Rare K and B Decays in Warped Extra Dimensions with Custodial Symmetry

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Outline

Based on:

M. Albrecht, M. Blanke, A. Buras, BD, K. Gemmler, [arXiv:0903.2415]
M. Blanke, A. Buras, BD, S. Gori, A. Weiler, [arXiv:0809.1073]
M. Blanke, A. Buras, BD, K. Gemmler, S. Gori, [arXiv:0812.3803]
A. Buras, BD, S. Gori, [in preparation]

1. **The RS Model with Custodial Protection**
   - Motivation
   - Basic Features
   - EWPT
2. **Flavor Physics**
   - K and B Mixing
   - Rare K and B Decays
3. **Conclusions**

Motivation I: The Gauge Hierarchy Problem

The Gauge Hierarchy Problem

- Large Hierarchy between the electroweak and the Planck scale,

$$v/M_{Pl} \approx 10^{-16}$$

- Naturally, radiative corrections drag lower scales towards higher scales

Is there a natural way to stabilize the EW scale?

- Supersymmetry
- Large Extra Dimensions
- Technicolor
- (...)

Motivation II: The Flavor Problem

The Flavor Problem

- Quark masses range over five orders of magnitude,

$$m_u \approx 5\mathrm{MeV} \text{ while } m_t \approx 172.5\mathrm{GeV}$$

- CKM matrix elements are vastly different,

$$|V_{ud}| \approx 1 \text{ while } |V_{us}| \simeq 0.226,\ |V_{cb}| \simeq 0.041,\ |V_{ub}| \simeq 0.0038$$

Is there a natural explanation for these hierarchies?

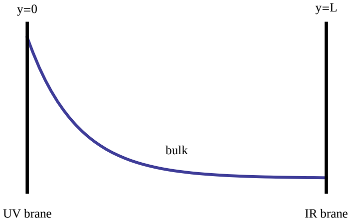
Widely used: flavor symmetries.

But: Many choices for the symmetry group, explicit breaking or multi-Higgs, auxiliary symmetries necessary (...)

The Randall-Sundrum (RS) Model

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

Randall, Sundrum, hep-ph/9905221



5D space-time with a warped, non-factorizable metric

- Metric solves 5D Einstein equations
- Energy scales warped down from UV $\rightarrow$ IR
- Higgs localized at IR brane + proper choice of geometric parameters

$\Longrightarrow$ **Explanation of EW-Planck hierarchy**

Particles in the Bulk

In the original RS setup, only the graviton propagates into the bulk, now allow also for **gauge bosons** and **fermions** to do so.

Chang et al., hep-ph/9912498
Grossman, Neubert, hep-ph/9912408
Gherghetta, Pomarol, hep-ph/0003129

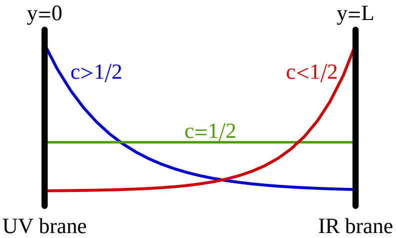
- Gauge zero modes are flat
- Fermion zero modes depend on 5D bulk masses ($\rightarrow$ next slide)
- All Kaluza-Klein (KK) modes are strongly localized towards the IR brane
- All KK modes have masses $\mathcal{O}(ke^{-kL}) \sim \mathcal{O}(\mathrm{TeV})$

Fermion Localization

Zero mode localization depends exponentially on the 5D bulk mass parameter $c = m^{5D}_{\text{Dirac}}/k$:

$$f^{(0)}(y,c) \propto e^{(\frac{1}{2}-c)ky}$$

- $c > \frac{1}{2} \rightarrow$ localized towards UV brane
- $c < \frac{1}{2} \rightarrow$ localized towards IR brane
- $c = \frac{1}{2} \rightarrow$ conformally flat



Effective Yukawa Couplings

Arkani-Hamed, Grossman, Schmaltz, hep-ph/9912265

A Froggatt-Nielsen-like Scenario

Yukawa matrices Y_{ij} are anarchical

+

IR-brane values F_i of quark wave functions are hierarchical

Both are natural assumptions:

- Y_{ij} are input parameters and expected to be $\mathcal{O}(1)$
- slightly different c-parameters of $\mathcal{O}(1)$ lead to a large hierarchy in F_i

Example:

$$(c_{Q_1} = 0.66, c_{Q_2} = 0.59, c_{Q_3} = 0.41) \Longrightarrow (F_{Q_1} = 0.0017, F_{Q_2} = 0.017, F_{Q_3} = 0.42)$$

- Resulting effective Yukawa matrices are very hierarchical
- Hierarchy of quark masses and mixings is explained by a purely geometric approach

Flavor problem is solved!

EW Precision Tests: S, T

Higgs VEV repels W, Z gauge zero modes from the IR brane
(Alternative description: Zero modes mix with KK modes after EWSB)

Csaki, Erlich, Terning, hep-ph/0203034

$\Longrightarrow$ **Corrections to EW observables**

S parameter

Bound on KK mass scale $M_{\text{KK}} \gtrsim (2-3)\text{TeV}$ Agashe et al., hep-ph/0308036

T parameter

Bound on KK mass scale $M_{\text{KK}} \gtrsim 10\text{TeV}$

Agashe, Delgado, May, Sundrum, hep-ph/0308036

Csaki, Grojean, Pilo, Terning, hep-ph/0308038

Enlarge gauge group to $SU(2)_L \times SU(2)_R \times U(1)_X$

$\Longrightarrow$ With this **custodial symmetry**, the model is consistent with EW precision data for M_{KK} as low as 3TeV

Carena, Ponton, Santiago, Wagner, hep-ph/0701055

EW Precision Tests: $Zb\bar{b}$

Gauge zero modes are distorted near the IR brane,
fermion zero modes mix with KK modes after EWSB

$\Longrightarrow$ **Non-universalities in gauge couplings arise**

- Third generation is affected most
- Naturally, corrections arise at the $(1-2)\%$ level

In particular Zbb is measured very precisely:

$$-2\cdot 10^{-3} \lesssim \delta g_{Zb_L\bar{b}_L} \lesssim 6\cdot 10^{-3} \quad (95\%\mathrm{C.L.}).$$

$\Longrightarrow$ **Corrections to $Zb\bar{b}$ are in general too large**

Discrete Parity

Agashe, Contino, Da Rold, Pomarol, hep-ph/0605341

Pattern of EWSB in the presence of a custodial symmetry:

$$O(4) \to O(3) \quad \sim \quad SU(2)_L \times SU(2)_R \times P_{LR} \to SU(2)_V \times P_{LR}$$

Consider $Zb_L\bar{b}_L$ coupling:

$$g_{Zb_L\bar{b}_L} = \frac{g}{\cos\theta_W}\left(Q_L^3 - Q\sin^2\theta_W\right) Z^\mu \bar{b}_L \gamma_\mu b_L$$

- Before EWSB, $Q_L^3 = T_L^3$, after EWSB $Q_L^3 \to Q_L^3 + \delta Q_L^3$
- Electric charge $U(1)_V$ is conserved, $\delta Q = \delta Q_L^3 + \delta Q_R^3 = 0$
- If P_{LR} symmetry is imposed, $\delta Q_L^3 = \delta Q_R^3$

$\Rightarrow$ $\delta Q_L^3 = \delta Q_R^3 = 0 \quad \Rightarrow \quad$ **$Zb_L\bar{b}_L$ coupling is protected**

A Realistic Model: Particle Content

- Higgs transforms as a $(\mathbf{2},\mathbf{2})$ of $SU(2)_L \times SU(2)_R$
- For b_L, $T^3_R(b_L) = T^3_L(b_L)$ must hold
  $\Rightarrow$ Left-handed SM quark doublet is embedded into a $(\mathbf{2},\mathbf{2})$
- Right-handed SM up-quark is embedded into a $(\mathbf{1},\mathbf{1})$ singlet
- Right-handed SM down-quark is embedded into a $(\mathbf{1},\mathbf{3}) \oplus (\mathbf{3},\mathbf{1})$

Possible (gauge-invariant and P_{LR} symmetric) Yukawa interactions

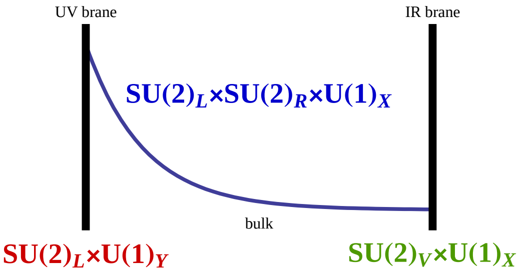
$$\overline{(\mathbf{2},\mathbf{2})}(\mathbf{2},\mathbf{2})(\mathbf{1},\mathbf{1}) \qquad \overline{(\mathbf{2},\mathbf{2})}(\mathbf{2},\mathbf{2})(\mathbf{1},\mathbf{3}) \qquad \overline{(\mathbf{2},\mathbf{2})}(\mathbf{2},\mathbf{2})(\mathbf{3},\mathbf{1})$$

Contino et al., hep-ph/0612048, Carena et al., hep-ph/0607106

- Necessity to implement P_{LR} leads to new particles of electrical charge $-1/3$, $2/3$, and $5/3$ with masses possibly $\lesssim$ 1TeV

$\Rightarrow$ **Smoking gun signature**

Gauge Structure



- Symmetry breaking on the UV brane by boundary conditions

$$SU(2)_R \times U(1)_X \to U(1)_Y$$

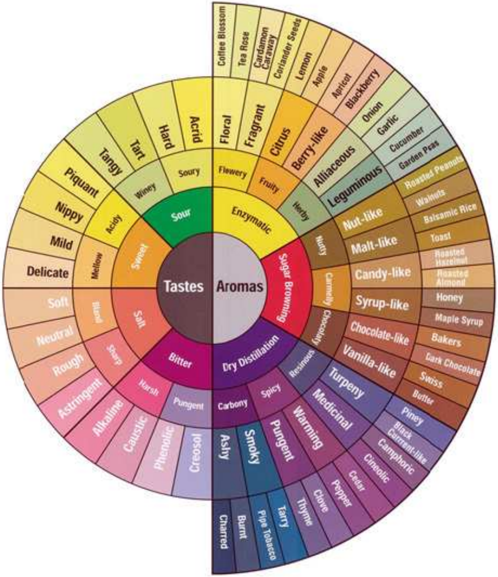
- Symmetry breaking on the IR brane by the Higgs vev

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

- Low energy symmetry structure

$$SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$$

Flavor Physics



Parameters in the Flavor Sector

Agashe, Perez, Soni, hep-ph/0408134

Sources of flavor violation in the RS model are...

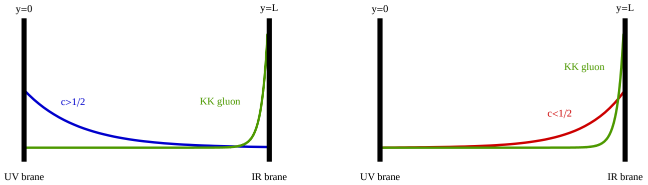
Hermitian 3×3 bulk mass matrices c_Q, c_u, c_d	3×6 real parameters 3×3 complex phases
Complex 3×3 Yukawa matrices λ_u, λ_d	2×9 real parameters 2×9 complex phases
	36 real parameters 27 complex phases
$U(3)^3$ flavor symmetry	-9 real parameters -17 complex phases
Physical flavor parameters (SM + RS)	**27 real parameters** **10 complex phases**

Flavor Violation

- 4D gauge couplings are determined by overlap integrals

$$\sim \frac{1}{L^{3/2}} \int_0^L dy\, f_{ferm}(y) f_{ferm}(y) f_{gauge}(y)$$

- Couplings of SM fermions to SM gauge bosons are universal
- Couplings of SM fermions to KK gauge bosons are non-universal



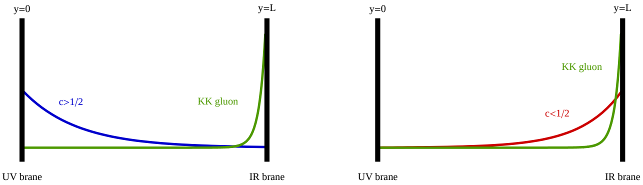
- When going to the quark mass eigenstate basis:

Non-universalities $\Rightarrow$ Flavor off-diagonal couplings

- Analog of GIM is active: “RS-GIM”

The RS-GIM Mechanism

- Both KK gauge and Higgs profiles are localized close to (or on) the IR brane
- This suggests that KK gauge couplings and quark masses are related



Small mass $\Longleftrightarrow$ small KK gauge coupling

Large mass $\Longleftrightarrow$ large KK gauge coupling

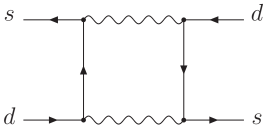
- The flavor off-diagonal couplings are proportional to the mass splitting:

$$\Delta_{ij} \sim (m_i - m_j) U_{ij}$$

Impact on K and B Meson Mixing

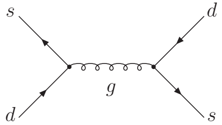
$$\Delta M_K\,,\ \epsilon_K\,,\ \Delta M_{B_{d,s}}\,,\ S_{\psi\phi},\ S_{\psi K_S},\ A^s_{SL},\ A^d_{SL}$$

SM: $\Delta F = 2$ processes proceed through boxes



Q_1^{VLL}

RS: $\Delta F = 2$ processes already at tree level



Q_1^{VLL}, Q_1^{VRR}, Q_1^{LR}, Q_2^{LR}

Particles exchanged at tree level:

- KK gluons
- KK photons
- $Z\,, Z_H\,, Z'$

Operators:

$$Q_1^{VLL} = (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu P_L d)$$
$$Q_1^{VRR} = (\bar{s}\gamma_\mu P_R d)(\bar{s}\gamma^\mu P_R d)$$
$$Q_1^{LR} = (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma^\mu P_R d)$$
$$Q_2^{LR} = (\bar{s}P_L d)(\bar{s}P_R d)$$

$\Delta F = 2$ Issues

KK gauge bosons contribute to $\Delta F = 2$ processes at the tree level.

On the other hand, the RS-GIM mechanism protects most observables from large corrections very efficiently.

But ϵ_K is special:

- In the SM, ϵ_K is accidentally suppressed by a factor 10^2 by CKM elements ($Re(M_{12}^K)/Im(M_{12}^K) \sim \mathcal{O}(10^2)$)
- In the RS model, new operators contribute to ϵ_K
- These operators are chirally and RG enhanced

Csaki, Falkowski, Weiler, 0804.1954

$\Rightarrow$ Tension between anarchic Yukawas, ϵ_K, and a low KK-scale

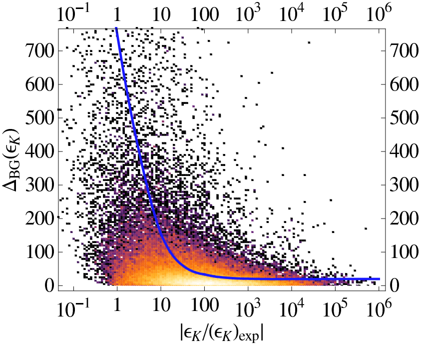
($M_{\text{KK}} \geq 21\text{TeV}$)

Fine-Tuning in ϵ_K

$$\Delta_{BG}(Obs.) = \max_i \left| \frac{d \ln(Obs.)}{d \ln(x_i)} \right| = \max_i \left| \frac{x_i}{Obs.} \frac{dObs.}{dx_i} \right|$$

Barbieri, Giudice

M. Blanke, A. Buras, BD, S. Gori, A. Weiler, [arXiv:0809.1073]



$M_{KK} \simeq 2.45\text{TeV}$

- Generically, $\epsilon_K \simeq 10^2 \epsilon_K^{exp}$
- Δ_{BG} decreases with increasing ϵ_K
- Parameter sets with moderate Δ_{BG} and $\epsilon_K \approx \epsilon_K^{exp}$ exist

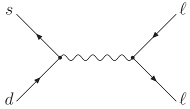
$\Delta F = 2$ Summary

- There is a **generic tension** between anarchic Yukawas, small RS contributions to ϵ_K and LHC-reachable M_{KK}
- But: The structure of the RS model is rich enough to **still achieve all of the above** for small or moderate fine-tuning in sizeable areas of parameter space
- All other $\Delta F = 2$ observables, in particular in the B system require nearly no fine-tuning at all
- EW gauge bosons contribute significantly to $\Delta B = 2$ processes
- **Large effects in the** B **system** are possible
- Use the data sets that fulfill all constraints for the analysis of rare decays

Plan for Rare K and B Decays

- Investigate impact of custodial protection
- Estimate relative sizes of NP effects in the K and $B_{d,s}$ systems
- What would happen if the custodial protection was removed?
- Are there correlations between observables?

Loop Functions X,Y,Z



- Tree level contributions from $Z, Z_H, Z', A^{(1)}$
- Effective Hamiltonian (e.g. for $K \to \pi\nu\bar{\nu}$)

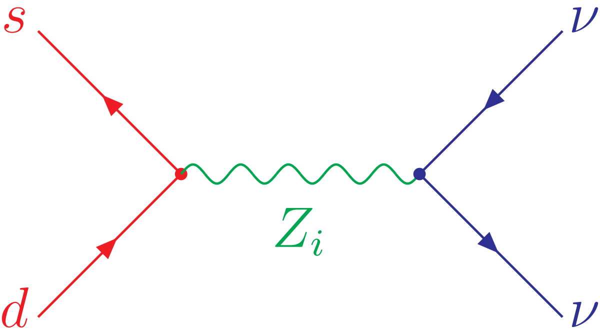
$$\left[\mathcal{H}_{\text{eff}}^{\nu\bar{\nu}}\right]^K \propto V_{ts}^* V_{td} \sum_{\ell=e,\mu,\tau} \left[X_{\text{SM}} + X_K^{V-A}\right] (\bar{s}d)_{V-A} (\bar{\nu}_\ell \nu_\ell)_{V-A}$$
$$+ V_{ts}^* V_{td} \sum_{\ell=e,\mu,\tau} X_K^{V-A} (\bar{s}d)_V (\bar{\nu}_\ell \nu_\ell)_{V-A}$$

- Coefficients of SM operators are modified, new operators are present

Modified properties of the loop functions X, Y, Z:

loop-induced	$\rightarrow$	tree-induced
flavor-universal	$\rightarrow$	non-universal
real	$\rightarrow$	complex

Anatomy of X,Y,Z



$$\left(X^K_{Z_i}\right)^{V-A} = \frac{1}{\lambda_t}\frac{1}{4g^2_{\text{SM}}}\left[\Delta^{sd}_L(Z_i) - \Delta^{sd}_R(Z_i)\right]\frac{1}{M^2_{Z_i}}\Delta^{\nu\nu}_L(Z_i)$$

EW Gauge Bosons and Custodial Protection

M. Albrecht, M. Blanke, A. Buras, BD, K. Gemmler, [arXiv:0903.2415]

M. Blanke, A. Buras, BD, S. Gori, A. Weiler, [arXiv:0809.1073]

- Z, Z', Z_H are linear combinations of $Z^{(0)}$, $Z^{(1)}$ and $Z_X^{(1)}$
- Coefficients enter couplings to quarks accordingly

$$Z \simeq Z^{(0)} - \frac{g^2 v^2 \mathcal{I}_1}{4L^2 M^2 \cos^2\psi} \times \left[Z^{(1)} - \cos\psi \cos\phi Z_X^{(1)}\right]$$

$$\Delta_L(Z), \Delta_L(Z') \sim g_L(d) - \kappa_1(d) \cos\psi \cos\phi$$

- The above linear combination of coupling constants is zero

  $\Rightarrow$ Suppressed flavor off-diagonal couplings
  - of Z,Z' to left-handed down-quarks
  - of Z,Z' to right-handed up-quarks

- Cancellation is not complete due to symmetry breaking effects on the UV brane
- Cancellation is weaker in the case of Z'
- Z_H couplings are unsuppressed


INTERMISSION

Summary: Which Quantities are Protected?

- T-Parameter — Agashe, Delgado, May, Sundrum, hep-ph/0308036; Csaki, Grojean, Pilo, Terning, hep-ph/0308038
- $Zb_L\bar{b}_L$ — Agashe, Contino, DaRold, Pomarol, hep-ph/0605341
- $Zd^i_L\bar{d}^j_L$ — Blanke, Buras, BD, Gori, Weiler, arXiv:0809.1073; Blanke, Buras, BD, Gemmler, Gori, arXiv:0812.3803
- $Zu^i_R\bar{u}^j_R$ — Buras, BD, Gori, arXiv:0903.soon

Unprotected however are

$$Zd^i_R\bar{d}^j_R, \qquad Zu^i_L\bar{u}^j_L, \qquad W^+u^i_Ld^j_L, \qquad W^+u^i_Rd^j_R$$

Impact of KK-Fermions

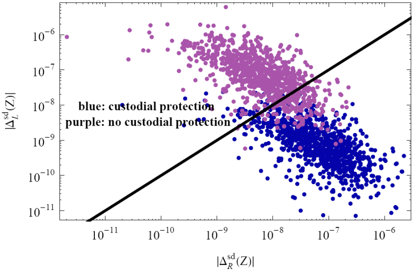
An effective Lagrangian approach Buras, BD, Gori, arXiv:0903.soon

- Derivation of general formulae for SM gauge-fermion interactions
- Explicit demonstration that custodial protection is **not spoilt** by mixing with KK-fermions
- Brute force calculation of above SM couplings and validation of general formulae
- Study violation of CKM unitarity

End of Intermission

Off-diagonal Z Couplings and Custodial Protection

Compare the situation with custodial protection (blue points) to the situation without custodial protection (purple points)



- With custodial protection: $\langle \Delta_R(Z) \rangle \sim \mathcal{O}(10^2) \langle \Delta_L(Z) \rangle$
- Without custodial protection: $\langle \Delta_R(Z) \rangle \sim \mathcal{O}(10^{-1}) \langle \Delta_L(Z) \rangle$

For Z' and active custodial protection: $\langle \Delta_R(Z') \rangle \sim \mathcal{O}(10^1) \langle \Delta_L(Z') \rangle$

Relative Contributions of Z, Z' and Z_H

$\Delta F = 2$: Suppressed quark couplings enter twice in processes with four external quarks

$\Rightarrow$ Z, Z' hardly contribute, Z_H clearly dominates

$\Delta F = 1$: Several effects have to be considered:

- Custodial Protection for some quark couplings (Z, Z')
- Mass suppression (Z_H, Z')
- Volume-suppression for some lepton couplings (Z_H, Z')

Upshot: In rare K and B decays...

Couplings of Z to right-handed down-quarks dominate

Estimate for NP effects in K and B Systems

With custodial protection: **Coupling of Z to RH quarks dominates**

- Hierarchy between meson systems in couplings is (roughly)
  $\Delta_R^{sd}(Z) : \Delta_R^{bd}(Z) : \Delta_R^{bs}(Z) \approx 1 : 5 : 10$
- Hierarchy between CKM factors:
  $\lambda_t^{(K)} : \lambda_t^{(d)} : \lambda_t^{(s)} \simeq 1 : 25 : 100$

Size of NP effects expected to be **largest in the K system**, by factor 4 smaller in B_d system and by another factor of 2 smaller in the B_s system.

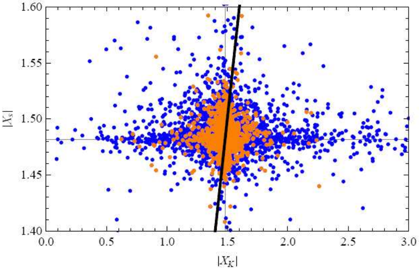
Without custodial protection: **Coupling of Z to LH quarks dominates**

- Hierarchy between meson systems in couplings is (roughly)
  $\Delta_L^{sd}(Z) : \Delta_L^{bd}(Z) : \Delta_L^{bs}(Z) \approx 1 : 15 : 100$

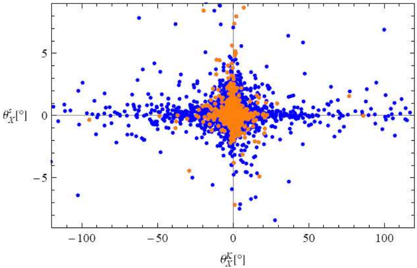
Size of NP effects expected to be **similar in the K and $B_{d,s}$ systems**.

Breakdown of Universality in X,Y,Z

...in moduli



...as well as in phases



blue points: arbitrary fine-tuning, orange points: moderate fine-tuning

Where e.g

$$X_K \equiv X_{\text{SM}} + X_K^{V-A} + X_K^V \equiv |X_K|\, e^{i\theta_K}$$

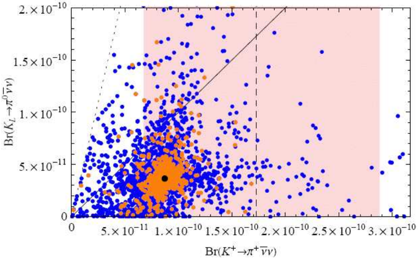
As estimated:

CP-conserving and CP-violating effects in the K system can be much larger than in the $B_{d,s}$ systems

Rare K Decays

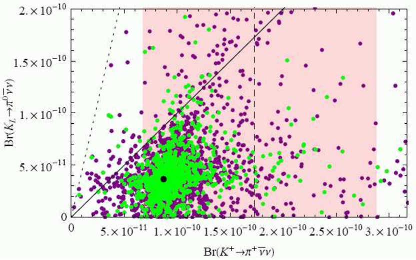
e.g. $Br(K_L \to \pi^0 \nu\bar{\nu})$ vs $Br(K^+ \to \pi^+ \nu\bar{\nu})$

With custodial protection:



- $Br(K_L \to \pi^0 \nu\bar{\nu})$: Enhancement by factor 5 possible
- $Br(K^+ \to \pi^+ \nu\bar{\nu})$: Enhancement by factor 2 possible

Without custodial protection:

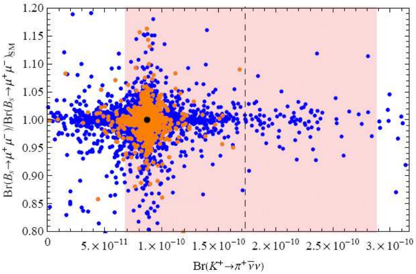


- Enhancement by another factor of 2 is possible

Correlation between Rare K and B Decays

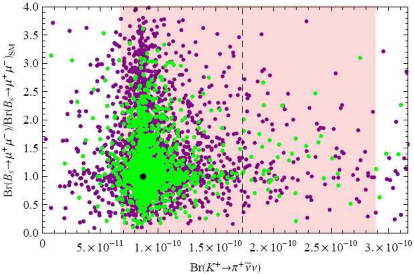
e.g. $Br(B_s \to \mu^+\mu^-)$ vs $Br(K^+ \to \pi^+\nu\bar{\nu})$

With custodial protection:



- $Br(B_s \to \mu^+\mu^-)$: Deviation from SM by 15% possible
- Much larger effects in $Br(K^+ \to \pi^+\nu\bar{\nu})$

Without custodial protection:



- $Br(K^+ \to \pi^+\nu\bar{\nu})$: Enhancement by another factor of 2
- $Br(B_s \to \mu^+\mu^-)$: Large enhancement
- Effects in K and B have roughly the same size

MFV Correlations between Decay Modes

Buras, hep-ph/0310208

Correlations between decay modes arise because...

- ...they are based on the same universal loop functions,

  e.g. $K_L \to \pi^0 \nu\bar{\nu} \quad \leftrightarrow \quad B \to X_{d,s} \nu\bar{\nu}$
- ...they are based on different loop functions, but in which NP is expected to enter in a similar manner,

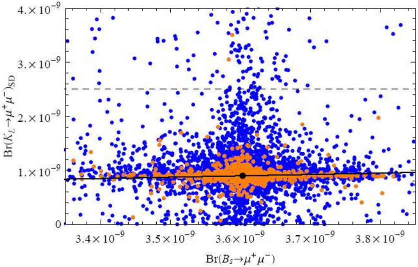
  e.g. $K_L \to \pi^0 \nu\bar{\nu} \quad \leftrightarrow \quad B_s \to \mu^+\mu^-$

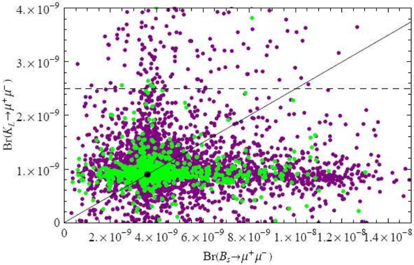
Besides that there are **correlations between ratios of observables** that arise because universal loop functions cancel out,

e.g. $Br(B_s \to \mu^+\mu^-)/Br(B_d \to \mu^+\mu^-) \quad \leftrightarrow \quad \Delta M_s/\Delta M_d$

In the RS model, we expect these MFV correlations to be clearly broken.

$K_L \to \mu^+\mu^-$ and $B_s \to \mu^+\mu^-$

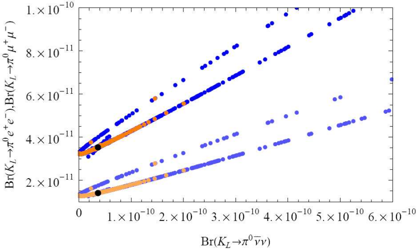




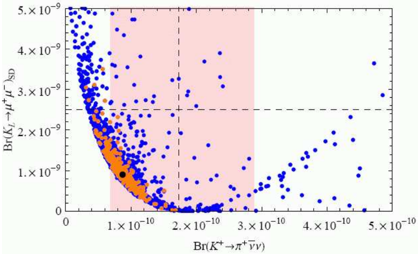
- Effects in K are larger than in B
- MFV correlation is strongly broken

- Effects in K and B similar in size
- Large effects in K and B **not simultaneously**
- MFV correlation broken even more strongly

$K_L \to \pi^0 \ell^+ \ell^-$, $K \to \pi \nu \bar{\nu}$ and $K_L \to \mu^+ \mu^-$



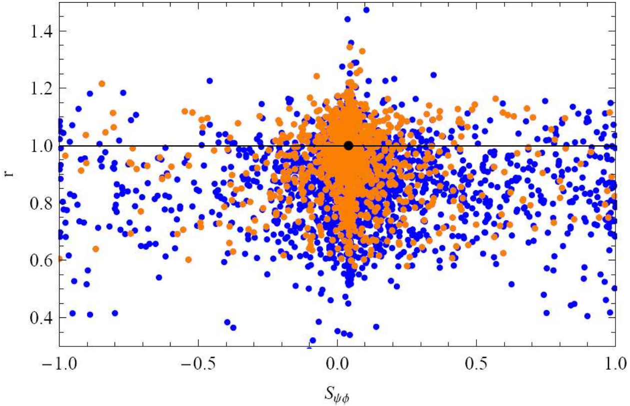
- Constructive interference assumed
- $Br(K_L \to \pi^0 \ell^+ \ell^-)$ enhanced by at most 60%
- Strong correlation



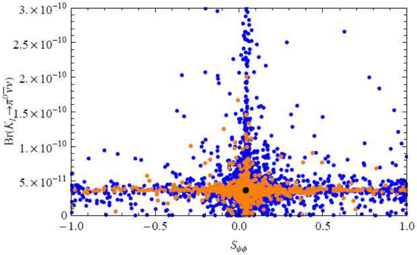
- Only short distance contribution shown
- Both CP conserving decays
- NP enters with opposite sign due to different operator structure

The Golden MFV Relation

$$\frac{Br(B_s \to \mu^+\mu^-)}{Br(B_d \to \mu^+\mu^-)} = \frac{\hat{B}_{B_d}}{\hat{B}_{B_s}} \frac{\tau(B_s)}{\tau(B_d)} \frac{\Delta M_s}{\Delta M_d} r$$



Rare K Decays and Mixing in the B System



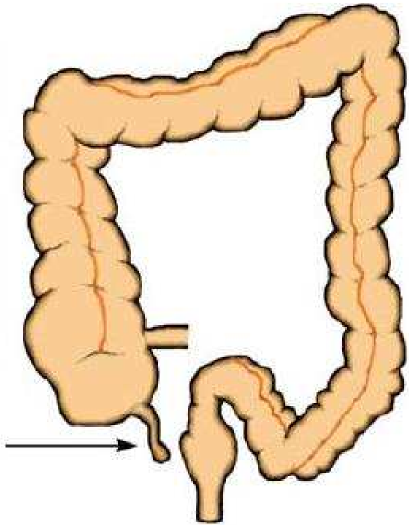
- $S_{\psi\phi}$ can be strongly enhanced
- Simultaneous large effects in both observables unlikely

$\Rightarrow$ **Clear experimental signature!**

Conclusions

- The RS model addresses the gauge hierarchy as well as the flavor problem
- Contributions to $\Delta F = 2$ processes can be consistent with experimental data
- In rare decays, significant enhancements in the K system are possible
- Effects in the $B_{d,s}$ systems are more modest
- Clear distinction from models of MFV is possible
- Simultaneous effects in K decays in $S_{\psi\phi}$ are unlikely

Backup Slides



Explicit Multiplets

- Higgs transforms as a (self-adjoined) bi-doublet of $SU(2)_L \times SU(2)_R$

$$H = \begin{pmatrix} \pi^+/\sqrt{2} & -(h^0 - i\pi^0)/2 \\ (h^0 + i\pi^0)/2 & \pi^-/\sqrt{2} \end{pmatrix}$$

- Left-handed SM quark doublet is embedded into a bi-doublet

$$\xi^i_{1L} = \begin{pmatrix} \chi^{u_i}_L(-+)_{5/3} & q^{u_i}_L(++)_{2/3} \\ \chi^{d_i}_L(-+)_{2/3} & q^{d_i}_L(++)_{-1/3} \end{pmatrix}_{2/3}$$

- Right-handed SM down-quark is embedded into a singlet

$$\xi^i_{2R} = u^i_R(++)_{2/3}$$

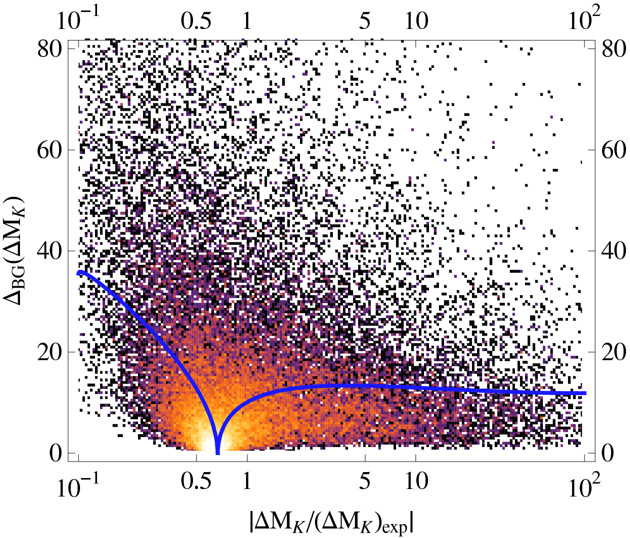
- Right-handed SM down-quark is embedded into a $(\mathbf{3},\mathbf{1}) \oplus (\mathbf{1},\mathbf{3})$

$$\xi^i_{3R} = \begin{pmatrix} \psi'^i_R(-+)_{5/3} \\ U'^i_R(-+)_{2/3} \\ D'^i_R(-+)_{-1/3} \end{pmatrix}_{2/3} \oplus \begin{pmatrix} \psi''^i_R(-+)_{5/3} \\ U''^i_R(-+)_{2/3} \\ D^i_R(++)_{-1/3} \end{pmatrix}_{2/3}$$

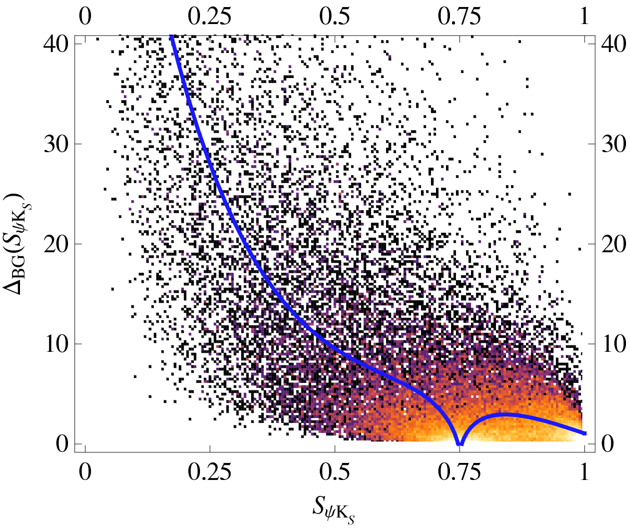
Froggat-Nielsen Equations

$$
\begin{aligned}
m_b &= \frac{v}{\sqrt{2}} \lambda^d_{33} \frac{e^{kL}}{kL} f^Q_3 f^d_3 \\
m_s &= \frac{v}{\sqrt{2}} \frac{\lambda^d_{33}\lambda^d_{22} - \lambda^d_{23}\lambda^d_{32}}{\lambda^d_{33}} \frac{e^{kL}}{kL} f^Q_2 f^d_2 \\
m_d &= \frac{v}{\sqrt{2}} \frac{\det(\lambda^d)}{\lambda^d_{33}\lambda^d_{22} - \lambda^d_{23}\lambda^d_{32}} \frac{e^{kL}}{kL} f^Q_1 f^d_1
\end{aligned}
$$

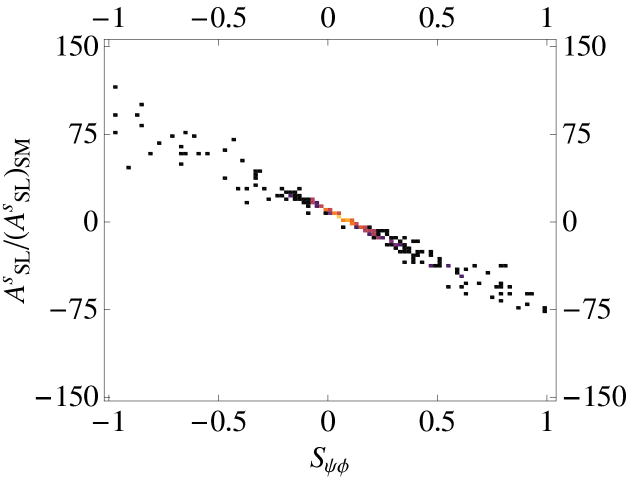
$$
(\mathcal{D}_L)_{ij} = \begin{cases} \omega^d_{ij} \frac{f^Q_i}{f^Q_j} & (i < j) \\ 1 & (i = j) \\ \omega^d_{ij} \frac{f^Q_j}{f^Q_i} & (i > j) \end{cases}
\qquad
(\mathcal{D}_R)_{ij} = \begin{cases} \rho^d_{ij} \frac{f^d_i}{f^d_j} & (i < j) \\ 1 & (i = j) \\ \rho^d_{ij} \frac{f^d_j}{f^d_i} & (i > j) \end{cases}
$$



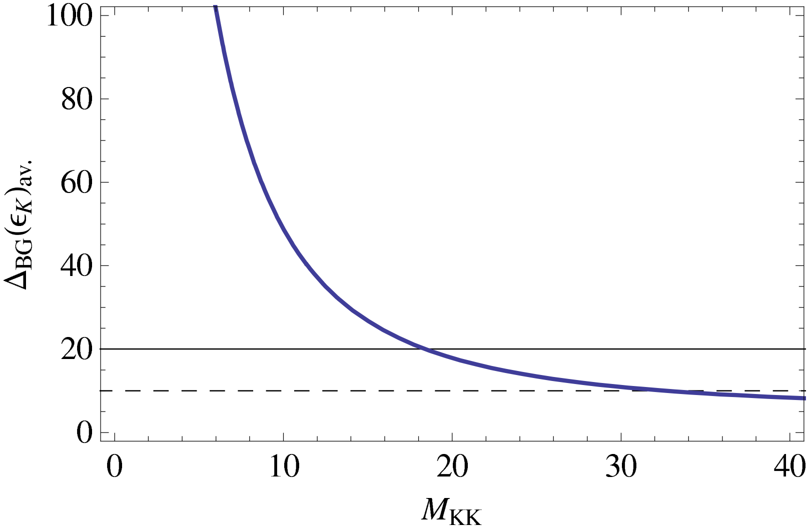
10^{-1}
0.5
1
5
10
10^{2}
80
60
40
20
0
$\Delta_{BG}(\Delta M_K)$
$|\Delta M_K/(\Delta M_K)_{exp}|$



0
0.25
0.5
0.75
1
40
30
20
10
0
$\Delta_{BG}(S_{\psi K_S})$
$S_{\psi K_S}$



−1
−0.5
0
0.5
1
150
75
0
−75
−150
$A^s_{SL}/(A^s_{SL})_{SM}$
$S_{\psi\phi}$



100
80
60
40
20
0
0
10
20
30
40
$\Delta_{BG}(\epsilon_K)_{av.}$
M_{KK}

Explicit Expressions for X,Y,Z

$$
\begin{aligned}
\left(X^K_{Z_i}\right)^{V-A} &= \frac{1}{\lambda_t}\frac{\Delta_L^{\nu\nu}(Z_i)}{4M^2_{Z_i}g^2_{\text{SM}}}\left[\Delta_L^{sd}(Z_i)-\Delta_R^{sd}(Z_i)\right] \\
\left(X^K_{Z_i}\right)^{V} &= \frac{1}{\lambda_t}\frac{\Delta_L^{\nu\nu}(Z_i)}{2M^2_{Z_i}g^2_{\text{SM}}}\Delta_R^{sd}(Z_i) \\
\left(Y^K_{Z_i}\right)^{V-A} &= -\frac{1}{\lambda_t}\frac{\left[\Delta_L^{\ell\ell}(Z_i)-\Delta_R^{\ell\ell}(Z_i)\right]}{4M^2_{Z_i}g^2_{\text{SM}}}\left[\Delta_L^{sd}(Z_i)-\Delta_R^{sd}(Z_i)\right] \\
\ldots
\end{aligned}
$$

Ranges for X,Y,Z

$$X(x_t) = 1.48\,, \quad Y(x_t) = 0.94\,, \quad Z(x_t) = 0.65$$

$$0.60 \le \frac{|X_K|}{X(x_t)} \le 1.30\,, \quad 0.90 \le \frac{|X_d|}{X(x_t)} \le 1.12\,, \quad , \quad 0.95 \le \frac{|X_s|}{X(x_t)} \le 1.08$$

$$0.45 \le \frac{|Y_K|}{Y(x_t)} \le 1.60\,, \quad 0.85 \le \frac{|Y_d|}{Y(x_t)} \le 1.20\,, \quad , \quad 0.93 \le \frac{|Y_s|}{Y(x_t)} \le 1.12$$

$$0.35 \le \frac{|Z_K|}{Z(x_t)} \le 2.05\,, \quad 0.80 \le \frac{|Z_d|}{Z(x_t)} \le 1.30\,, \quad , \quad 0.90 \le \frac{|Z_s|}{Z(x_t)} \le 1.17$$