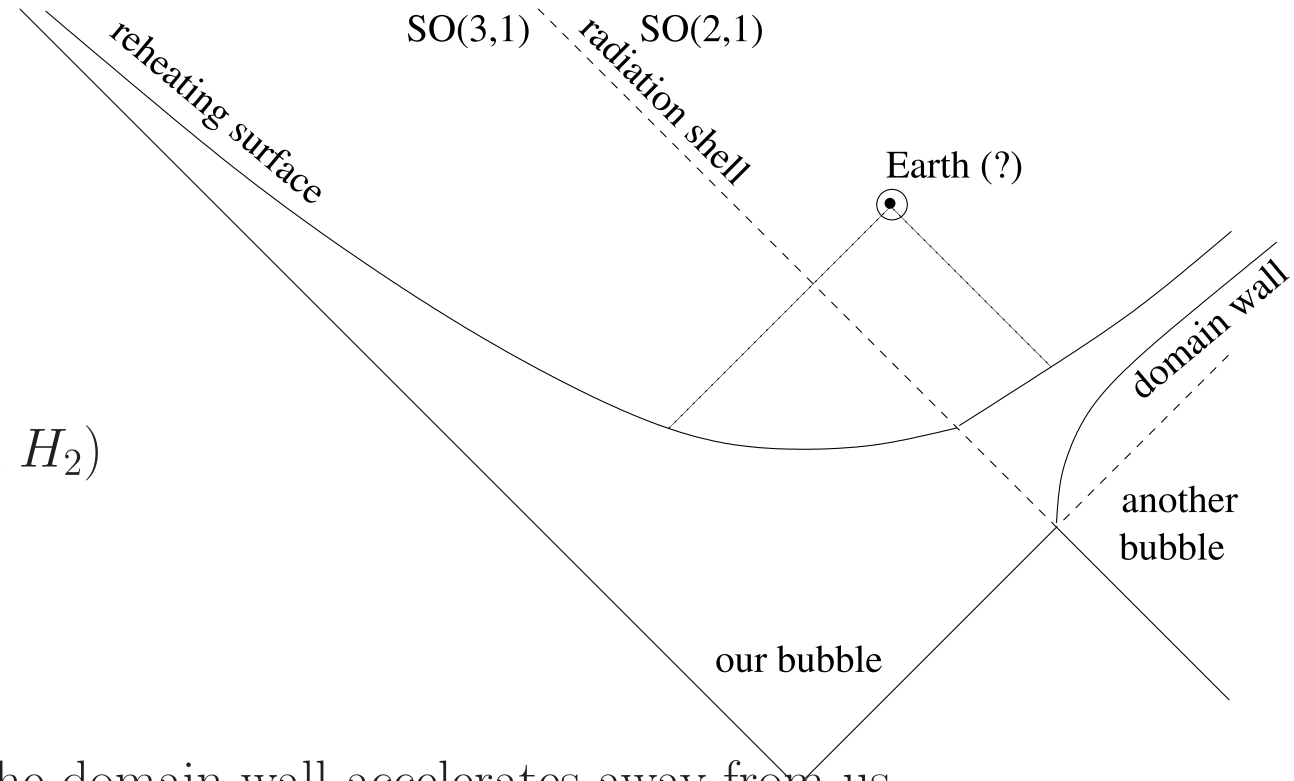
Preliminaries to ① – a single bubble



- This is a complete FRW universe.
- If we inhabit this bubble, we need slow-roll inflation inside it.
- It is most natural to identify the inflaton with the tunneling field.
- The reheating surface is a level set of the field.



① A bubble collision

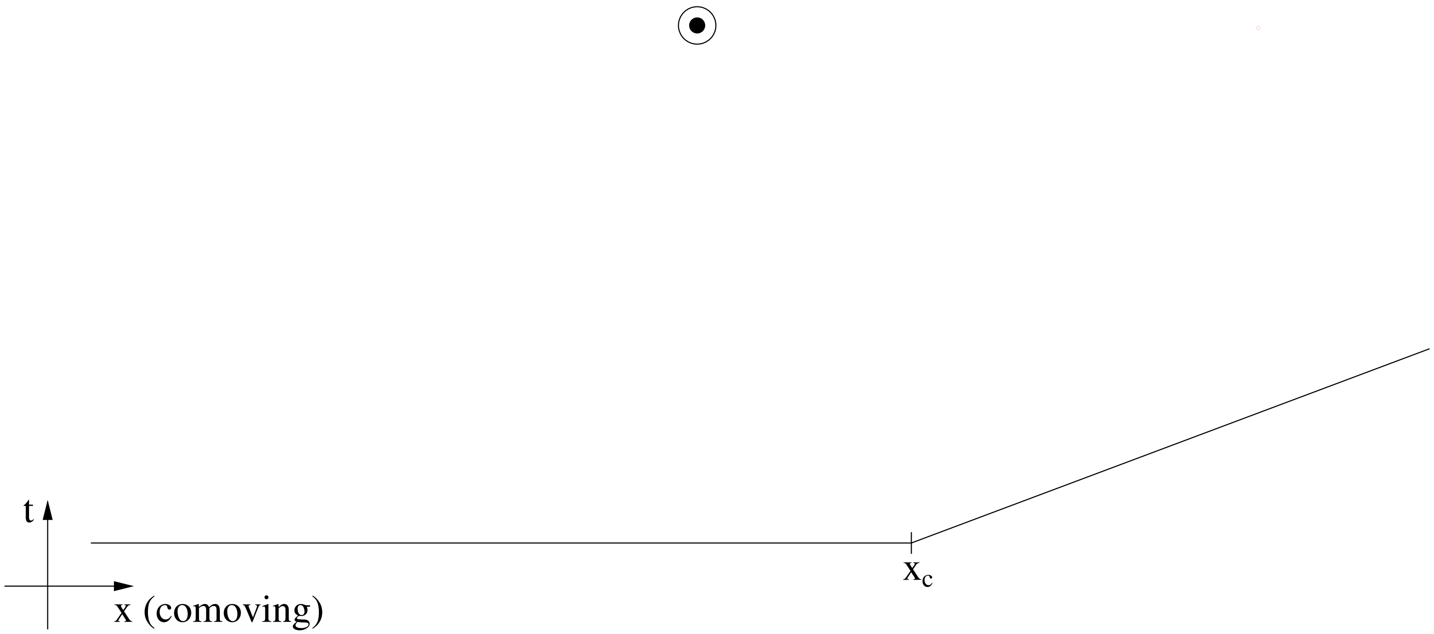


(each point is an H_2)

- Assume that the domain wall accelerates away from us
- Use Israel junction conditions to solve for the spacetime (Freivogel, Horowitz, Shenker, and Chang, Kleban, Levi 2007)
- Solve the scalar equation to find the reheating surface (Chang, Kleban, Levi 2008)
- Locate Earth, so Earthians see small effects of a collision
- To the future of the reheating surface, inflation has diluted curvature, so substitute $H_2 \to \mathbb{R}^2$ and $H_3 \to \mathbb{R}^3$

This leads to the following picture of the reheating surface:

① From the reheating surface to a cold / hot spot



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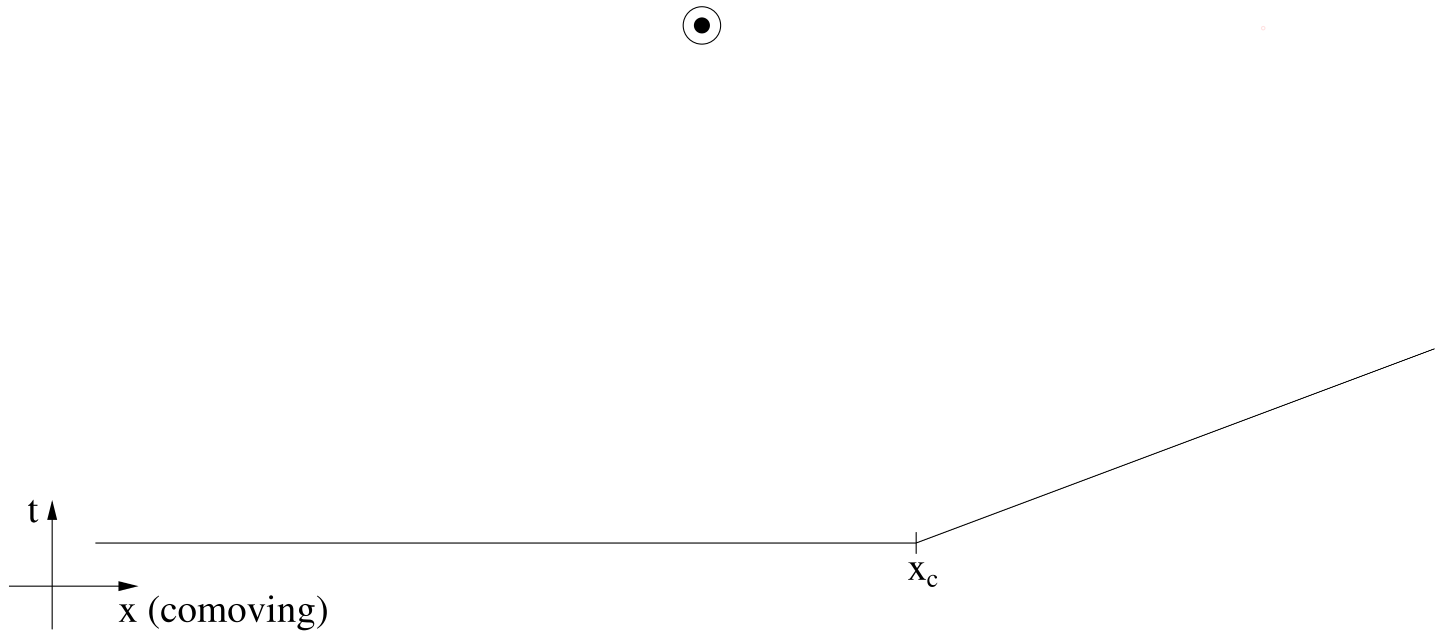

- But it is more convenient to pretend that the reheating surface is a straight line and package the effect of the collision into a temperature profile:

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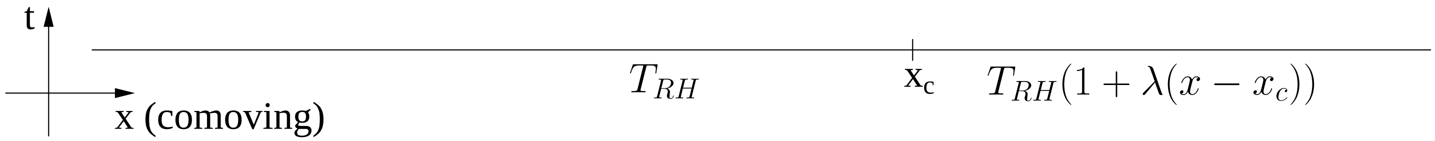

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- N.B. λ determines the magnitude of the effect.

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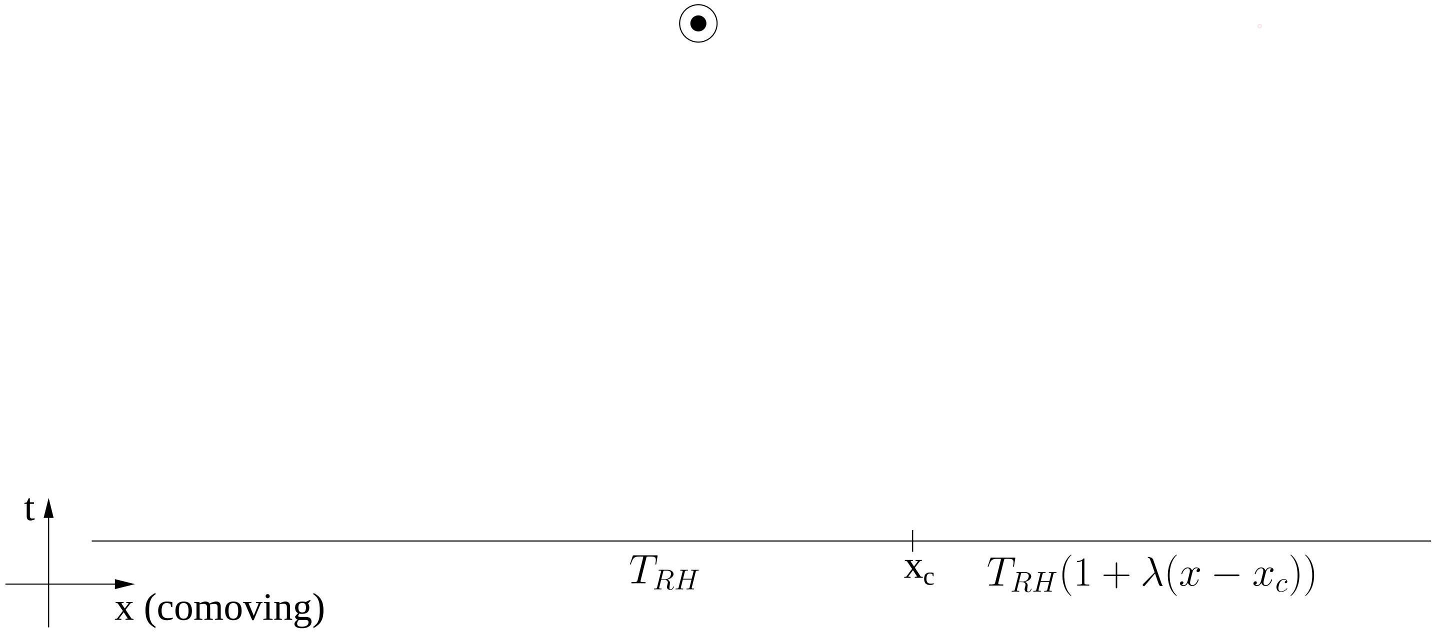

- But it is more convenient to pretend that the reheating surface is a straight line and package the effect of the collision into a temperature profile.
- N.B. λ determines the magnitude of the effect.
- Propagate the profile to the decoupling surface:

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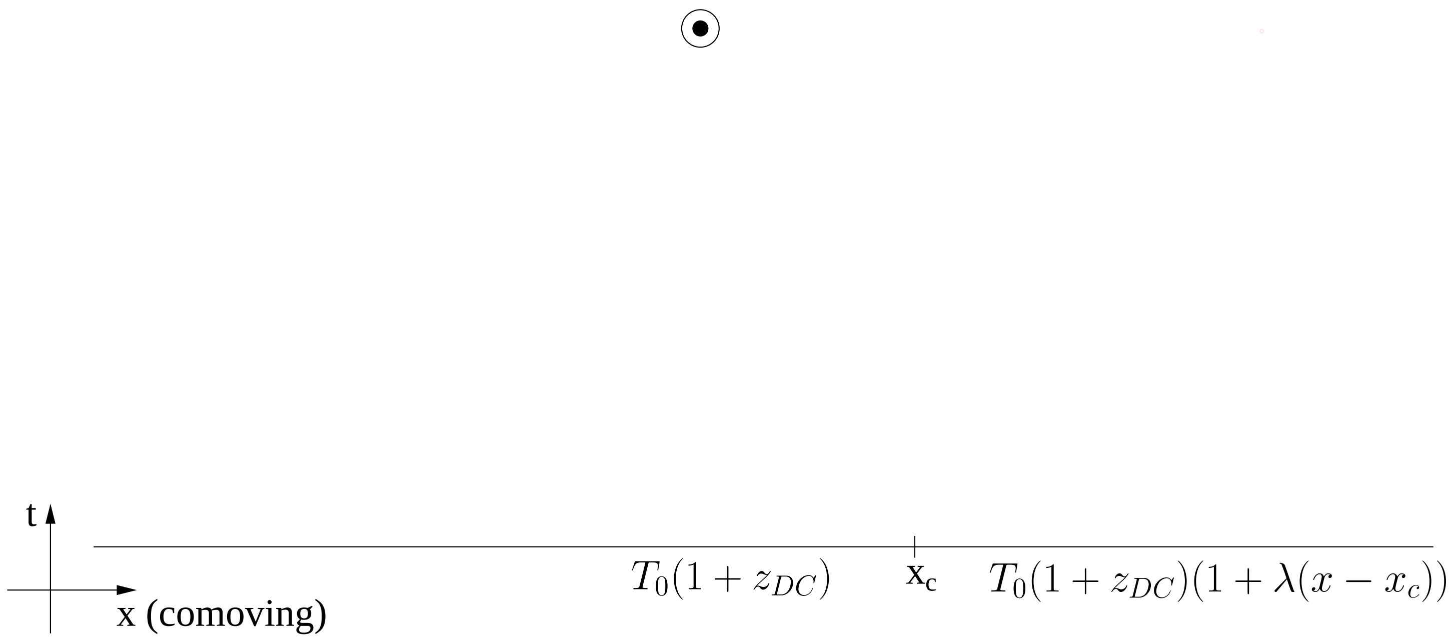

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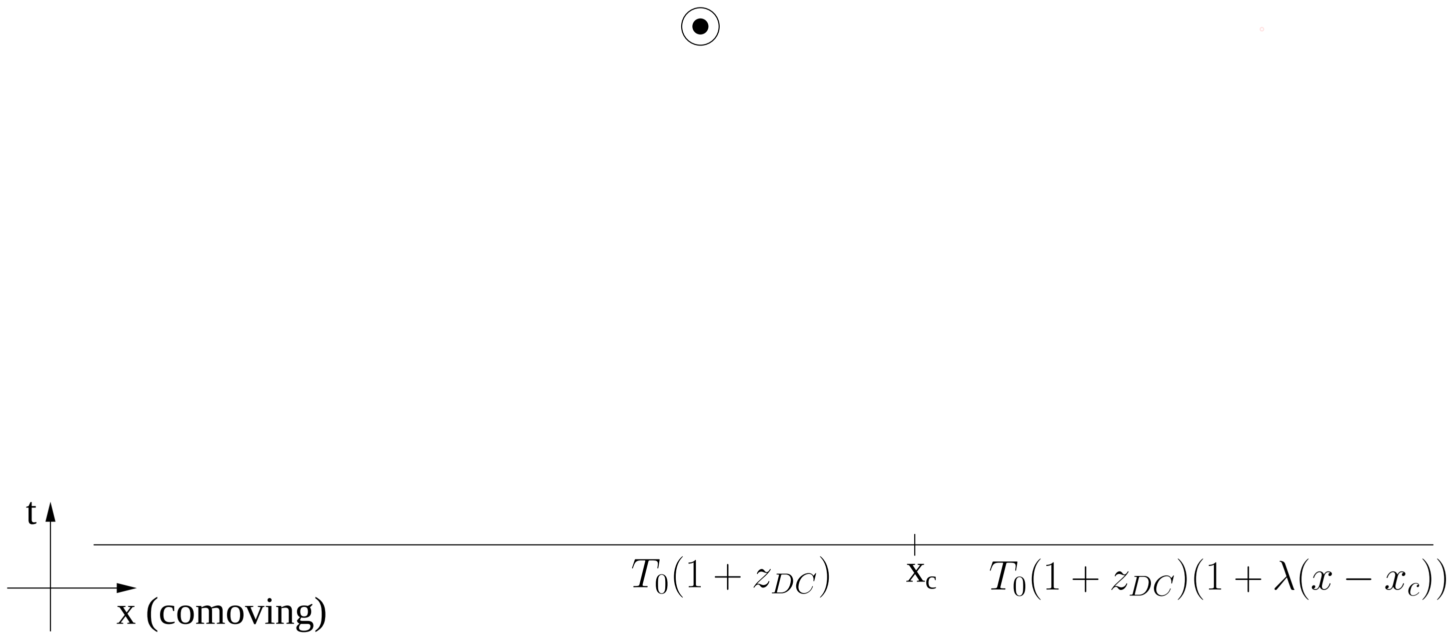

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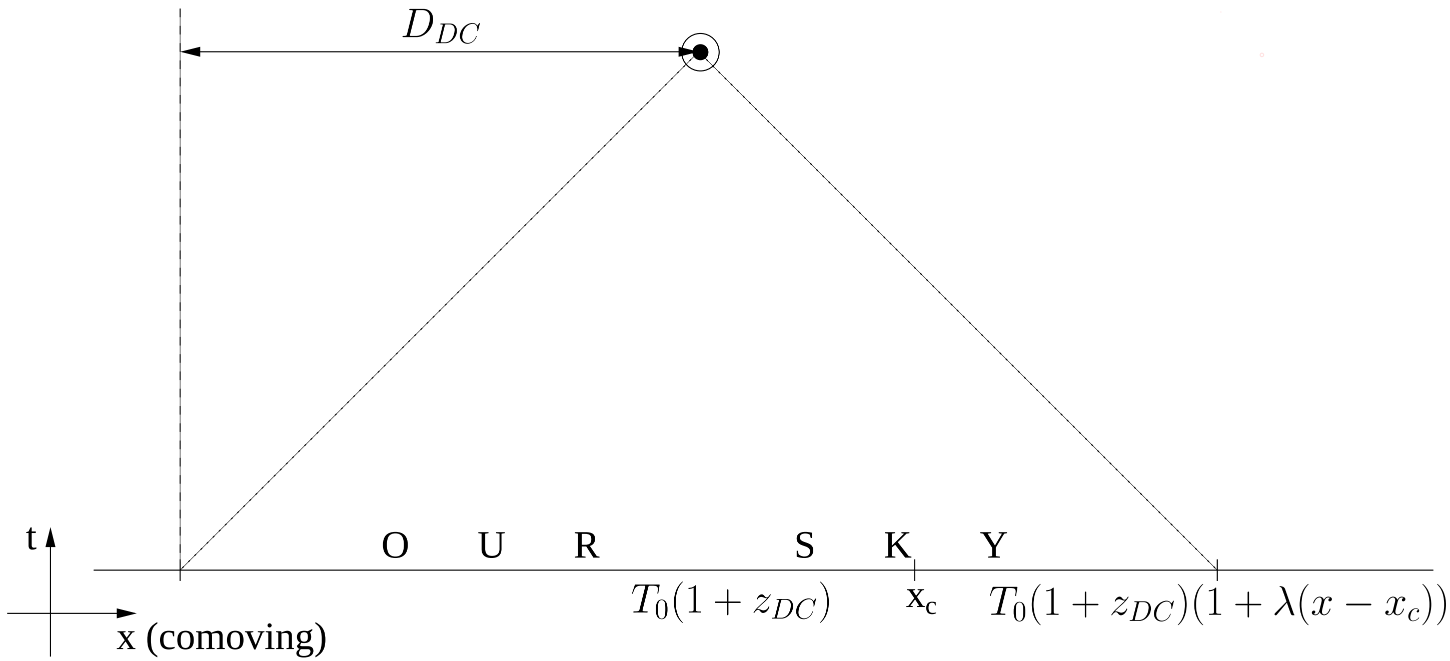
- Locate our Sky:

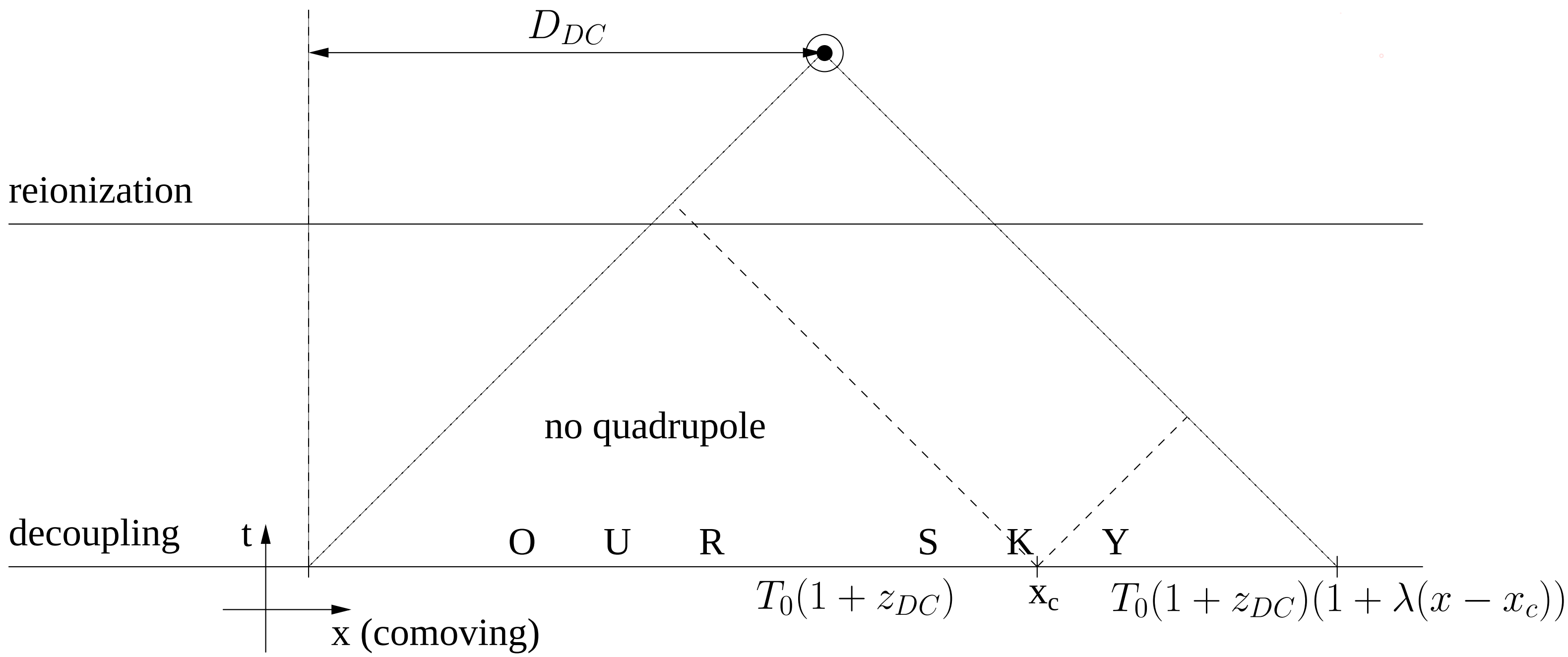
① From the reheating surface to a cold / hot spot



- But it is more convenient to pretend that the reheating surface is a straight line and package the effect of the collision into a temperature profile.
- N.B. λ determines the magnitude of the effect.
- Propagate the profile to the decoupling surface.
- Locate our Sky: each point on this segment is an azimuthal circle.

∴ A collision results in a cold / hot spot on our Sky.
(There is already a candidate in the CMB.)

① Toward CMB Polarization



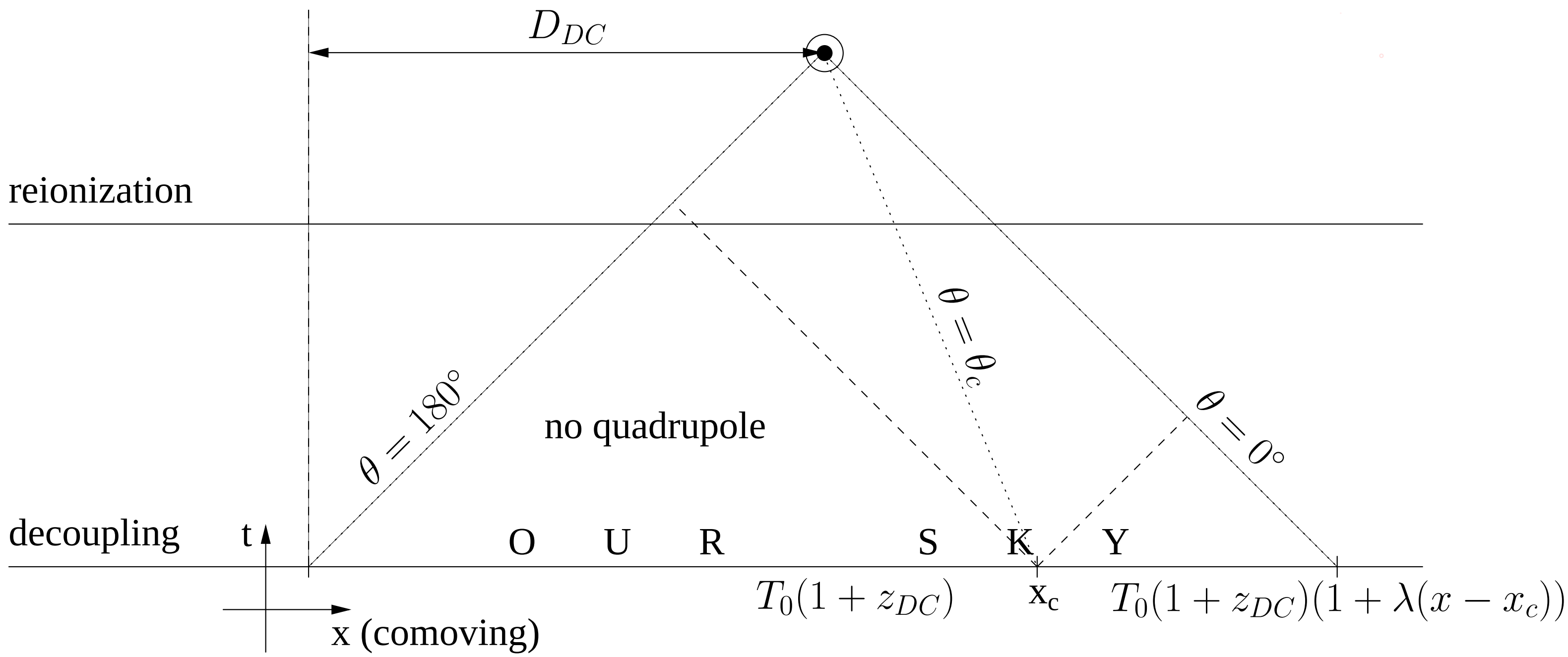
- Polarization comes from Thomson scattering off electrons that see a quadrupole temperature anisotropy.
- It only depends on θ, so it is fully E-mode (Stokes parameter Q):

$$Q(\theta) = \frac{\sqrt{6}}{10} \sum_{m=-2}^{2} {}_{\pm 2}Y_{2m} \int_{D_{DC}}^{0} dD g(D)\, T_{2m}(D\hat{n}_\theta)$$

- Integrate over θ-rays

- Measure is the "visibility function" – peaked at decoupling and reionization

① CMB Polarization

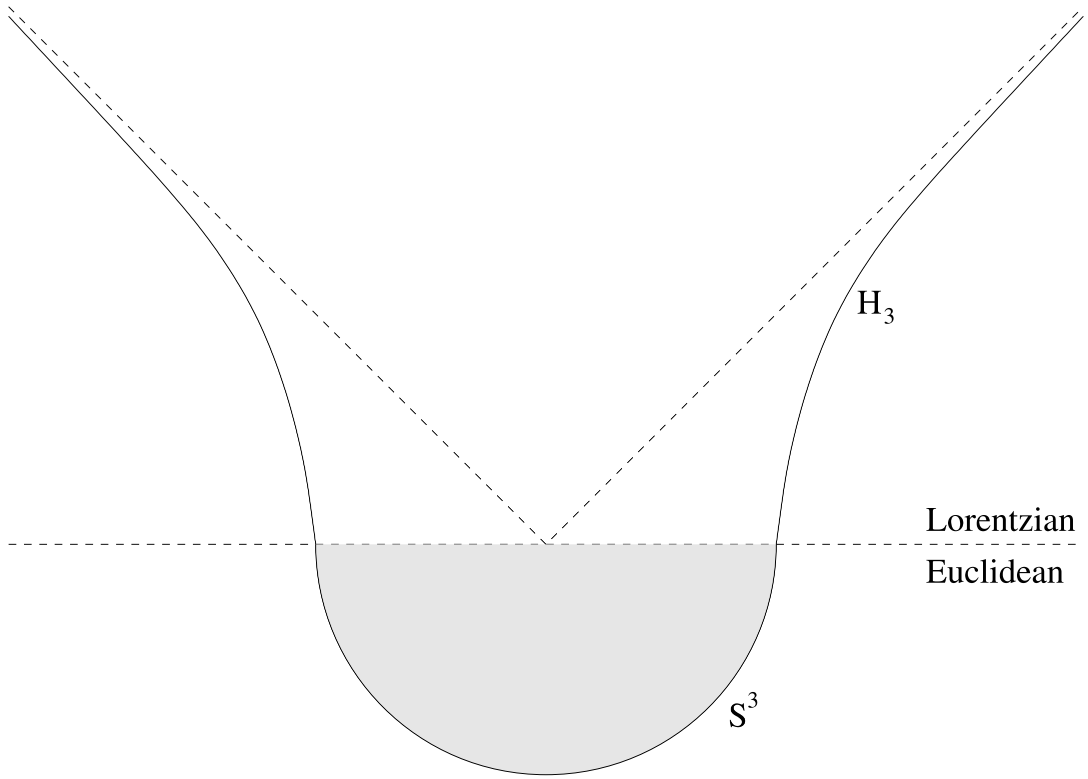


∴ There are two azimuthal peaks:

- narrow, cold / hot spot-sized, from decoupling
- broad from reionization (this one spills over the whole Sky for small spots)

This will be measured by Planck in the near future.

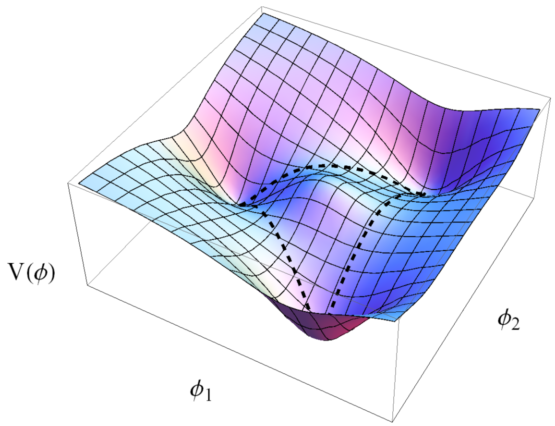
② Are spherical bubbles the whole story?

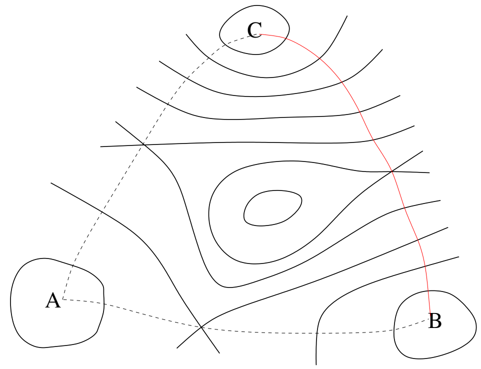


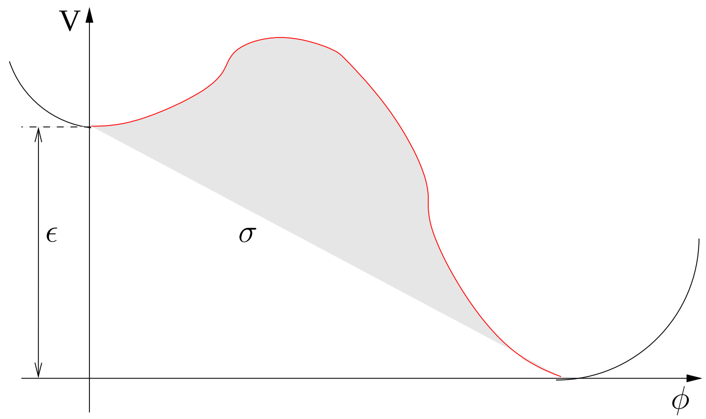
SAGREDO: Yes! Coleman, Glaser, and Martin told us so.
SALVIATI: But their proof only applies when the field space is one-dimensional. This is very different from the string landscape.

- More general instantons could significantly alter our picture of eternal inflation.
- From ①, their effects might even be observable.

② Setup







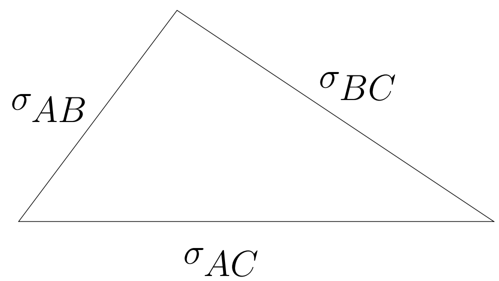
$$\{\epsilon_{AB}, \epsilon_{AC}, \epsilon_{BC},\ \sigma_{AB}, \sigma_{AC}, \sigma_{BC}\}$$

$$\{\epsilon, \sigma\}$$

- As a first step, just do field theory.
- Work in the thin wall approximation.
- The thin wall parameters are subject to relations:

$$\epsilon_{AC} = \epsilon_{AB} + \epsilon_{BC}$$

$$\sigma_{AC} = \min_{A\to C} \int_A^C dl \sqrt{V(l)} \quad \Rightarrow \quad \text{triangle inequality:}$$

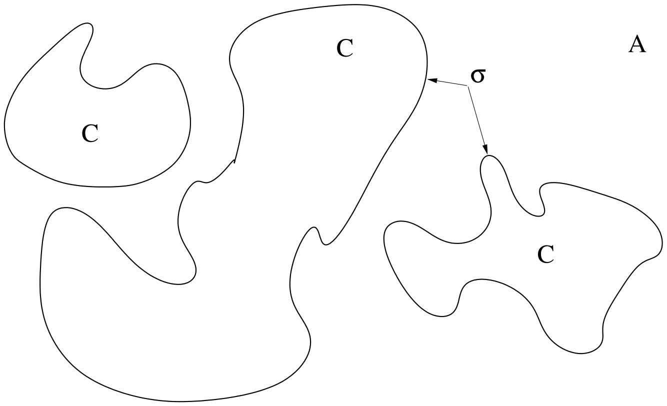


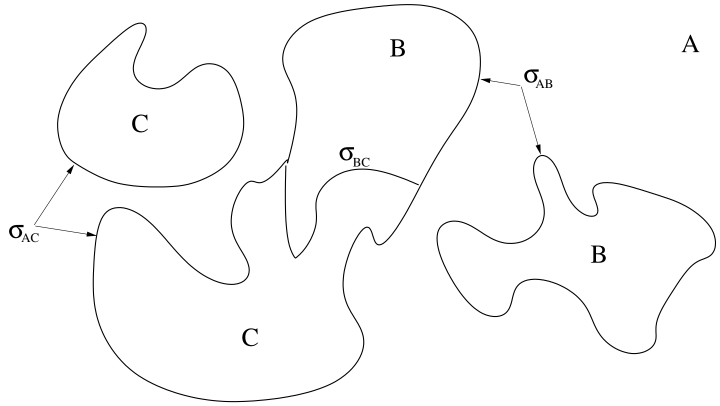
② Ansatz

2-vacuum problem

3-vacuum problem

- regions of 2 / 3 vacua separated by walls

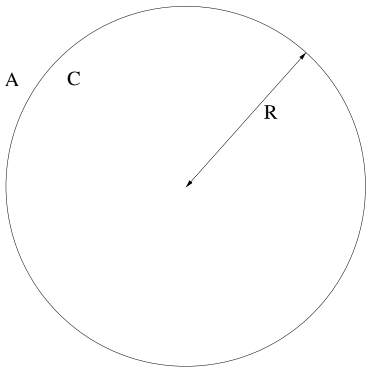


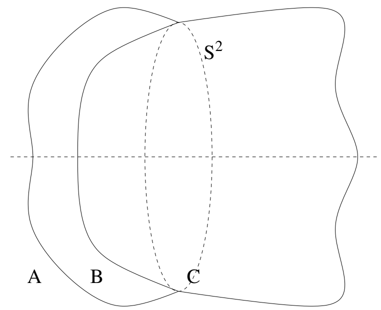


- take a single region

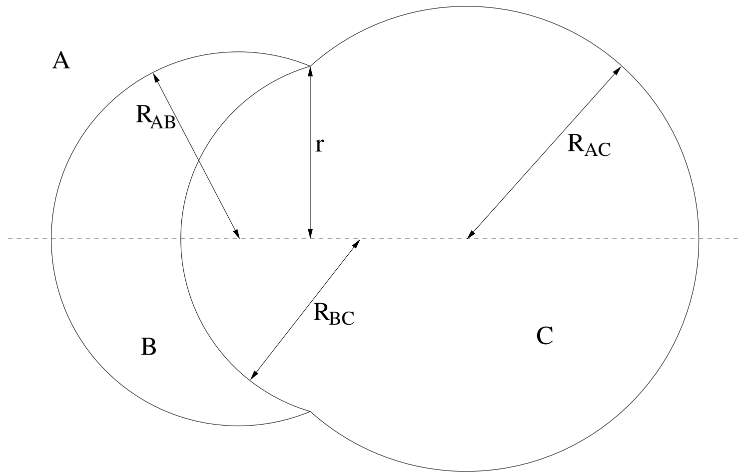
. . . with a single BC-interface

- form a maximally (spherically / cylindrically) symmetric object

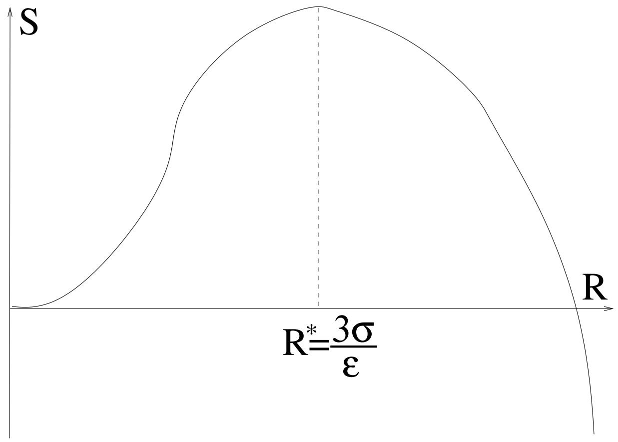




- find optimal surfaces with an S^2 boundary (junction)



② Calculation

	2-vacuum problem	3-vacuum problem
parameters:	R	$R_{AB}, R_{AC}, R_{BC}, r$ (junction radius)
action:	$-\epsilon R^4 \,\mathrm{vol}(B^4) + \sigma R^3 \,\mathrm{area}(S^3)$	$-\epsilon_{AB} \,\mathrm{vol}(AB) + \sigma_{AB} \,\mathrm{area}(AB)$ $-\epsilon_{AC} \,\mathrm{vol}(AC) + \sigma_{AC} \,\mathrm{area}(AC)$ $-\epsilon_{BC} \,\mathrm{vol}(BC) + \sigma_{BC} \,\mathrm{area}(BC)$
extremize:		$R^*_X = \frac{3\sigma_X}{\epsilon_X}$ (same as in the 2-vacuum case) $r = 0$ (spherical bubble) and $r = r^*$ (new)
negative modes:	one - R	Hessian is diagonal: $\frac{\partial^2 S}{\partial R_X \partial R_Y} = 0$ $\frac{\partial^2 S}{\partial r \partial R_X} \propto R_X - \frac{3\sigma_X}{\epsilon_X} = 0$ (by E.O.M.)
$\therefore$		count negative modes:

② Negative modes

$$\frac{\partial^2 S}{\partial R_X^2} \begin{cases} < 0 \;\text{ if } X \text{ is bigger than a hemisphere} \\ > 0 \;\text{ if } X \text{ is smaller than a hemisphere} \end{cases}$$

$$\frac{\partial^2 S}{\partial r^2} \quad \text{- obtain by analyzing } S(r):$$

- Because $S(r)$ has two extrema at 0 and r^*:

$\frac{\partial^2 S}{\partial r^2}|_{r=r^*} > 0\,(< 0) \quad \Leftrightarrow \quad r = 0$ is a local max (min) of $S(r)$

- But $\frac{\partial^2 S}{\partial r^2}|_{r=0} = 0 \quad \Rightarrow \quad$ this requires explanation

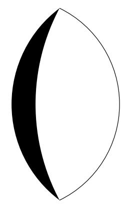
$\Rightarrow \quad$ we must go to cubic order:

- $\frac{1}{8\pi}\frac{\partial^3 S}{\partial r^3}|_{r=0} = \pm\sigma_{AB} \pm \sigma_{AC} \pm \sigma_{BC} \gtrless 0 \quad \Leftrightarrow \quad \frac{\partial^2 S}{\partial r^2}|_{r=r^*} \lessgtr 0$

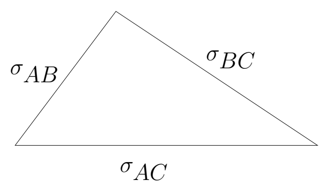
  + (-) sign for regions smaller (bigger) than a hemisphere

- We want exactly 1 negative mode:

case (1): $S_{rr} < 0 \Rightarrow$ all three $S_{RR} > 0 \Rightarrow$
three smaller-than-hemisphere regions



case (2): one $S_{RR} < 0 \Rightarrow$ exactly two $S_{RR} > 0 \Rightarrow$
two smaller-, one bigger-than-hemisphere region



$\therefore$ All non-trivial saddle points have 2 or more negative modes.

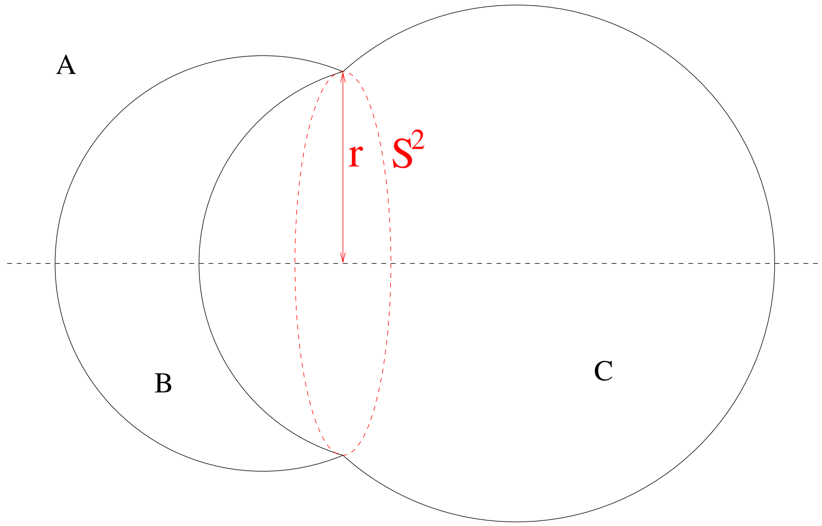
② Loose end

$$\frac{\partial^2 S}{\partial r^2}\Big|_{r=0} = 0$$

SAGREDO: $r = 0$ is the good, old spherical instanton.
Does this mean that it has a non-translational zero mode?
Does it enhance the nucleation rate?

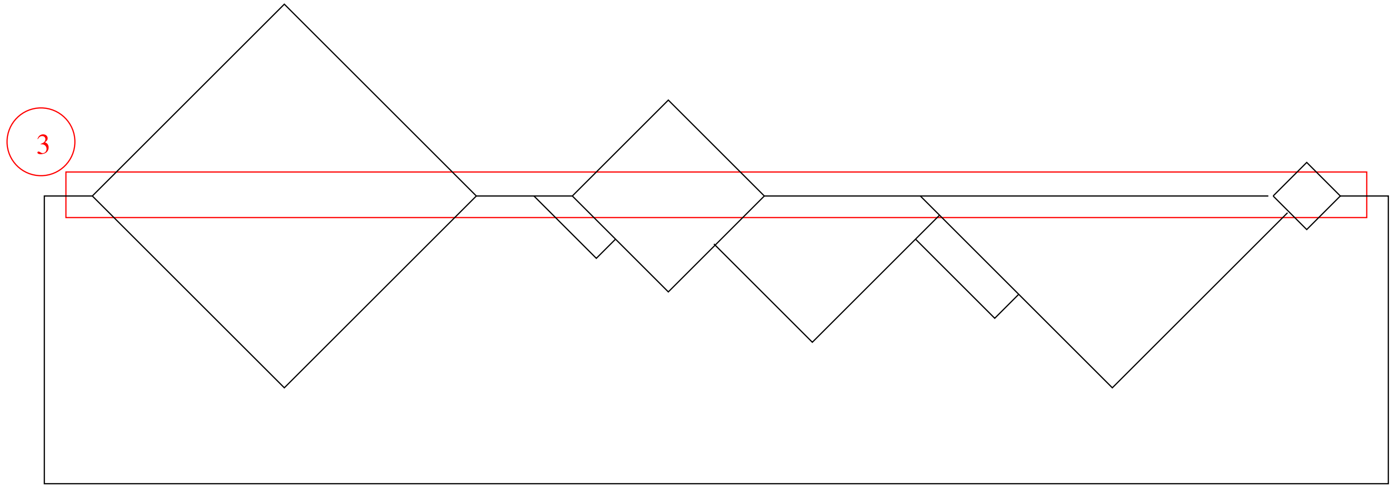
SALVIATI: No, because we neglected a quadratic piece of the action.
It arises from the cost of creating a junction:

$$S = S_{\text{before}} + \kappa r^2$$



- In the thin-wall approximation, codimension-2 junctions generalize objects of codimension-1 (walls) and codimension-0 (vacua).
- Microscopically, junction tensions depend on hills in the landscape.
- They are necessary to resolve the apparent zero modes.

③ Topology at future infinity

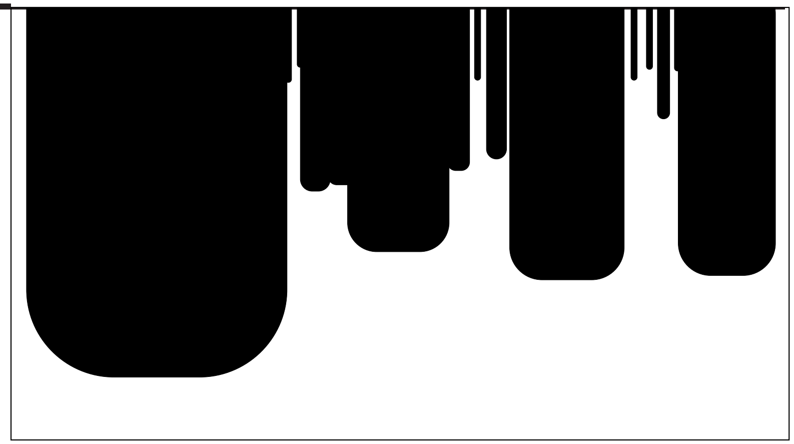


Motivation:

- well-defined (independent of slicing)
- independent of the measure problem
- theoretical significance (e.g. for FRW / CFT)
- mathematically fun

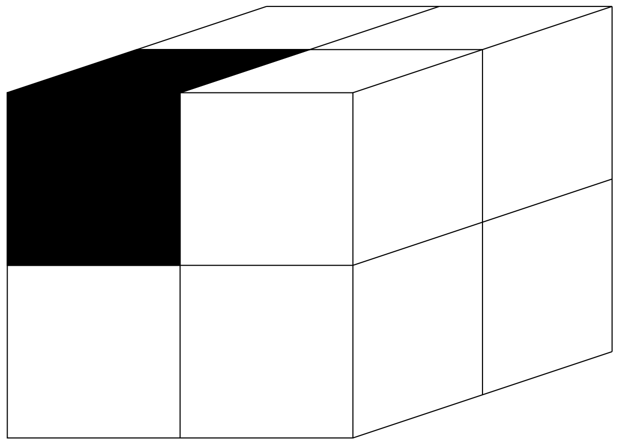
③ Discretization

- Re-draw diagram in comoving coordinates:
- Bubbles attain a fixed comoving size:
  $c = 1 = Ha\Delta x = \dot{a}\Delta x$
  (Hubble radius in comoving coordinates)
  $\Delta x = (\dot{a})^{-1} \propto e^{-t}$ in de Sitter



- Re-draw diagram with discrete cells:
- Set $\Delta x = (\dot{a})^{-1} \propto e^{-t}$
- After time Δt, the spatial cell size decreases by a factor $\frac{\dot{a}(t)^{-1}}{\dot{a}(t+\Delta t)^{-1}}$
- Set Δt so this ratio is a natural number.
- Here $N = 3$.

- This defines the Mandelbrot model (in 3 dimensions)
- 2 colors $\leftrightarrow$ vacua; 2 parameters:
  $N^3 = \#$ of daughter cells $\sim e^{3H\Delta t}$
  $p =$ prob. of coloring / nucleation $\sim \Gamma(\Delta x)^3 \Delta t$



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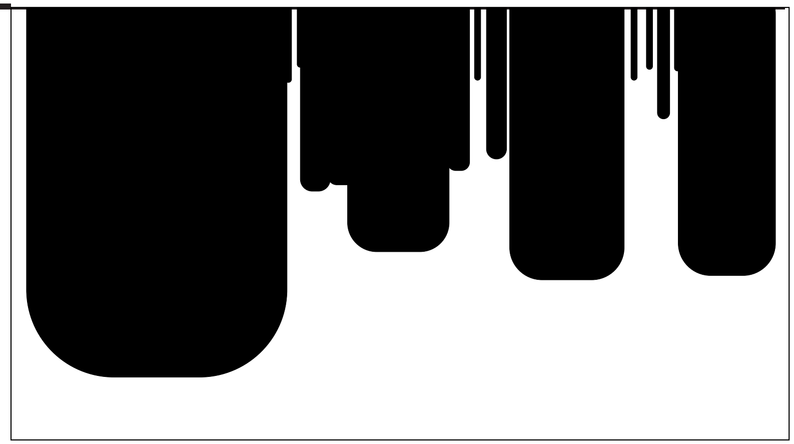

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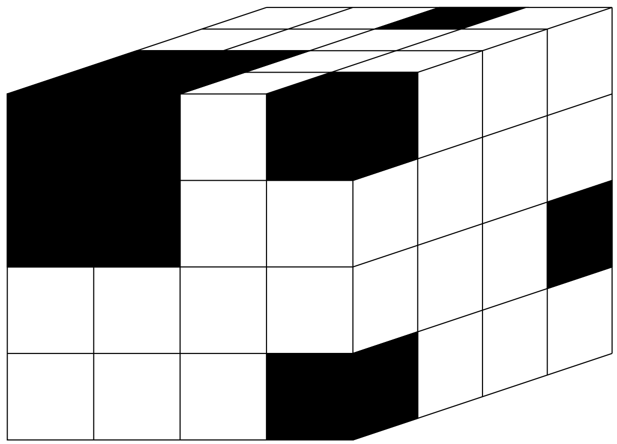

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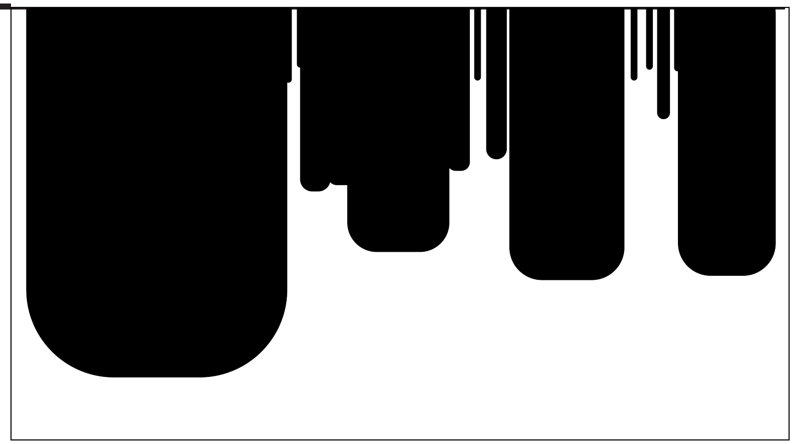

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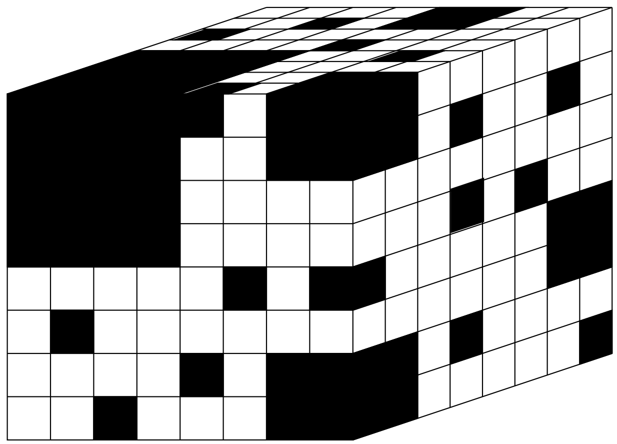

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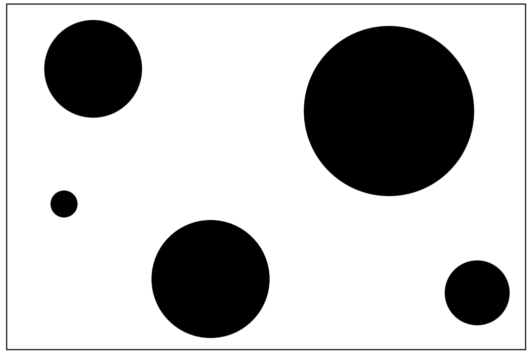
$\therefore$ What is the topology after infinitely many steps?

③ Previous results – 2-vacuum phase structure

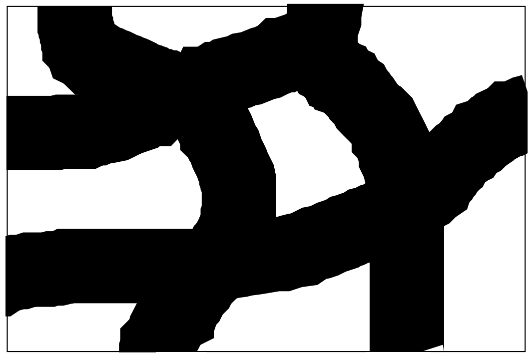
I. Black Island Phase

- Contains white crossing surfaces (infinite white screens).
- Open FRW universes.
- BW boundary has many disconnected components, occasionally finite genus.



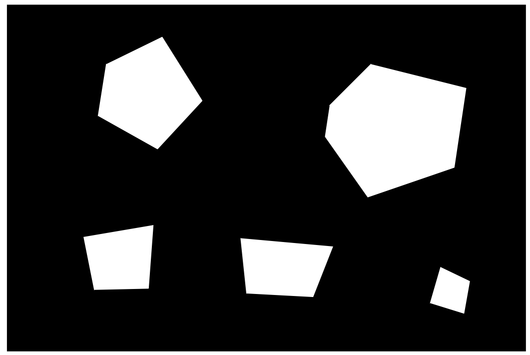
II. Tubular Phase

- Contains crossing curves (infinite tubes) of both colors.
- BW boundary is connected and has infinite genus.
- Observers in black regions see boundary genus grow without bound.



III. White Island Phase

- Contains black crossing surfaces (infinite black screens).
- BW boundary is again disconnected, now due to
- cracking: a process of tearing apart white regions, which produces singularities in black regions.

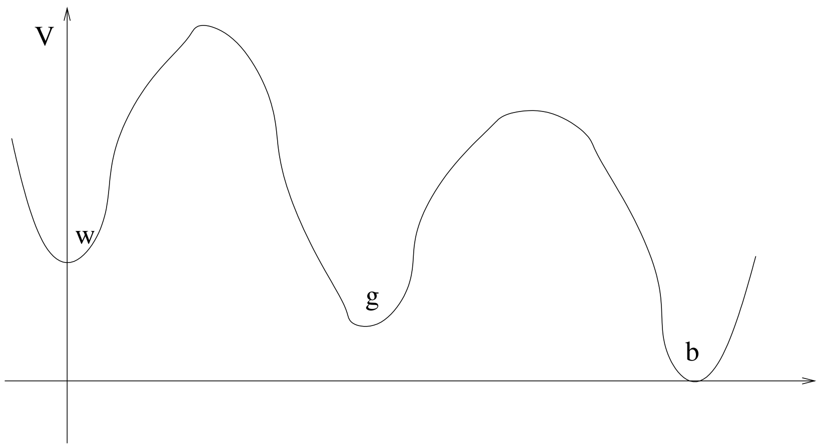


IV. Aborted Phase

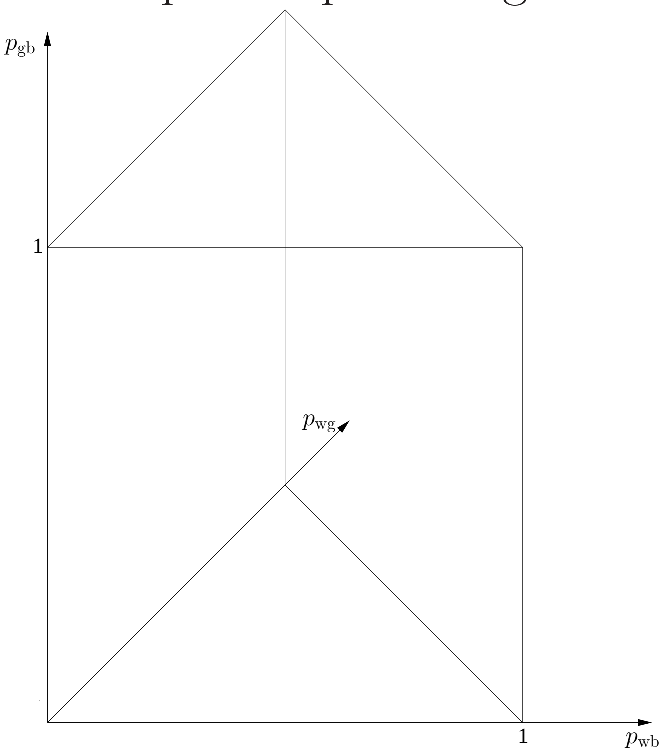
(Chayes et al. 1992; Sekino, Shenker, Susskind 2010)

③ Generalize to three vacua

- Consider the 3-vacuum system.
- There are 5 parameters: $p_{\rm wg}, p_{\rm wb}, p_{\rm gb}, N_w, N_g$
- But a shift in N_w, N_g can always be undone by a compensating shift in the probabilities



∴ The phase space diagram will look like this:

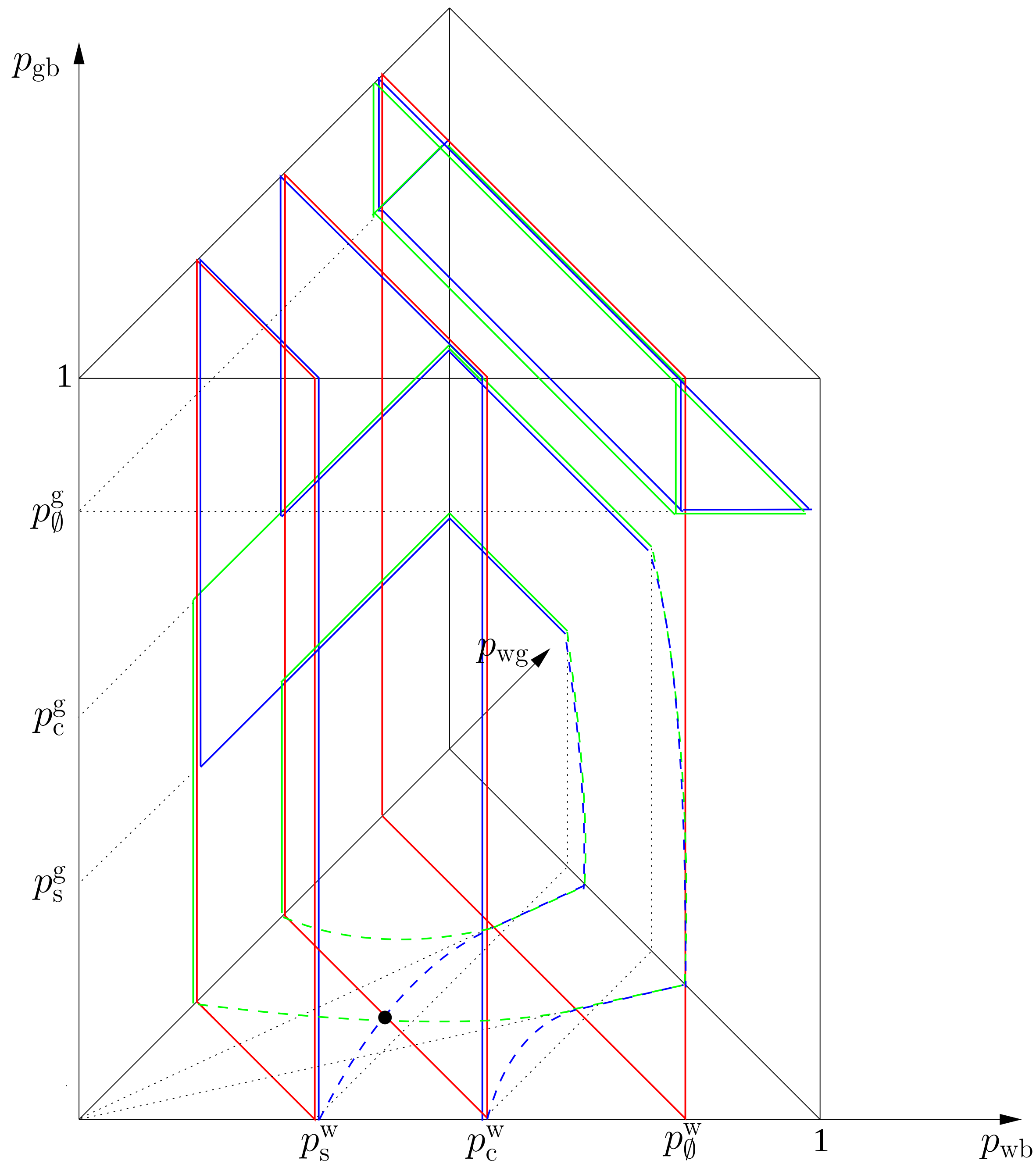


- Proceed by bootstrapping results from the 2-vacuum system.
- Example: white islands:

2-vacuum: $p_c \leq p_{\rm wb} \leq p_\emptyset$

3-vacuum: $p_c \leq p_{\rm wb} + p_{\rm wg} \leq p_\emptyset$

③ The 3-vacuum phase diagram



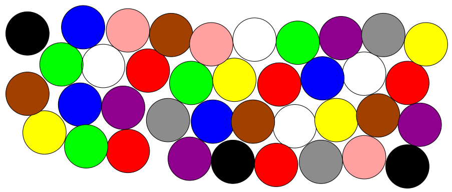
③ Lessons

- In the 2-vacuum case, we had crossing surfaces or two colors of crossing curves.
- In the 3-vacuum case, much of the phase diagram is occupied by phases:

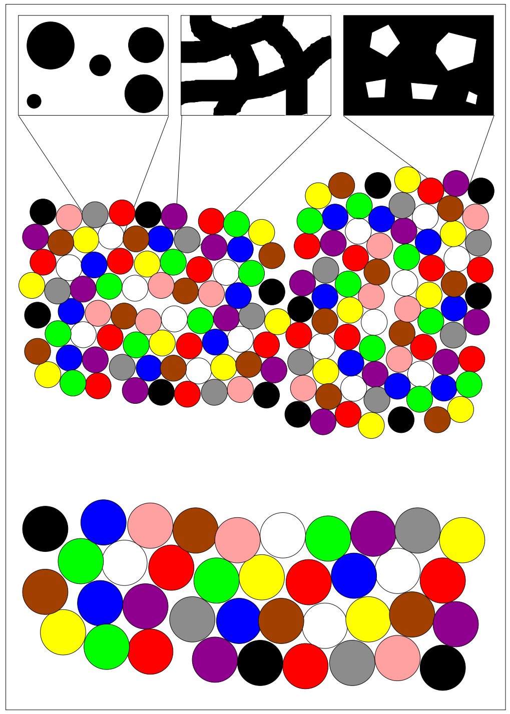
1)	white crossing curves	gray islands	black islands
2)	white islands	gray crossing curves	black islands
3)	white islands	gray islands	black crossing curves

∴ In the many-vacuum case, all colors will be generically present in island form.

∴ The “grainy phase” is generic.



∴ This leads to the following picture of eternal inflation:



Summary

① We predicted CMB polarization patterns, which could corroborate the string landscape.

② We excluded previously unconsidered, putative instantons, which would combine regions of two true(-r) vacua.

② We appreciated the role of "junctions" for regulating zero modes in thin-wall calculations of nucleation rates.

③ We saw that interesting topology may arise in eternal inflation, but mostly in the later generations and on the intra-bubble scale.

THANK YOU!