

# RR photons

Pablo G. Cámara

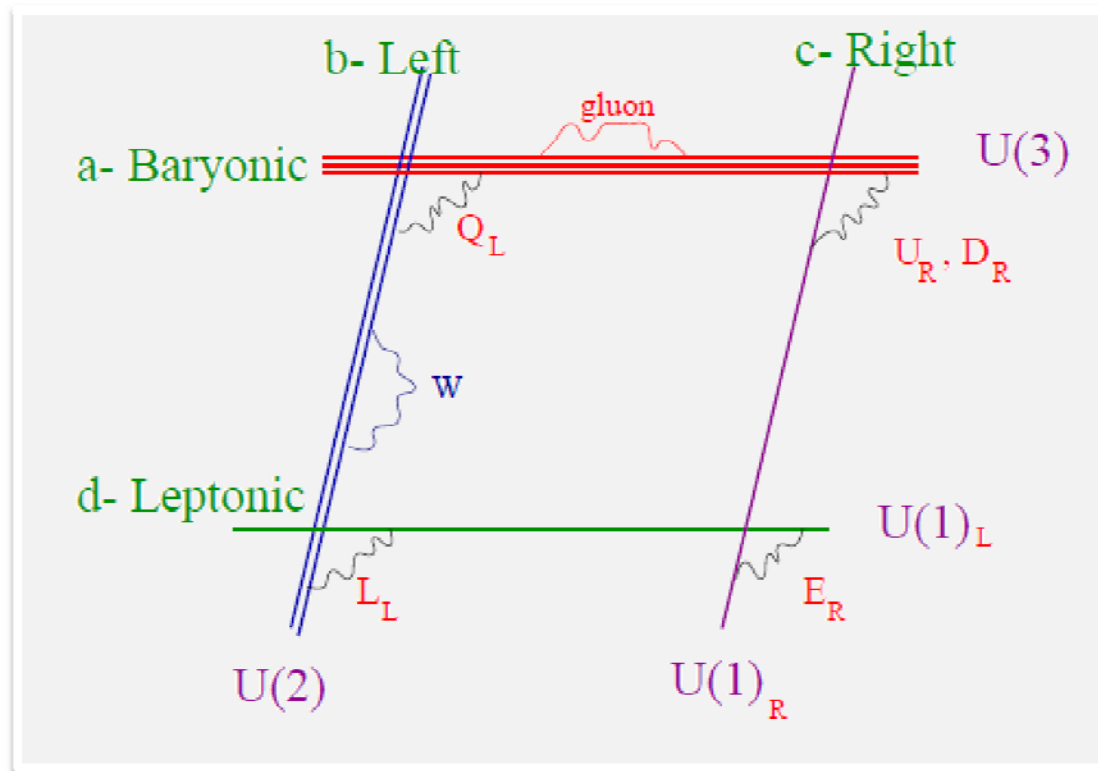


with L. E. Ibáñez and F. Marchesano, [arXiv:1106.0060](https://arxiv.org/abs/1106.0060) [hep-th].

Cornell University, 31 August 2011

# 1. Motivation

String theory compactifications with a semi-realistic spectrum generically lead to  $U(1)$  gauge symmetries beyond  $U(1)_Y$



[Cremades, Ibanez, Marchesano '02]

# 1. Motivation

- Some of these extra U(1) gauge symmetries acquire masses via the Stückelberg mechanism

$$\mathcal{L} \supset C_2 \wedge F_2 \quad \Rightarrow \quad \mathcal{L}_{\text{Stk}} = \frac{1}{2}(d\rho + qA)^2 \quad (d\rho = *_4 dC_2)$$

$$M_{U(1)_X} \sim M_s \quad \Rightarrow$$

global symmetries

broken by non-perturbative effects to discrete subgroups (e.g. matter parity, baryon triality...)

[Berasaluce et al. '11]

Only detectable at experiments if  $M_s \sim 1 \text{ TeV}$  (WIMPs)

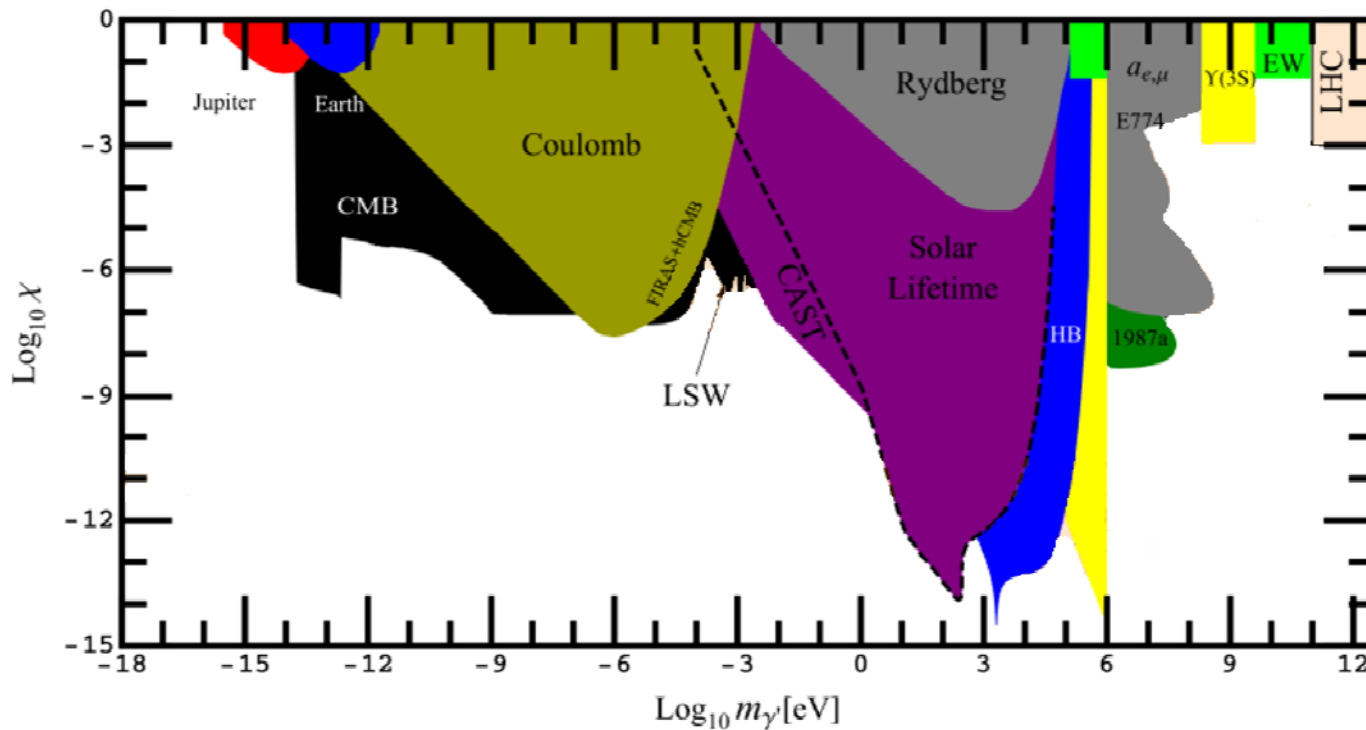
[Ghilencea et al. '02]

- Other U(1)'s however may remain massless or very light (WISPs) and lead to **light hidden U(1) gauge symmetries.**

# 1. Motivation

Light hidden U(1) gauge symmetries are a window of opportunity to hidden sector physics, even at large string scale

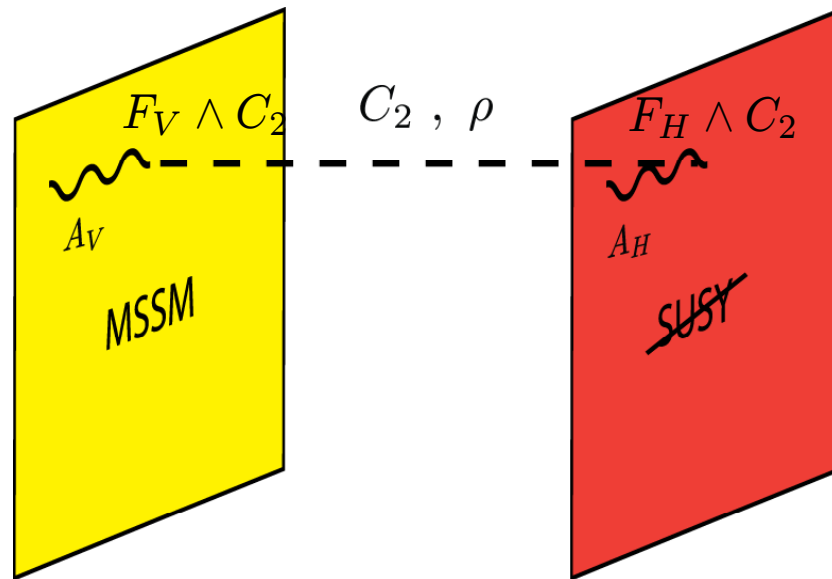
$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\chi}{2}X_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_{\gamma'}^2 X_\mu X^\mu$$



# 1. Motivation

- Hidden U(1)'s are also a possible mechanism for **mediating SUSY breaking** in a flavor independent way:

[Langacker et al '07]  
[Verlinde et al. '07]



$$\mathcal{L} \supset \frac{1}{2} |d\rho + eA_V + qA_H|^2$$



$$A_Y \sim eA_V - qA_H$$
$$A_X \sim eA_V + qA_H$$

# 1. Motivation

In type II string theory compactifications there are two sources of hidden U(1) gauge symmetries:

- D-branes located 'far away' from the MSSM D-brane sector
- Bulk U(1)'s arising from KK reduction of the Ramond-Ramond closed string fields  $\Rightarrow$  no massless matter charged under them

*It is therefore natural to ask:*

- *Can RR U(1)'s mix with the hypercharge??*
- *If so, can we compute  $\chi$  and  $m_{\gamma'}$  ??*
- *Can we obtain new phenomenological scenarios ??*

# 1. Motivation

- *Can RR  $U(1)$ 's mix with the hypercharge??*
- *If so, can we compute  $\chi$  and  $m_{\gamma'}$  ??*
- *Can we obtain new phenomenological scenarios ??*

Moreover, the **distinction between RR and D-brane  $U(1)$ 's is arbitrary at strong coupling**: in M-theory / F-theory both correspond to KK  $U(1)$ 's

Another related question is therefore:

- *Is there a geometric understanding of the Stueckelberg mechanism??*

# Outline of the talk

1.  $U(1)$ 's in type IIA compactifications
2. 'Kinetic mixing' with RR photons
3. 'Mass mixing' with RR photons
4. Some phenomenological implications
5. The unified M-theory picture
6. Concluding remarks



## 2. U(1)'s in type IIA compactifications

Type IIA string theory on a CY orientifold  $\mathbb{R}^{1,3} \times \mathcal{M}_6/\Omega_p(-1)^{F_L}\sigma$

$$\sigma J = -J, \quad \sigma \Omega = \bar{\Omega}$$

- **Closed string spectrum:** one-to-one correspondence between massless 4d closed-string fields and **harmonic forms**

$$h_-^{1,1} + h^{1,2} + 1 \quad N = 1 \text{ chiral multiplets}$$

$$h_+^{1,1} \quad N = 1 \text{ vector multiplets}$$

## 2. U(1)'s in type IIA compactifications

$$h_-^{1,1} + h^{1,2} + 1 \quad N = 1 \text{ chiral multiplets}$$

Scalar components parametrize compactification moduli space:

$$J_c \equiv B_2 + iJ = T^{\hat{i}} \omega_{\hat{i}}, \quad \Omega_c \equiv C_3 + i\text{Re}(C\Omega) = N^I \alpha_I$$

Real parts of complex structure moduli  axions

- Invariant under shift symmetries
- Can participate in Stückelberg mechanism

Dual 2-forms: 
$$C_5 = \sum_I C_2^I \wedge \beta^I + \dots$$

## 2. U(1)'s in type IIA compactifications

$$h_+^{1,1} \quad N = 1 \text{ vector multiplets}$$

RR U(1) gauge bosons from the expansions:

$$C_3 = \sum_I \text{Re}(N^I) \alpha_I + \sum_i A^i \wedge \omega_i$$

Dual magnetic d.o.f. from  $C_5$

Gauge kinetic function

[Grimm, Louis '04]

$$f_{ij} = -i\mathcal{K}_{ij\hat{k}} T^{\hat{k}}$$

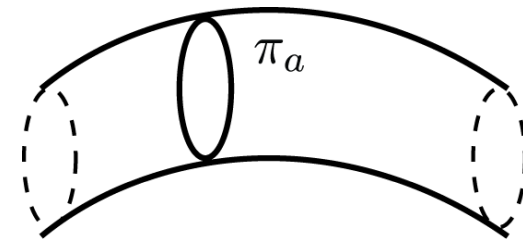
Each massless  $U(1)_{\text{RR}}$  has an element of  $H_2^+(\mathcal{M}_6, \mathbb{R})$  associated to it.

## 2. U(1)'s in type IIA compactifications

*D6-brane  $N = 1$  vector & chiral multiplets*

D6-branes wrap special Lagrangian 3-cycles in the CY

$$\mathcal{J}|_{\pi_a} = 0, \quad \text{Im}(\Omega)|_{\pi_a} = 0$$



Standard Model located in this sector

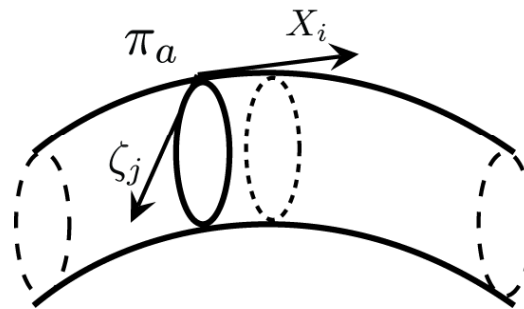
$N_a$  D6-branes



$$SU(N_a) \times U(1)_a$$

$$f_a = -iN_a \int_{\pi_a} \Omega_c$$

Deformations preserving sLag parametrized by  $b_1(\pi_a)$  open string moduli:



[McLean '98]

$$\Phi_a^j = \theta_a^j + \lambda_i^j \phi_a^i$$

$$\theta_a = \theta_a^j \zeta_j, \quad \phi_a = \phi_a^i X_i, \quad \iota_{X_i} \mathcal{J}_c|_{\pi_a} = \lambda_i^j \zeta_j$$

## 2. U(1)'s in type IIA compactifications

*D6-brane N = 1 vector & chiral multiplets*

There is a Stückelberg mechanism for some of the D6-brane U(1)'s:

$$\int_{\mathbb{R}^{1,3} \times \pi_a} C_5 \wedge F_2^a = -c_a^I \int_{\mathbb{R}^{1,3}} C_2^I \wedge F_2^a \quad \Rightarrow \quad Q^I = \sum_a c_a^I N_a Q^a \quad \text{is massive}$$

$$c_a^I = - \int_{\pi_a} \beta^I$$

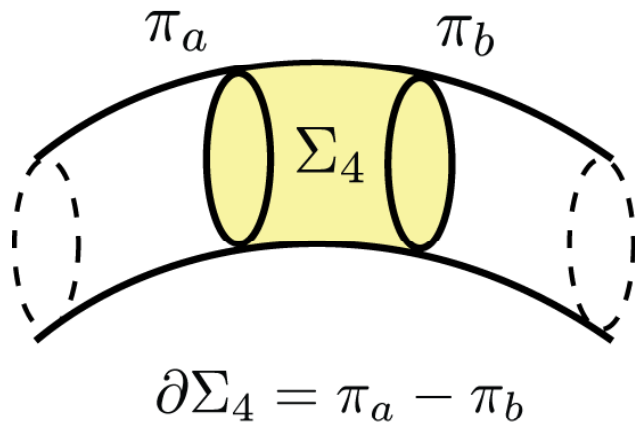
**Nice interpretation geometric interpretation.** Each D6-brane U(1)<sub>a</sub> gauge symmetry has an element of  $H_3^-(\mathcal{M}_6, \mathbb{R})$  associated to it,  $\pi_a - \pi_a^*$

$$Q^b = \sum_a n_a^b Q^a \quad \text{massless} \quad \longleftrightarrow \quad \pi_b^- = \sum_a n_a^b N_a (\pi_a - \pi_a^*) \quad \text{trivial}$$

( $\partial \Sigma_4 = \pi_b^-$ )

## 2. U(1)'s in type IIA compactifications

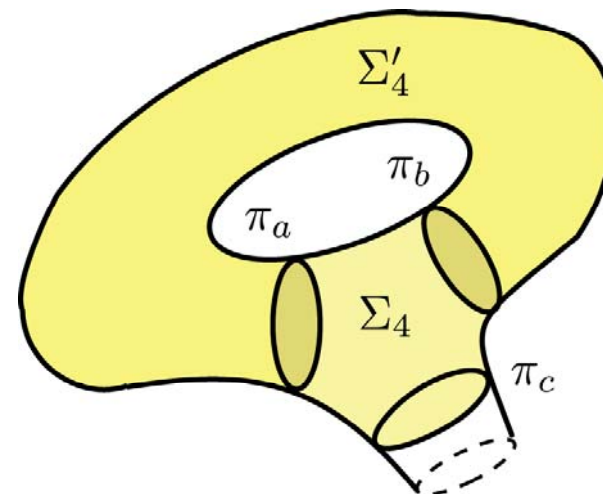
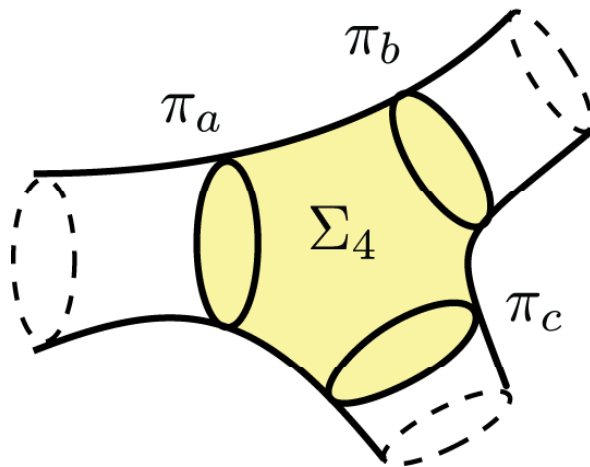
*D6-brane N = 1 vector & chiral multiplets*



$$U(2) \rightarrow U(1)_a \times U(1)_b$$

$$U(1)_a - U(1)_b \quad \text{massless}$$

$$U(1)_a + U(1)_b \quad \text{massive}$$



...

### 3. Kinetic mixing with RR photons

Can D6-brane and RR U(1)'s mix kinematically ??

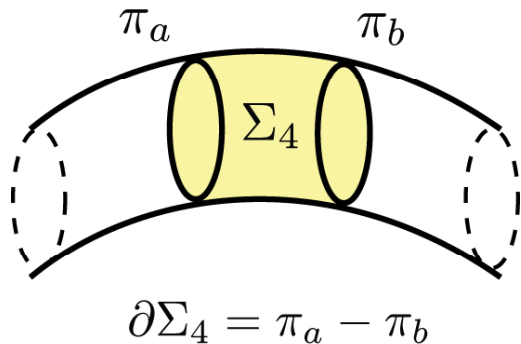
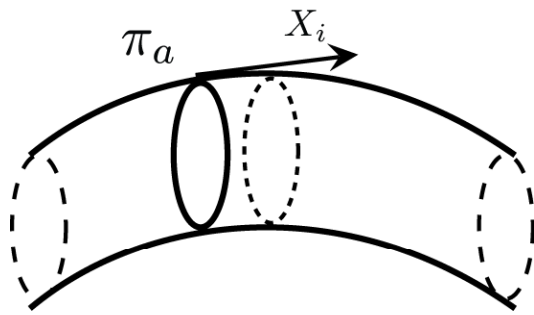
$$S_{4d,\text{mix}} = - \int_{\mathbb{R}^{1,3}} [\text{Re}(f_{ia}) F_{\text{RR}}^i \wedge *_4 F_2^a + \text{Im}(f_{ia}) F_{\text{RR}}^i \wedge F_2^a]$$

From the D6-brane CS action:

$$\mathcal{F}_2^a \wedge C_5 + \frac{1}{2} \mathcal{F}_2^a \wedge \mathcal{F}_2^a \wedge C_3 \quad \Rightarrow \quad f_{ia} = \Phi_a^j \int_{\pi_a} \omega_i \wedge \zeta^j + \dots$$

Requires non-trivial 2-cycle in  $\pi_a$  and  $\mathcal{M}_6$

Well-defined for massless U(1)'s:



$$f_{i(a-b)} = (\Phi_a^j - \Phi_b^j) \int_{\rho_j} \omega_i + \dots \quad \Rightarrow \quad f_{ib} = \int_{\Sigma_4} (J_c + F_2^b) \wedge \omega_i$$

## 4. Mass mixing with RR photons

We have seen the following U(1) charge assignment:

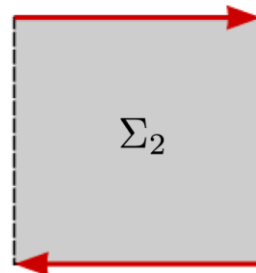
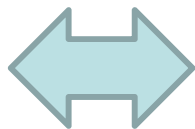
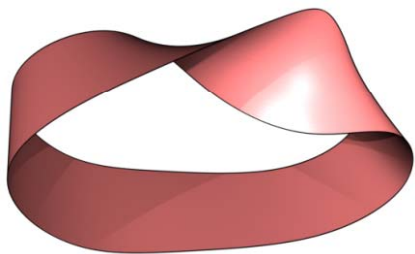
$$H_3^-(\mathcal{M}_6, \mathbb{R}) \quad \Rightarrow \quad \text{D6-brane U(1)'s}$$

$$H_2^+(\mathcal{M}_6, \mathbb{R}) \quad \Rightarrow \quad \text{RR U(1)'s}$$

But shouldn't be  $H_r(\mathcal{M}_6, \mathbb{Z})$  the relevant classes??

$$H_r(\mathcal{M}_6, \mathbb{Z}) = \underbrace{\mathbb{Z} \oplus \dots \oplus \mathbb{Z}}_{b_r} \oplus \mathbb{Z}_{k_1} \oplus \dots \oplus \mathbb{Z}_{k_n}$$

$$\partial \Sigma_{r+1} = k \pi_r^{\text{tor}}$$



$$H_1(\mathcal{M}, \mathbb{Z}) = \mathbb{Z}_2$$

Torsional cycles cannot be detected by 4d massless fields because

$$\int_{\pi_r^{\text{tor}}} \omega_r = \frac{1}{k} \int_{\Sigma_{r+1}} d\omega_r = 0$$




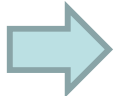
## 4. Mass mixing with RR photons

Some useful results in algebraic topology (UCT + Poincaré duality):

$$\text{Tor } H_3(\mathcal{M}_6, \mathbb{Z}) \simeq \text{Tor } H_2(\mathcal{M}_6, \mathbb{Z})$$

$$\text{Tor } H_1(\mathcal{M}_6, \mathbb{Z}) \simeq \text{Tor } H_4(\mathcal{M}_6, \mathbb{Z})$$

D2-brane wrapping  $\pi_2^{\text{tor}}$   4d particle

D4-brane wrapping  $\pi_3^{\text{tor}}$   4d string

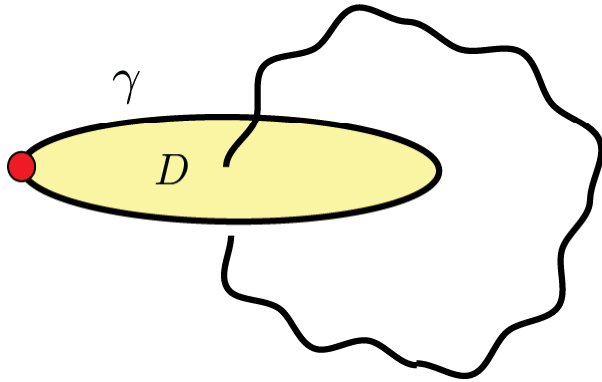
Non-BPS objects in 4d, but stable mod  $k$

Aharonov-Bohm strings and particles

[Alford, Krauss, Wilczek '89]

## 4. Mass mixing with RR photons

$$dF_4 = \delta_5$$



They satisfy  $\mathbb{Z}_k$  holonomies:

$$\frac{1}{2\pi i} \log [\text{hol}(\gamma, [\pi_2^{\text{tor}}])] \equiv_{\text{mod } 1} \frac{1}{k} \int_{D \times k\pi_2^{\text{tor}}} F_4 = \frac{1}{k} \int_{D \times \Sigma_3} \delta_5 = \frac{p}{k}$$

Linking number  $\equiv_{\text{mod } 1} L([\pi_2^{\text{tor}}], [\pi_3^{\text{tor}}])$

A-B strings and particles are the smoking gun of massive U(1)'s higgsed down to a discrete  $\mathbb{Z}_k$  gauge symmetry via the Stuckelberg mechanism

[Banks, Seiberg '10]

We can see this more explicitly from dimensional reduction.

## 4. Mass mixing with RR photons

For that we introduce the set of forms which correspond to the generators of  $\text{Tor } H^4(\mathcal{M}_6) \simeq \text{Tor } H_3(\mathcal{M}_6)$  and  $\text{Tor } H^3(\mathcal{M}_6) \simeq \text{Tor } H_2(\mathcal{M}_6)$

$$d\omega_\alpha^{\text{tor}} = k_\alpha^\beta \alpha_\beta^{\text{tor}}, \quad d\beta^{\text{tor},\beta} = -k^\beta_\alpha \tilde{\omega}^{\text{tor},\alpha} \quad L([\pi_{2,\alpha}^{\text{tor}}], [\pi_3^{\text{tor},\beta}]) = (k^{-1})_\alpha^\beta$$

Expanding in these,

$$C_3 = \sum_\alpha \text{Re}(N^\alpha) \alpha_\alpha^{\text{tor}} + A^\alpha \wedge \omega_\alpha^{\text{tor}} + \dots$$



$$dC_3 = [\text{Re}(dN^\beta) + k^\beta_\alpha A^\alpha] \wedge \alpha_\beta^{\text{tor}} + dA^\alpha \wedge \omega_\alpha^{\text{tor}} + \dots$$

**Massive RR U(1) gauge symmetries**

Electric charges: A-B particles

Magnetic charges: A-B strings

## 4. Mass mixing with RR photons

### Massless RR U(1)'s

$$H_2^+(\mathcal{M}_6, \mathbb{R})$$

Hodge duality:  $H_2^+(\mathcal{M}_6, \mathbb{R}) \simeq H_4^-(\mathcal{M}_6, \mathbb{R})$

Intersection number

Electric charges: D2 (4d particles)

Magnetic charges: D4 (4d monopoles)

$U(1)$  gauge symmetry

### Massive RR U(1)'s

$$\text{Tor } H_2^+(\mathcal{M}_6, \mathbb{Z})$$

UCT+Poinc.:  $\text{Tor } H_2^+(\mathcal{M}_6, \mathbb{Z}) \simeq \text{Tor } H_3^-(\mathcal{M}_6, \mathbb{Z})$

Linking number

Electric charges: D2 (4d A-B particles)

Magnetic charges: D4 (4d A-B strings)

$\mathbb{Z}_k$  gauge symmetry

$$H_2^+(\mathcal{M}_6, \mathbb{Z})$$

## 4. Mass mixing with RR photons

*Can D6-brane and RR U(1)'s mix through the Stuckelberg mechanism ??*

We have seen that a D4-brane wrapping a torsional 3-cycle

$$[\pi_b^-] = \sum_{\beta} c_b^{\beta} [\pi_3^{\text{tor},\beta}]$$

develops a coupling,

$$S_{4d} \supset \sum_{\beta} c_b^{\beta} \int_{\mathbb{R}^{1,3}} C_2^{\beta} \quad C_2^{\beta} \equiv \int_{\pi_3^{\text{tor},\beta}} C_5$$

Similarly, a D6-brane wrapping the same 3-cycle develops a Stuckelberg coupling in its worldvolume,

$$- \sum_{\beta} c_b^{\beta} \int_{\mathbb{R}^{1,3}} C_2^{\beta} \wedge F_2^b$$

It can also be seen from dim. reduction of the CS D6-brane action

Therefore, massive RR U(1)'s couple to the same complex structure axions than D6-branes do.

## 4. Mass mixing with RR photons

Massive RR U(1)'s therefore may mix with D6-brane U(1)'s.

$$Q^I = \sum_a c_a^I N_a Q^a$$
$$Q^\beta = \sum_\alpha k^\beta{}_\alpha Q_{RR}^\alpha + \sum_a c_a^\beta N_a Q^a$$

Each linear combination of D6-brane and torsional RR U(1) gauge symmetries has an element of  $H_3^-(\mathcal{M}_6, \mathbb{Z})$  associated to it. Massless combinations of U(1)'s are trivial elements in *integer* homology.

$$Q_0 = \sum_a n_a Q^a + \sum_\alpha \check{n}_\alpha Q_{RR}^\alpha \quad \text{massless}$$



$$\sum_a \frac{N_a n_a}{2} ([\pi_a] - [\pi_a^*]) + \sum_{\alpha, \gamma} \check{n}_\alpha k^\alpha{}_\gamma [\pi_3^{\text{tor}, \gamma}] = 0$$

Elements which are also trivial in de Rham do not mix with RR U(1)'s

# 5. Some phenomenological implications

Some examples: Type IIA orientifold of the Enriques CY

[Aspinwall '95]

	1						
	0	0					
0	11	0					
1	11	11	1				
	0	11	0				
	0	0					
	1						

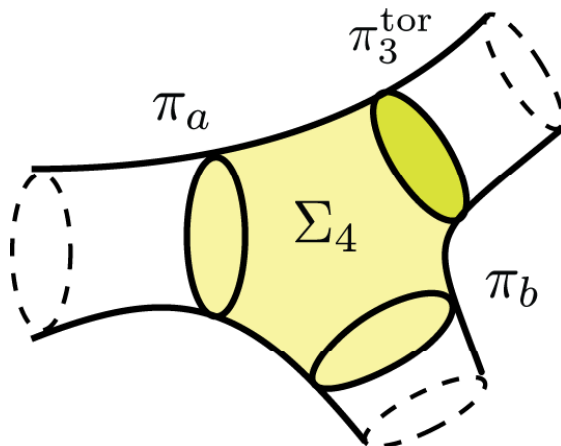
  

$H_0(\mathcal{M}_6)$	$H_1(\mathcal{M}_6)$	$H_2(\mathcal{M}_6)$	$H_3(\mathcal{M}_6)$	$H_4(\mathcal{M}_6)$	$H_5(\mathcal{M}_6)$	$H_6(\mathcal{M}_6)$
$\mathbb{Z}$	$(\mathbb{Z}_2)^3$	$(\mathbb{Z})^{11} \oplus \mathbb{Z}_2$	$(\mathbb{Z})^{24} \oplus \mathbb{Z}_2$	$(\mathbb{Z})^{11} \oplus (\mathbb{Z}_2)^3$	0	$\mathbb{Z}$

Freely-acting  $T^6 / (\mathbb{Z}_2 \times \mathbb{Z}_2)$

RR U(1)'s allow for new phenomenological scenarios:

- Two stacks of fractional D6-branes which differ by  $\pi_3^{\text{tor}}$



*Massless:*  $U(1)_Y \sim 2U(1)_a - 2U(1)_b + U(1)_{\text{RR}}$

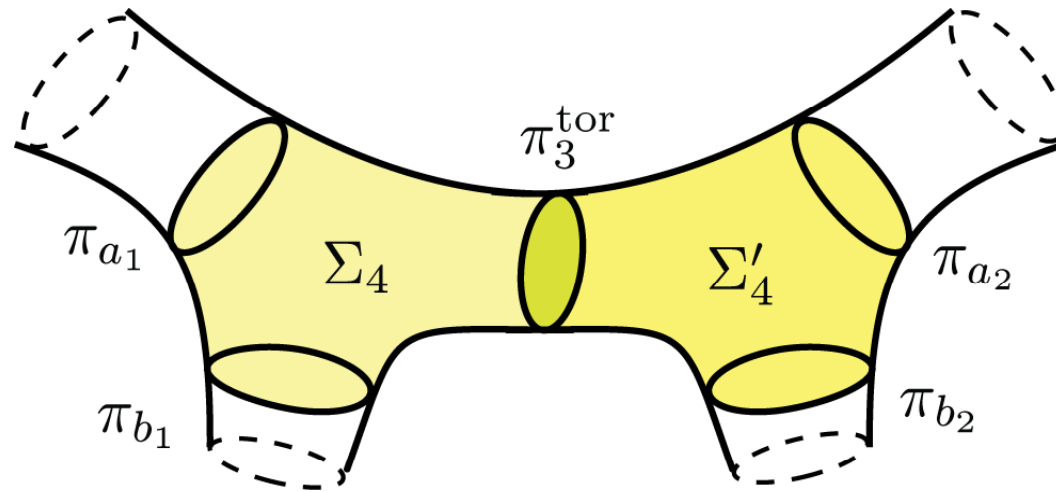
*Massive:*  $U(1)_{G_1} \sim U(1)_a + U(1)_b$

$U(1)_{G_2} \sim U(1)_a - U(1)_b - 4U(1)_{\text{RR}}$

$$f_{YG_2} = -\frac{4i}{27} \sqrt{\frac{10}{3}} (N^0 - T^{\hat{1}})$$

## 5. Some phenomenological implications

- Two mutually hidden brane sectors which communicate via RR photons



*Massless:*

$$U(1)_{Y_k} \sim 2U(1)_{a_k} - 2U(1)_{b_k} + U(1)_{\text{RR}}, \quad k = 1, 2$$

*Massive:*

$$U(1)_{G_k} \sim U(1)_{a_k} + U(1)_{b_k}$$

$$U(1)_{G_3} \sim U(1)_{a_1} - U(1)_{b_1} + U(1)_{a_2} - U(1)_{b_2} - 4U(1)_{\text{RR}}$$

$$f_{Y_1 Y_2} = -\frac{i}{80} (8T^{\hat{1}} - 9f_1 - 9f_2)$$



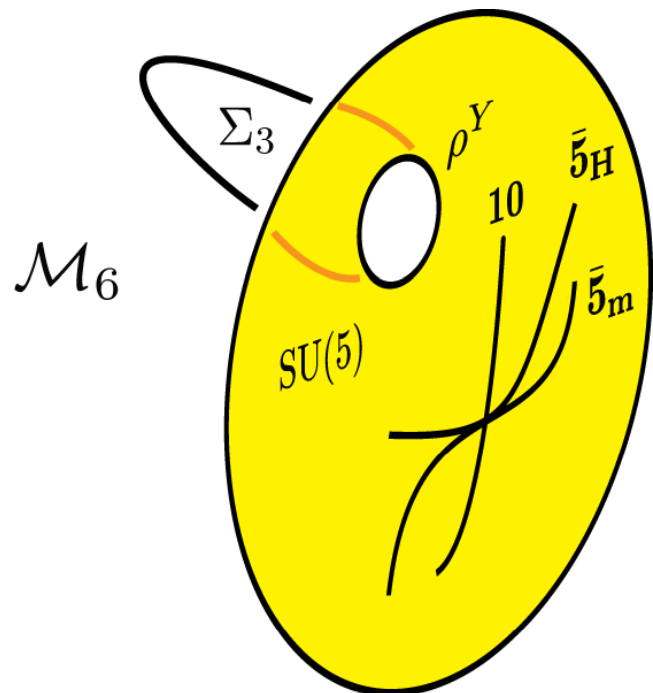
## 5. Some phenomenological implications

RR U(1)'s may also lead to new scenarios in the context of GUT models:

- Similar results for type IIB orientifolds with magnetized D7-branes (or their F-theory extension). RR photons arise from reduction of the RR 4-form on  $H_3^+(\mathcal{M}_6, \mathbb{Z})$

- Let us consider **SU(5) GUT models**

[Beasley, Heckman, Vafa '08]  
[Donagi, Wijnholt '08]



SU(5) 7-brane wrapping 4-cycle S, matter fields localized at intersections...

Hypercharge flux breaking

$$SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$$

$$\int_{\mathbb{R}^{1,3} \times S} C_4 \wedge F_Y \wedge \bar{F}_Y \rightarrow \int_{\mathbb{R}^{1,3}} C_2^Y \wedge F_Y$$

$$C_2^Y \equiv \int_S C_4 \wedge \bar{F}_Y = \int_{\rho^Y} C_4$$

## 5. Some phenomenological implications

- 2-cycle  $\rho^Y$  trivial in the  $CY_3$  in order  $U(1)_Y$  to remain massless.
- Thresholds ( $F^4$ ) lead to wrong ordering of fine structure constants at  $M_s$ :

$$\frac{1}{\alpha_3} < \frac{1}{\alpha_1} < \frac{1}{\alpha_2}$$

[Blumenhagen '08]

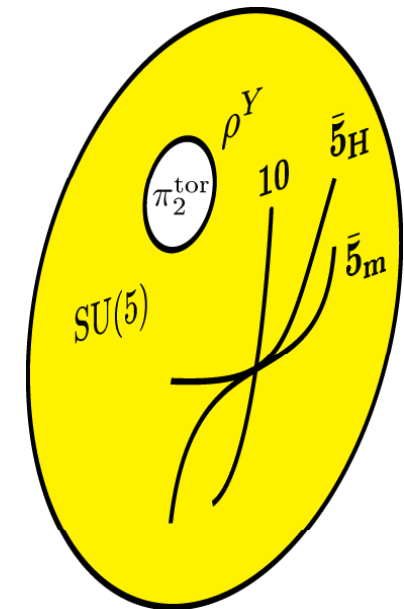
The above condition can be relaxed. We can take  $\rho^Y$  to be trivial in  $H_2^+(\mathcal{M}_6, \mathbb{R})$  but still non trivial in  $H_2^+(\mathcal{M}_6, \mathbb{Z})$

I.e,  $\rho^Y$  can be a torsional 2-cycle of the  $CY_3$ .



Mixing of the “hypercharge” with a  $U(1)_{RR}$

$$\mathcal{L} \supset -\frac{1}{2} \left( \text{Re}(dT) + k_{RR} A_{RR} + \frac{5k_Y}{3} A_Y \right)^2$$



## 5. Some phenomenological implications

Mass eigenstates:

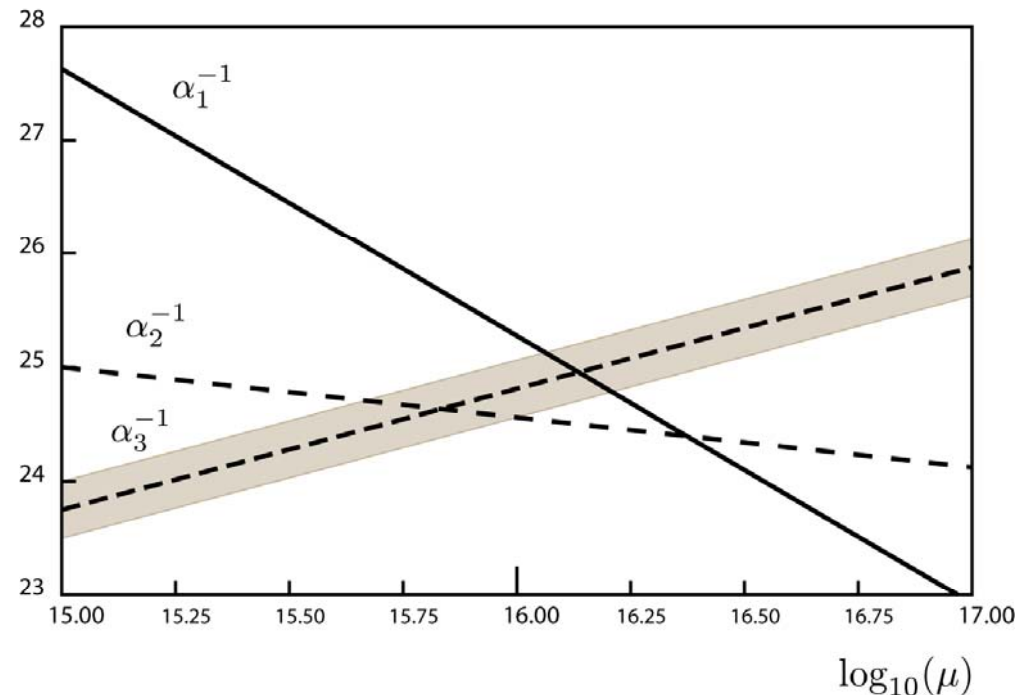
$$\begin{aligned}
 \text{Massless: } A_1 &= \cos(\theta) \tilde{A}_Y - \sin(\theta) \tilde{A}_{RR} \\
 \text{Massive: } A_X &= \sin(\theta) \tilde{A}_Y + \cos(\theta) \tilde{A}_{RR}
 \end{aligned}
 \quad \sin(\theta) \equiv \frac{g_Y k_Y}{\sqrt{g_{RR}^2 k_{RR}^2 + g_Y^2 k_Y^2}}$$

The inverse fine structure constant of the massless U(1) is

$$\frac{1}{\alpha_1} = \frac{3}{5\alpha_{SU(5)}} + \frac{k_Y^2}{k_{RR}^2 \alpha_{RR}}$$

Could explain the known few percent discrepancy in MSSM gauge coupling unification.

Similar to [Tatar, Watari '08]



## 6. The unified M-theory picture

M-theory provides a unifying picture for D-brane and RR U(1) gauge symmetries.

We consider **M-theory on a  $G_2$  manifold**  $\hat{\mathcal{M}}_7$  admitting at least one perturbative IIA CY<sub>3</sub> orientifold limit

$$\hat{\mathcal{M}}_7 \rightarrow (\mathcal{M}_6 \times S^1)/\hat{\sigma} \quad \hat{\sigma} = (\sigma, -1)$$

$b_2$  massless U(1)'s and  $b_3$  massless complex scalars:

$$A_3 = \text{Re}(M^I)\phi_I + A^\alpha \wedge \omega_\alpha \quad \Phi_3 = \text{Im}(M^I)\phi_I \quad \begin{array}{l} I = 1, \dots, b_3(\hat{\mathcal{M}}_7) \\ \alpha = 1, \dots, b_2(\hat{\mathcal{M}}_7) \end{array}$$

In the IIA perturbative limit they become the massless D6-brane and RR U(1)'s and the massless closed and open string moduli.

If  $\hat{\mathcal{M}}_7$  admits several IIA perturbative limits, **open / closed string dualities may exchange D6-brane and RR U(1)'s.**

[Kachru, McGreevy '01]

## 6. The unified M-theory picture

Gauge kinetic function described geometrically by the triple intersection numbers of  $\hat{\mathcal{M}}_7$

$$f_{\alpha\beta} = M^I \int_{\hat{\mathcal{M}}_7} \phi_I \wedge \omega_\alpha \wedge \omega_\beta$$

[Papadopoulos, Townsend '99]

**Massive U(1) gauge symmetries spontaneously broken to discrete gauge symmetries arise from**  $\text{Tor } H_2(\hat{\mathcal{M}}_7, \mathbb{Z}) \simeq \text{Tor } H_4(\hat{\mathcal{M}}_7, \mathbb{Z})$

M2-branes wrapping torsional 2-cycles  $\Rightarrow$  4d Aharonov-Bohm particles

M5-branes wrapping torsional 4-cycles  $\Rightarrow$  4d Aharonov-Bohm strings

$$\hat{k}_\alpha^\beta \phi_\beta^{\text{tor}} = d\omega_\alpha^{\text{tor}} \quad dA_3 = \left( \text{Re}(dM^\alpha) + \hat{k}^\alpha_\beta A^\beta \right) \wedge \phi_\alpha^{\text{tor}} + dA^\beta \wedge \omega_\beta^{\text{tor}}$$

In the IIA perturbative limit they become the massive D6-brane and RR U(1)'s.

Thus, in a general compactification massless U(1)'s and discrete gauge symmetries are both classified by  $H_2(\hat{\mathcal{M}}_7, \mathbb{Z})$

## 7. Concluding remarks

- We have considered the **interplay between open and closed string U(1) gauge symmetries.**
- RR U(1)'s can play a prominent role. **Mixing with the hypercharge** can occur either via direct kinetic mixing or via the mass terms induced by Stückelberg couplings. **Interesting phenomenological implications.**
- **We have provided a geometric description of mass mixing in terms of the torsional homology of the CY,** and developed the right tools to compute the mixing parameters in specific models.
- As a byproduct, we have provided a **stringy realization of discrete gauge symmetries and 4d A-B strings and particles in terms of the torsional homology.** In particular  $\text{Tor } H_2(\hat{\mathcal{M}}_7, \mathbb{Z})$  should contain the MSSM discrete symmetries of any semi-realistic model.